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ABSTRACT

Stiffened structures are very common in engineering applications, for instance the wing of an airplane or the pressure hull of a submarine; both are subject to an excitation due to their movement in the fluid around, namely the Turbulent Boundary Layer (TBL) excitation. For improving the knowledge about the interaction between stiffened structures and TBL, this paper deals with the modelling of infinite periodically rib-stiffened plates excited by TBL. The mathematical formulation of the problem is well established in the literature. The originality of the present work relies on the use of a reciprocity technique for evaluating the response of the plate to convected harmonic pressure waves. The present approach allows then to compute the vibro-acoustic response of the plate due to the TBL from the response of the stiffened plate to a point mechanical force and to an acoustic monopole. Results in the physical space and the wavenumber space show the filter effect of the stiffened plate and the role of the supersonic domain on the far field radiation. They allow us to interpret the influence of the periodicity of the stiffeners on the vibration and the far-field noise for plates excited by TBL.

1 INTRODUCTION

Stiffened structures excited by turbulent boundary layer (TBL) are very common for practical applications (airplane, ship, train). In order to reduce the noise radiated from these structures, it is then important to understand how a stiffened structure reacts to the TBL excitation. For improving the knowledge in this domain, the current paper deals with modelling of infinite periodically rib-stiffened plates excited by TBL. A focus is made on the influence of the stiffeners and their periodicity on the vibration of the plate and its radiated pressure in the fluid.

The use of the wavenumber-frequency formalism for evaluating the response of a panel randomly excited in time and space is well established in the litterature [1] [2] [3]. This formalism, which will be briefly described in section 2, permits to estimate the random response from the knowledge of the wavenumber-frequency spectrum of the excited pressure and from the responses of a panel for a set of harmonic plane waves. The presented study concerning the stiffened plate excited by TBL is based of this formalism. The originality of the works relies on the use of the

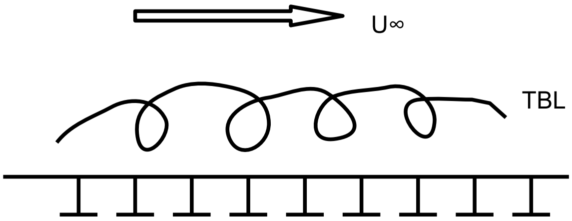


Figure 1: Panel excited by a TBL with flow speed U_∞

reciprocity technique [4] to calculate the plate response due to the harmonic plane waves. Indeed, the point to point reciprocity allows us to establish a reciprocity relation between the point response of the plate excited by a harmonic plane waves and the response in the wavenumber space of the plate excited by a point source. As for a periodically stiffened plate immerged in a fluid, this latter response can be calculated analytically [5], the calculation of the plate response to TBL excitation can be obtained easily. The numerical calculation time is relatively short and this approach allows us to analyse the filtering effect of the excitation by the stiffened plate as it will be discussed on one example.

2 ESTIMATION OF THE SPACE FREQUENCY SPECTRUM OF THE RESPONSE OF THE PLATE

2.1 Wavenumber-frequency formulation

Let us consider a baffle panel excited by a fully developed TBL as shown on figure 1.
One supposes that the wavenumber-frequency spectrum of the wall pressure is known.
The displacement of the plate on point $\mathbf{x}$ due to wall pressure p_b can be expressed as the convolution product

$$w(\mathbf{x},t) = \int_{S_p} \int_{-\infty}^{+\infty} h_w(\mathbf{x},\tilde{\mathbf{x}},t-\tau) p_b(\tilde{\mathbf{x}},\tau) \mathrm{d}\tau \mathrm{d}\tilde{\mathbf{x}}, \tag{1}$$

where $h_w(\mathbf{x},\tilde{\mathbf{x}},\tau)$ is the impulse response in displacement on point $\mathbf{x}$.
The inter-correlation function $R_{ww}(\mathbf{x},\mathbf{x}',t)$, which can be defined as

