Why ruin theory should be of interest for insurance practitioners and risk managers nowadays

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Ruin theory concerns the study of stochastic processes that represent the time evolution of the surplus of a stylized non-life insurance company. The initial goal of early researchers of the field, Lundberg (1903) and Cramér (1930), was to determine the probability for the surplus to become negative. In those pioneer works, the authors show that the ruin probability $\psi(u)$ decreases exponentially fast to zero with initial reserve $u \geq 0$ in numerous cases when the net profit condition is satisfied: if the insurance company receives premium continuously at a deterministic rate $c >$ and pays for claims that are described by a compound Poisson process, for all $u \geq 0$ we have an upper bound for the ruin probability $\psi(u) \leq e^{-Ru}$, as well as information on the asymptotic behaviour, because $\psi(u) \sim Ce^{-Ru}$ as $u \to +\infty$, where $0 < C \leq 1$. This result is valid for light-tailed claim amounts, i.e. when the probability of very large claims decreases fast enough. This condition is satisfied in the particular case where claims amounts are bounded, which is often true in practice. Following the approach of Gerber (1974), it is possible to link the Cramr-Lundberg adjustment coefficient R with the risk aversion coefficient a. If one measures a random claim amount X thanks to indifference pricing method (which means that the insurer does not show any preference between not insuring the risk and bearing the risk after receiving premium π), with exponential utility function $u(x) = (1 - e^{-ax})/a$, the insurer would ask for premium[1]

$$\pi = \frac{1}{a} \ln \left(E \left(e^{aX} \right) \right) .$$

Gerber (1974) notes that if the insurer determines the premium following this principle, then the Cramér-Lundberg adjustement coefficient R is identical to the risk aversion parameter a. Conversely, if the insurer wants the ruin probability to decrease exponentially fast, she can use

[1]Denote by $E(Y)$ the mathematical expectation of an integrable random variable X, by $Var(X)$ its variance if X is square integrable. Denote $VaR_\beta(X)$ the Value-at-Risk (quantile) of level $\beta \in [0,1]$ of a general random variable X.

indifference pricing principle with exponential utility function. Note that in this dynamic vision, at first order, the insurer uses a pricing principle that looks like the variance principle $\pi \simeq E(X) + \frac{a}{2}Var(X)$. This is different from the static framework, which consists (like in Solvency II) in studying the probability that the net asset value of the company is negative in one year. If one computes the risk margin thanks to the cost of capital approach, this leads to a theoretical pricing as $E(X) + b\,(VaR_{99,5\%}(X) - E(X))$. This corresponds at first order to the standard error coefficient pricing principle $\pi \simeq E(X) + bq\sqrt{Var(X)}$, where b is a parameter that quantifies cost of capital, and q is a factor that links the standard error coefficient and the 99.5%-Value-at-Risk of X (approximately 3 for a Gaussian distribution, 4 or 5 for heavier tails). Ruin theory thus provides more sustainable valuation principle than the Value-at-Risk approach, because it takes into account liquidity constraints and penalizes large position sizes.

In risk management, insurance companies start to set risk limits: more precisely, they want to guarantee that the Solvency Capital Requirement (SCR) coverage ratio stays above a certain level with a large enough probability. Modeling the evolution of the SCR coverage ratio is of course delicate. Internal models (that study the one year change in net asset value) are already very complex and require large computation times. On the average term, insurers often merely study solvency in some adverse scenarios, without trying to affect probability to each of those scenarios. Ruin theory does not offer a precise, miraculous answer to this question, but it may provide interesting insight thanks to different situations for which the ruin probability is known explicitly or can be approximated. Note that the zero surplus level corresponds then to the minimum SCR coverage ratio level in that case. Finite-time ruin probabilities have been studied among others by Picard and Lefèvre (1997), and Ignatov et al. (2001). The probability of ruin at inventory dates has been studied by Rullière and Loisel (2003). Researchers in ruin theory currently work on models with credibility adjusted premium, with tax payments, with correlations and correlation crises between claim amounts, as well as the ability for the insurer to invest into risky assets or to transfer part of its risks. Less binary risk and profit indicators are also considered. For regularly varying claim size distributions (Pareto distribution for example), Embrechts and Veraverbeke (1982) have shown that the ruin probability decreases more slowly with u:

$$\psi(u) \sim K u^{-\alpha+1},$$

where $\alpha > 1$. In several models with a non diversifiable and no compensable risk driver, Albrecher et al. (2011) and Dutang et al. (2012) show that the ruin probability admits a positive limit as $u \to +\infty$:

$$\psi(u) - A \sim \frac{B}{u},$$

where $0 < A < 1$ and $B > 0$ are constant numbers. Here, ruin should be understood in a broader sense, economic ruin or switch to run-off mode before being completely ruined. This corresponds to the idea that capital is not always the answer and that the capacity to react fast is a key element of efficient risk management. The book by Asmussen and Albrecher (2010) contains most references of papers dealing with ruin theory.

Another classical problem of ruin theory is to determine optimal dividend strategies. In Switzerland, if ruin were not a problem, it would not be efficient to pay dividends, because they are taxed.