$$R_{ww}(\mathbf{x},\mathbf{x}',t) = E(w(\mathbf{x},\tau)w(\mathbf{x}',t+\tau)), \tag{2}$$

is obtained when (1) is replaced in (2) and its expression is

$$R_{ww}(\mathbf{x},\mathbf{x}',t) = \iint_{S_p} \iint_{-\infty}^{+\infty} h_w^*(\mathbf{x},\tilde{\mathbf{x}},t-\tau) R_{p_b p_b}(\tilde{\mathbf{x}},\tilde{\mathbf{x}}',\tau-\tilde{\tau}) \\ h_w(\mathbf{x}',\tilde{\mathbf{x}}',t-\tilde{\tau}) \mathrm{d}\tau \mathrm{d}\tilde{\tau} \mathrm{d}\tilde{\mathbf{x}} \mathrm{d}\tilde{\mathbf{x}}'. \tag{3}$$

The space-frequency spectrum $S_{ww}(\mathbf{x},\mathbf{x}',\omega)$ is the obtained taking the temporal Fourier transform of the inter-correlation function and can be written as

$$S_{ww}(\mathbf{x},\mathbf{x}',\omega) = \iint_{S_p} H_w^*(\mathbf{x},\tilde{\mathbf{x}},\omega) S_{pp}(\tilde{\mathbf{x}},\tilde{\mathbf{x}}',\omega) H_w(\mathbf{x}',\tilde{\mathbf{x}}',\omega) \mathrm{d}\tilde{\mathbf{x}} \mathrm{d}\tilde{\mathbf{x}}' \tag{4}$$

where $H_w(\mathbf{x},\tilde{\mathbf{x}},\omega)$ is the transfer function in displacement of the structure observed on point $\mathbf{x}$, driven by a point force on point $\tilde{\mathbf{x}}$, and $S_{pp}(\tilde{\mathbf{x}},\tilde{\mathbf{x}}',\omega)$ is the space-frequency spectrum of the wall pressure.
Defining $\phi_{pp}(\mathbf{k},\omega)$ as the space Fourier transform of $S_{pp}(\tilde{\mathbf{x}},\tilde{\mathbf{x}}',\omega)$, that is

$$S_{pp}(\tilde{\mathbf{x}},\tilde{\mathbf{x}}',\omega) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} \phi_{pp}(\mathbf{k},\omega) e^{-j\mathbf{k}(\tilde{\mathbf{x}}'-\tilde{\mathbf{x}})} \mathrm{d}^2\mathbf{k} \tag{5}$$

one can write

$$S_{ww}(\mathbf{x},\mathbf{x}',\omega) = \frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} \phi_{pp}(\mathbf{k},\omega) \iint_{S_p} H_w^*(\mathbf{x},\tilde{\mathbf{x}},\omega)e^{j\mathbf{k}\tilde{\mathbf{x}}}\mathrm{d}\tilde{\mathbf{x}} \times \iint_{S_p} H_w(\mathbf{x}',\tilde{\mathbf{x}}',\omega)e^{-j\mathbf{k}\tilde{\mathbf{x}}'}\mathrm{d}\tilde{\mathbf{x}}'\mathrm{d}^2\mathbf{k}. \quad (6)$$

In this expression, the space Fourier transforms of $H_w^*(\mathbf{x},\tilde{\mathbf{x}},\omega)$ and $H_w(\mathbf{x}',\tilde{\mathbf{x}'},\omega)$ appear :

$$\tilde{H}_w^\omega(\mathbf{x},\mathbf{k}) = \iint_{S_p} H_w(\mathbf{x},\tilde{\mathbf{x}},\omega)e^{j\mathbf{k}\tilde{\mathbf{x}}}\mathrm{d}\tilde{\mathbf{x}}. \quad (7)$$

$\tilde{H}_w^\omega(\mathbf{x},\mathbf{k})$ is here the frequency response observed on point $\mathbf{x}$ of the plate excited by a plane wave of wavevector $\mathbf{k}$. Indeed it can be interpreted as the sum of the responses on point $\mathbf{x}$ of the plate due to excitations of amplitude $e^{j\mathbf{k}\tilde{\mathbf{x}}}$ on points $\tilde{\mathbf{x}}$.
One can write finally the space-frequency spectrum of displacement, depending on the transfer function $\tilde{H}_w^\omega(\mathbf{x},\mathbf{k})$ and wall-pressure spectrum $\phi_{pp}(\mathbf{k},\omega)$:

$$S_{ww}(\mathbf{x},\mathbf{x}',\omega) = \frac{1}{4\pi^2}\iint_{-\infty}^{+\infty} \tilde{H}_w^{\omega*}(\mathbf{x},\mathbf{k})\phi_{pp}(\mathbf{k},\omega)\tilde{H}_w^\omega(\mathbf{x}',\mathbf{k})\mathrm{d}^2\mathbf{k} \quad (8)$$

where $\mathbf{k} = [k_x, k_y]$, k_x the wavenumber in the flow direction and k_y the wavenumber in the transversal direction. If $\mathbf{x}$ and $\tilde{\mathbf{x}}$ are the same point, the spectral power density of the displacement at point $\mathbf{x}$ is

$$S_{ww}(\mathbf{x},\omega) = \frac{1}{4\pi^2}\iint_{-\infty}^{+\infty} |\tilde{H}_w^\omega(\mathbf{x},\mathbf{k})|^2\phi_{pp}(\mathbf{k},\omega)\mathrm{d}^2\mathbf{k}. \quad (9)$$

With the same approach the spectral power density of the pressure at point z_0 in the fluid is given by:

$$S_p(\mathbf{x},\omega) = \frac{1}{4\pi^2}\iint_{-\infty}^{+\infty} |\tilde{H}_p^\omega(z_0,\mathbf{k})|^2\phi_{pp}(\mathbf{k},\omega)\mathrm{d}^2\mathbf{k} \quad (10)$$

2.2 Use of the reciprocity principle

Expressions (9) and (10) allow us to estimate the vibrations of the plate and its radiated pressure in the fluid from the knowledge of the wall pressure spectrum, Φ_{pp}, and the transfer functions, $\tilde{H}_w$ and $\tilde{H}_p$. These transfer functions can be obtained by calculating the response of the panel excited by a harmonic plane wave of wavevector $\mathbf{k}$. As the number of wavevectors to be considered can be large, the computing time can become prohibitive. In this paper, to overcome this obstacle, one proposes to use the reciprocity principle [4]. This principle indicates that the ratio of the normal displacement of the plate at point x over the applied normal force at point $\mathbf{x}$ is equal to the ratio of normal displacement of the plate at point $\mathbf{x}$ over the applied normal force at point $\mathbf{x}$. With our notation, one can write :

$$H_w(\mathbf{x},\tilde{\mathbf{x}},\omega) = H_w(\tilde{\mathbf{x}},\mathbf{x},\omega). \quad (11)$$

Introducing this expression in (7), one has:

$$\tilde{H}_w^\omega(\mathbf{x},\mathbf{k}) = \iint_{S_p} H_w(\tilde{\mathbf{x}},\mathbf{x},\omega)e^{j\mathbf{k}\tilde{\mathbf{x}}}\mathrm{d}\tilde{\mathbf{x}} \quad (12)$$

which says that $\tilde{H}_w^\omega(\mathbf{x},\mathbf{k})$ is the space Fourier transform of the plate excited on point $\mathbf{x}$. Consequently, the power spectrum density of the displacement of the plate at point $\mathbf{x}$ excited by the TBL can be calculated with equation (9) from the knowledge of the response of the plate excited by a normal force at point $\mathbf{x}$. That is to say that the plate response at a given point due to TBL can be estimated from the vibratory field of the plate excited by a point force at the same point. The principle of reciprocity can be applied for an observation point in the fluid domain then, the transfer function $\tilde{H}_p^\omega(\mathbf{z},\mathbf{k})$ can be interpreted as the space Fourier transform of the normal displacement of the plate for the volume acceleration of an acoustic source located at point $\mathbf{z}$. The power spectrum density of the radiated pressure by the plate at point $\mathbf{z}$ when the plate is excited by the TBL can be calculated with equation (10) from the knowledge of the response of the plate excited by an acoustic source located at point $\mathbf{z}$. This view, using the reciprocity principle, allows us to save computing time compared to the direct approach described at the beginning of this section. Indeed, for the system considered in this paper (i.e. periodically stiffened plate immerged in water), the displacement of the plate expressed in the wavenumber domain can be obtained analytically as it will be remember in the next section. The computing time is then very short.