It would be better to let the stock price increase faster in the absence of dividends, because capital gains are not taxed. But as ruin may occur, the investor faces the problem of dividend optimization. De Finetti, who was also actuary at Generali, shew in a simplified model that the optimal dividend strategy consisted in paying dividends above some horizontal barrier (which of course increases ruin probability) and computed the optimal barrier level. Dubourdieu (1952) formalized several results on this issue and gave credit to De Finetti for the main ideas (see posterior paper by De Finetti (1957)). In a more general setting, optimal strategies might involve several bands instead of one single barrier. Since the works by Borch (1974) and by Gerber (see for example Gerber (1979)), this subject had been almost forgotten, but has been addressed by numerous papers in the recent years (see the survey by Avanzi (2009) on those issues and by Albrecher et Thonhauser (2009) on optimal control strategies).

This theory could be useful to address the problem of determining appropriate Solvency Capital Requirement coverage ratio target levels. In the new regulation framework Solvency II, in addition to the technical provisions (composed of best estimate of liabilities and of a risk margin), the insurer must have at least the so-called Solvency Capital Requirement (SCR). Most insurers have now to choose a target SCR level, usually comprised between 110% and 200%. Besides, they usually adopt a kind of dividend strategy that corresponds to a refraction strategy: if the SCR coverage ratio becomes higher than a threshold, then the insurer starts to pay part of the excess as dividends. If the SCR coverage ratio overshoots a certain level (250%, say), then all the excess is paid as dividends, which corresponds to reflection from a barrier. For Enterprise Risk Management purposes, it might be interesting to study the probability to become insolvent before 5 or 10 years in a steady regime to check whether the activity would be sustainable in a steady regime, in the absence of change of risk environment. With a first-order approximation, this corresponds to a finite-time ruin problem with a certain dividend strategy, where the ruin level is the 100% coverage ratio level (it is different from the economic ruin level where the net asset value of the company becomes negative). The dynamic balance sheet is illustrated in Figure 1 and the simplified ruin problem is illustrated in Figure 2.

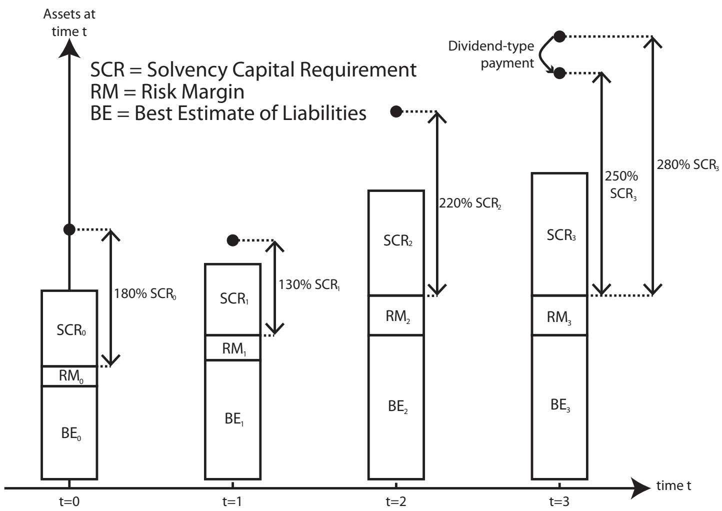

Figure 1: Evolution of economic balance sheet.

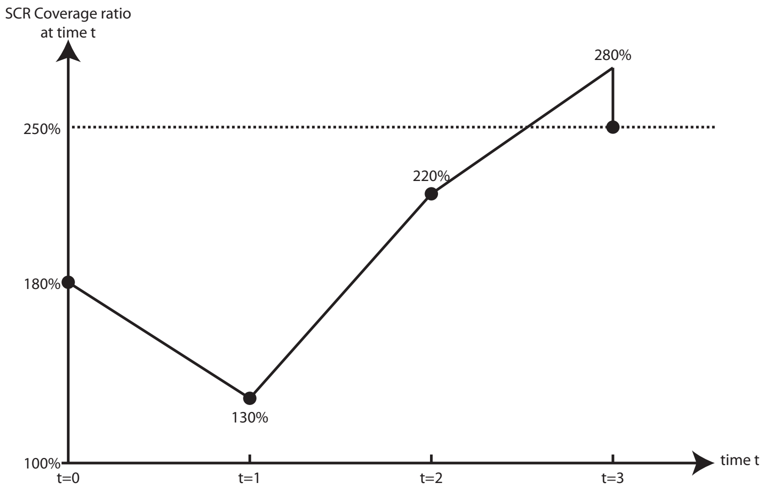

Figure 2: Evolution of SCR coverage ratio.

Nevertheless the simplified view is far from being perfect, because the insolvency threshold depends on the evolution of the assets and liabilities. Of course, the evolution of the economic balance sheet of a company is much more complicated than classical risk models. However, as multi-period risk models are often intractable on a 5-year time horizon in practice, it may be interesting to have benchmarks that come from ruin theory in mind while thinking about the risk appetite implementation.

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