3 WAVENUMBER SPACE RESPONSE OF THE RIBBED PLATE IMMERGED IN A FLUID

Let us consider an infinite thin plate of thickness h lying in the plane $z = 0$ and with identical beam-like stiffeners. The stiffeners are assumed to be straight and uniform. They are attached along the lines $x = nd$, n being an integer and d, the distance between two stiffeners. The connection between the plate and the stiffeners is assumed to be rigid. The materials are linearly elastic, homogeneous and isotropic. The plate is loaded by an acoustic fluid on one side and is excited by a harmonic excitation.

Two sorts of excitation are considered :

- A normal point force at $(x_0, 0)$ on the plate,
- A monopole source at $(0, 0, z_0)$ in the acoustic domain.

This problem can be solved in the wavenumber space. These developments have already been treated by Rumerman [6], Mace [7] and Maxit [5] for a point force excitation. It is extended here for a monopole source, the process being the same.

The developments in the wavenumber space lead to an analytical expression of the plate displacement :

$$\begin{aligned}\tilde{W}(k_x,k_y) = \frac{1}{Z(k_x,k_y)}[\Lambda \\ - \frac{Z_p(k_y)}{1+S_0Z_p(k_y)}[T_0 - \frac{S_1A_p(k_y)(T_1(1+S_0Z_p(k_y)) - S_1Z_p(k_y)T_0)}{((1+S_0Z_p(k_y))(1+S_2A_p(K_y)) - S_1^2Z_p(k_y)A_p(k_y))}] \\ - \frac{k_xA_p(T_1(1+S_0Z_p(k_y)) - S_1Z_p(k_y)T_0)}{((1+S_0Z_p(k_y))(1+S_2A_p(K_y)) - S_1^2Z_p(k_y)A_p(k_y))}]\end{aligned} \tag{13}$$

where Z, Z_p and A_p represent the flexural impedance of the plate and the stiffeners, respectively, S_p and T_p are such as :

$$\begin{cases} S_p = \displaystyle\sum_{n\in Z} \frac{(k_x + \frac{2\pi n}{d})^p}{Z(k_x + \frac{2\pi n}{d}, k_y)} \\ T_p = \displaystyle\sum_{n\in Z} \frac{(k_x + \frac{2\pi n}{d})^p\Lambda}{Z(k_x + \frac{2\pi n}{d}, k_y)} \end{cases} \tag{14}$$

and Λ is given by :

$$\begin{cases} \Lambda(k_x,k_y) = e^{-jk_xx_0} & \text{for a unit point force excitation at } (x_0,0), \\ \Lambda(k_x,k_y) = j\omega\rho_0\frac{e^{-k_zz_0}}{-k_z} & \text{for a monopole source at } (0,0,z_0) \text{ of strength } 1\mathrm{m}^3/\mathrm{s}. \end{cases} \tag{15}$$

The quantity $\tilde{H}_w^\omega(\mathbf{x},\mathbf{k})$ mentioned in the previous section with $\mathbf{k} = (k_x, k_y)$, corresponds to $\tilde{W}(k_x, k_y)$ obtained with Eq. (13) for the case of a point force at $\mathbf{x} = (x_0, 0)$ and the angular frequency ω . The same $\tilde{H}_p^\omega(\mathbf{z},\mathbf{k})$ corresponds to $j\omega\tilde{W}(k_x, k_y)$ for the case of a monopole source at $\mathbf{z} = (0, 0, z_0)$ and the angular frequency ω.

4 ANALYSIS OF RESULTS ON A TEST CASE

Some numerical examples will be now given to illustrate some of the features of the present approach. A naval-like structure is considered. This test case is composed of a steel plate, $50\mathrm{mm}$ thick, and steel stiffeners with a T cross section ($0.15\mathrm{m}$ / $0.08\mathrm{m}$ $\times$ $0.15\mathrm{m}$ / $0.08\mathrm{m}$). The damping loss factor is 0.02 for the plate and the stiffeners. The plate is loaded by the water and is periodically stiffened by the beam-like stiffeners. Two values of the stiffener spacing will be considered for the numerical simulation: $d = 1\mathrm{m}$ and $d = 1.35\mathrm{m}$. One considers a flow speed of $12\mathrm{m/s}$ which is supposed to induce a homogeneous TBL on the plate. The parameters of the TBL are: $0.047\mathrm{m}$ for the thickness and $9.6\mathrm{m}$ for the convection velocity. The interspectrum of the turbulent boundary layer is modelled using the 1987 model of Chase [8].

For the figures presented in this paper, the levels of displacement or pressure are expressed in dB using a reference of a unit in the International System of Units (SI).

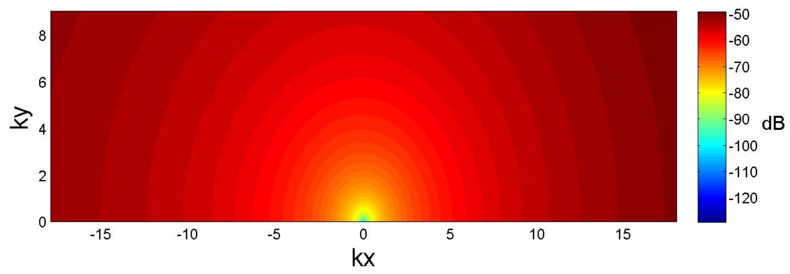
4.1 TBL wall pressure and plate displacement

In order to illustrate the two terms intervening in Eq. (9), one shows on figure 2, the wavenumber spectrums of the TBL wall pressure and the displacements of the stiffened plate (obtained with Eq. (13)).

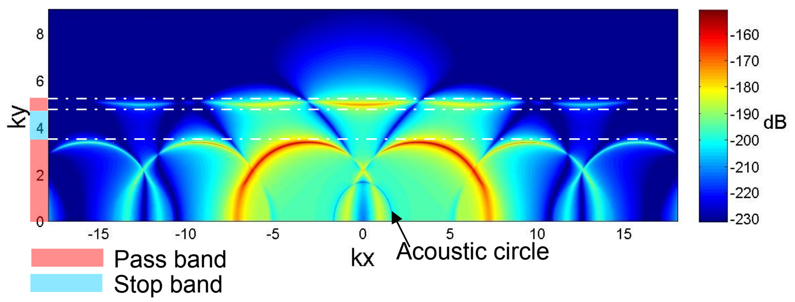
As the 1987 Chase model is considered, the TBL wall pressure shown on figure 2a is vanishing at zero wavenumber. As the wavenumbers considered on this figure are well below the convection wavenumber, the spectrum for these low-wavenumbers is thus proportional to the square of the wavenumber. Outside the zero wavenumber region, the pressure level (in dB) vary relatively slowly.

As it has been developed in section 2.2 with the reciprocity principle, one remembers that the term $H_w^\omega(\mathbf{x},\mathbf{k})$ in Eq. (9) corresponds to the space Fourier transform of the plate, $\tilde{W}(k_x, k_y)$ for a force excitation at $\mathbf{x}$. One notices on the spectrum of the plate displacement, on figure 2b:

- the acoustic circle in the bottom-center. It is known that only the inside of this acoustic circle will radiate in the far field;
- the regions of high displacement amplitude (in red). They will have the greatest contribution in the integral of Eq. (9). These regions are approximately contained in a circle having the flexural natural wavenumber of the plate as radius (around $6\mathrm{m}^{-1}$). They correspond to propagating Bloch-Flocquet waves which are the results of the interaction between the flexural waves of the plate and the flexural/torsional waves of the stiffeners (which have a periodical spacing). At a given frequency, for a considered wavenumber k_y (i.e. considering a one-dimensional system), the waves are either propagating or evanescent. The set of k_y wavenumber having propagating waves is called Pass band where as the one having evanescent waves is called Stop band. These bands have been symbolised with two colors on k_y-axis of figure 2b. For this frequency, one has two pass bands and one stop bands. These bands depend on the considered frequency. As we will show later, these bands and their location in the wavenumber space have an influence on the radiated pressure by the plate excited by TBL.



(a)



(b)

Figure 2: (2a),Wavenumber spectrum of the TBL wall pressure $\Phi_{pp}(k_x, k_y)$ at 400 Hz (Chase model). (2b), Wavenumber spectrum of the plate displacement $\tilde{W}(k_x, k_y)$ at 400 Hz for a force excitation on a rib. Stiffener spacing: $1\mathrm{m}$.

4.2 Vibroacoustic response of the plate excited by TBL

The response of the stiffened plate excited by TBL is obtained from Eq. (9) for the displacement and (10) for the radiated pressure. The integral is evaluated numerically by sampling and truncating the wavenumber space with appropriate criterions. The computations are achieved on a standard PC computer with MATLAB and take a handful of seconds for one frequency. The frequency spectrums of the plate displacement at $(0, 0)$ and the radiated pressure at $(0, 0, 100\mathrm{m})$ are proposed on figures 3 and 4, respectively. Three cases are considered on these figures: the unstiffened plate and two stiffened plate with different stiffener spacings: $1\mathrm{m}$ and $1.35\mathrm{m}$. One notices, as it can be expected, that the vibratory levels for the stiffened plates are lower than those of the unstiffened plate. Moreover, the vibratory response of the plate is smooth in frequency for the three cases. On the contrary, the radiated pressures by the stiffened plates are significantly greater than those of the unstiffened plate and the frequency responses of the stiffened plate have strong variations in frequency: the radiated pressure can be 15 dB higher in some frequencies bands than other ones. This result can be surprising at first sight if it is compared to the vibration level. However, by studying the integrand of Eq. (9) and (10), one can give some explanations. The values of this integrand are shown on figure 5, for two frequencies for the plate with $d = 1\mathrm{m}$: $400\mathrm{Hz}$ and $600\mathrm{Hz}$. The radiated pressures at $(0, 0, 100)$ are respectively -31 dB and -44 dB, respectively for these two frequencies. Figure 5a is fairly similar to figure 2b especially because the TBL spectrum (i.e. figure 2a) does not vary much in the wavenumber domain of interest, at the exception of the low wavenumbers close to zero. This observation could be made for the others cases presented in this paper. On figure 5b and 5d, corresponding to the observation point in the far field, one observes that the values of the integrand are significant, only for wavenumbers inside the acoustic circle or inside periodic copies of the acoustic circle. The periodicity of the copies is $\frac{2p}{d}$. The acoustic circle results of the acoustic filtering of the fluid domain and the periodic copies results of the stiffener periodicity. At $400\mathrm{Hz}$, a pass band (as defined previously) is present in the k_y acoustic domain (i.e. wavenumbers domain corresponding to $k_y < k_0$). It results high levels in the periodic copies (see

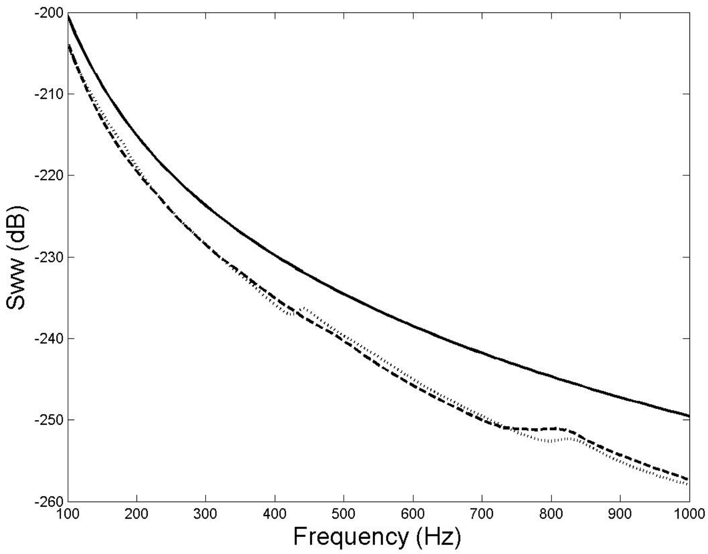


Figure 3: Frequency spectrum of the plate displacement at (0,0) for a TBL excitation. without stiffeners (solid line); stiffener spacing: 1m (dash line); stiffener spacing: 1.35 m (dotted line)

figure 5b), and lead to a high radiated noise. At contrary, at $600\mathrm{Hz}$, a stop band is present in the ky acoustic domain. The levels in the periodic copies are lower than at $400\mathrm{Hz}$. This explains the difference of radiated noise between the two frequencies. Moreover, as the effect of the acoustic circle and its periodic copies intervenes only for observation points in the far field, it explains why these frequencies variations are not observed on the vibratory response. For a stiffener spacing of $1.35\mathrm{m}$, one observes that the number of frequency band showing high radiated noise is greater than the one for $1\mathrm{m}$ spacing. It is due to the increasing number of pass band when the spacing increases.

5 CONCLUSION

This study has proposed an original way based on the reciprocity principle to compute the vibro-acoustic response of a periodically stiffened plate under a turbulent layer excitation. The method is in pratice easily implemented as it consists in an integration of a quantity over the wavenumber domain; the harder part consists in calculating the wavenumber response of the plate to elementary excitations (i.e. force, monopole).

Analysis of results on naval test cases shows the difference in behavior of the vibratory response and the far field pressure response for TBL excitation : while adding stiffeners reduces the vibration, it enhances the noise in the far field. Moreover, the effect of the pass band and stop band has also been highlighted. Analytical expression has been obtained for periodically stiffened plate. This approach can easily be applied to other panel systems. In this case, the wavenumber responses to elementary excitations can be estimated by discrete Fourier transforms

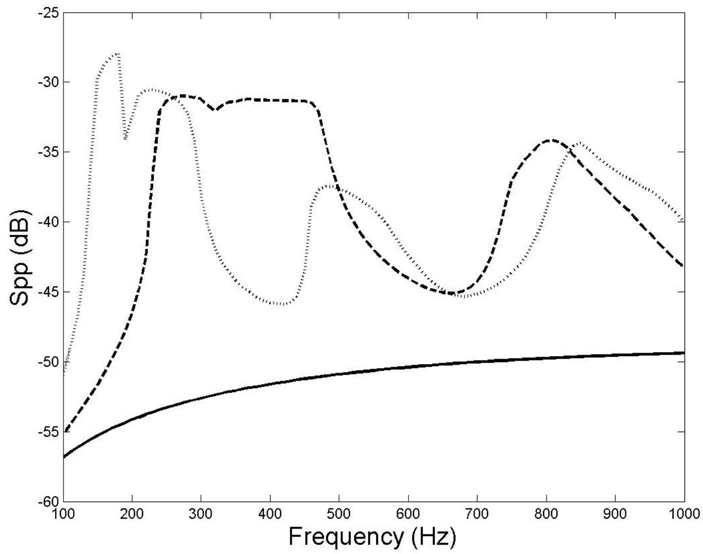


Figure 4: Frequency spectrum of the radiated pressure at (0,0,100m) for a TBL excitation. without stiffeners (solid line); stiffener spacing: 1m (dash line); stiffener spacing: 1.35 m (dotted line)

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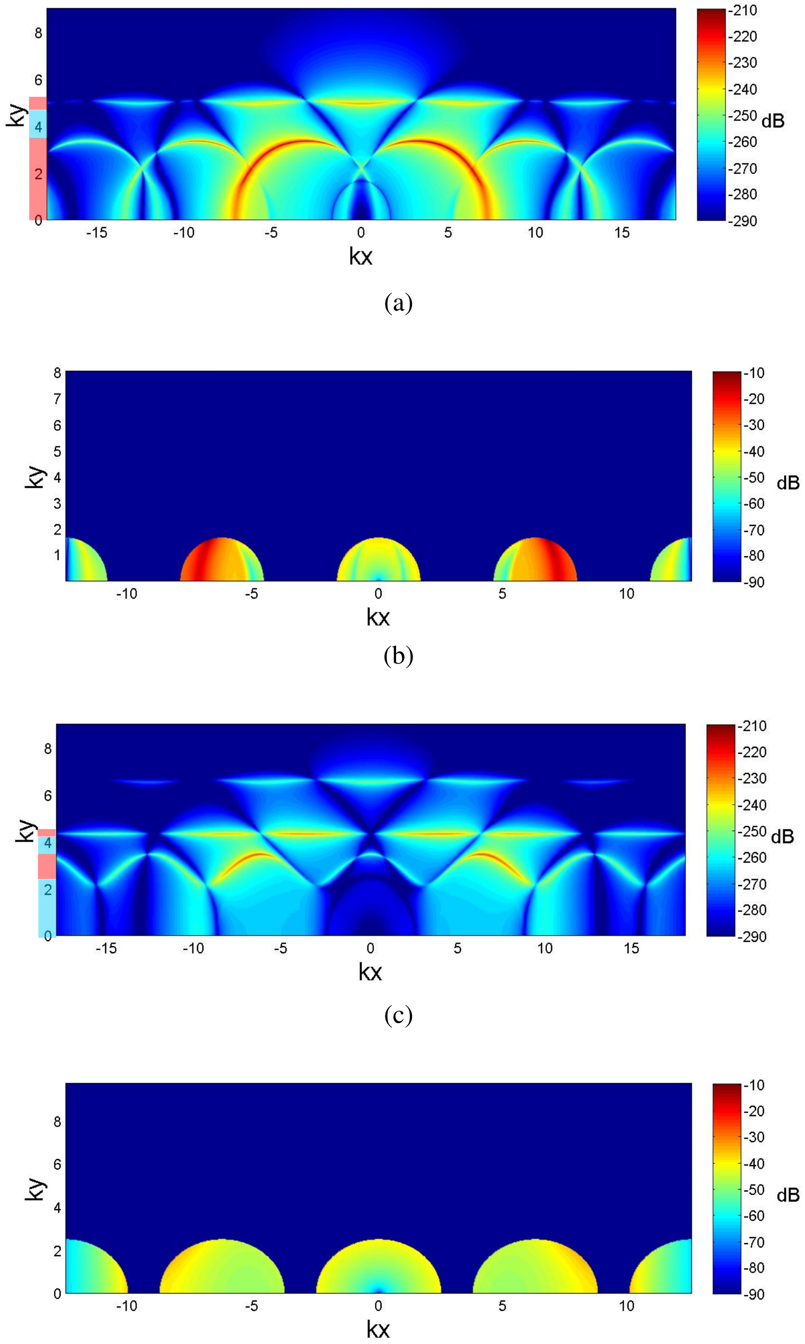

Figure 5: Values of the integrand of Eq. (9), observation point at (0,0) on the plate: (5a), (5c); of Eq. (2.10) , observation point at (0,0,100) in the acoustic domain: (5b), (5d). Two frequencies: (5a), (5b), 400 Hz; (5c), (5d), 600 Hz. Stiffener spacing: $d = 1\mathrm{m}$.