Production planning for a ramp-up process with learning in production and growth in demand

Christoph H. Glock, Mohamad Y. Jaber, Saeed Zolfaghari

Production planning for a ramp-up process with learning in production and growth in demand

Journal:	*International Journal of Production Research*
Manuscript ID:	TPRS-2011-IJPR-0212.R1
Manuscript Type:	Original Manuscript
Date Submitted by the Author:	23-Jun-2011
Complete List of Authors:	Glock, Christoph; University of Wuerzburg, Chair of Business Management and Industrial Management Jaber, Mohamad; Ryerson University, Department of Mechanical and Industrial Engineering Zolfaghari, Saeed; Ryerson University, Department of Mechanical and Industrial Engineering
Keywords:	LOT SIZING, LOT STREAMING, PRODUCTION PLANNING
Keywords (user):	learning, production ramp-up

Production planning for a ramp-up process with learning in production and growth in demand

Christoph H. Glock[a1], Mohamad Y. Jaber[b], Saeed S. Zolfaghari[b]

[a]Chair of Business Management and Industrial Management, University of Würzburg, Sanderring 2, 97070 Würzburg, Germany

[b]Department of Mechanical and Industrial Engineering, Ryerson University, 350 Victoria Street, Toronto, Ontario, M5B 2K3 Canada

Abstract

This paper presents a production-planning model for a manufacturing process that undergoes a ramp-up period with learning in production and growth in demand. The labour production and demand functions assumed in this paper are validated using available empirical data. A mathematical programming model is developed with numerical examples presented. The results of the paper indicate that the total costs of production can be minimised if the facility produces without interruption during the ramp-up phase and if the production and demand rates are synchronised as much as possible. The latter can be achieved by producing with the lowest possible production rate and by frequently re-structuring the workforce assigned to the production line.

Keywords: Production planning, learning in production, growth in demand, labour requirement, worker assignment, production ramp-up

---

[1] Corresponding author

1. Introduction

These days, customers are continually demanding products of higher quality and functionality that more go well with their needs. Examples of the rapid rate of innovation exist in the high technology, consumer electronics, and personal computer industries. To be responsive to changes in customers' consumption behaviours, firms have to be continually innovative in designing their products and processes. Being innovative may require a manufacturing facility to either add a new production line or to implement new technology. These actions subject a manufacturing facility to undergo a ramp-up phase, which is characterised by increases in output and product quality and a reduction in unit production cost (Almgren, 1999a; Matta et al., 2007; Winker and Slamanig, 2008; Fjällström et al., 2009).

Ramp-up phases in production processes are the result of learning, which could be reflected in a better utilisation of added equipment, in continuous improvement in product quality, and reduction in labour requirements. Furthermore, these processes could also experience the diffusion of a product into the market place (Naim, 1993; Almgren, 1999b; Cantemessa and Valentini, 2000), where the demand for this product experiences a growth, or learning effect (Jørgensen et al., 1999).

The ramp-up phase is critical for the successful introduction of a product to a market. Terwiesch and Bohn (2001) noted that customers are often willing to pay a premium price during the ramp-up phase, wherefore effectively managing production ramp-up may lead to higher profit margins and an advantage over competitors. Other authors have argued that the ramp-up phase may constitute a significant fraction of the total life cycle of a product (see, e.g., House and Price, 1991; Matta et al., 2007; Gross and Renner, 2010), which renders it an essential component of the sales period. Yelle (1980) earlier stated that being an inventor is not enough to assure success, and further added that lower costs, achieved through careful planning, are needed to maximise market success.

A closer look at the literature reveals that researchers have both concentrated on identifying important characteristics of ramp-up processes as well as on developing decision models that help production planners to effectively control the ramp-up process. Baloff (1970) and Almgren (1999a), for example, studied factors that influence the success of a ramp-up process and concluded that work methods, work pace, process disturbances and product conformance are the most critical factors in managing the ramp-up process. In a follow-up study, Almgren (2000) analysed disturbances that affect performance during the ramp-up period and identified four areas that are critical for volume and quality improvement. Other authors, such as Cohen et al. (1996) and Bayus (1997) studied the trade-off between overall product quality and time-to-market, while Nembhard and Birge (1998), Xu and Albin (2002) and Matta et al. (2007) focused on the reconfiguration of production systems and process adjustments during the ramp-up phase. Learning in production in the context of a ramp-up process has explicitly been considered by Ngwenyama et al. (2007) and Plaza et al. (2010), who studied the timing of Software upgrades and implementations when the performance of the users is subject to learning, and Terwiesch and Bohn (2001) and Terwiesch and Xu (2004), who focused on the trade-off between investments in learning and capacity utilisation and investments in learning and process improvements, respectively, in the ramp-up period.

What has received a relatively little attention in the literature is that demand may also grow during the ramp-up phase. After a product has been introduced to the market, consumption experiences, such as habit formation, word-of-mouth or carry-over-effects, may result in increases in demand (see, e.g., Jørgensen et al., 1999). If the production rate of an individual or a production facility increases as well with the production output, capacity and demand have to be matched carefully to avoid situations like excess inventory or stock-out periods. While the effect of learning on inventory policies has been investigated in the past (see Jaber and

Bonney (1999, 2011) and Jaber (2006) for reviews of related literature), interdependencies between learning effects in production and demand have not been studied before.

Consequently, this paper extends earlier research by investigating the lot size problem in conjunction with learning in production, as a form of technological change, and growth in demand. The focus is on the ramp-up period of a production process, since some researchers advocated that the learning curve only applies to ramp-up periods, which are usually followed by periods of steady production rates where no further improvements are possible unless new methods or technologies are introduced (e.g., Baloff, 1970; Pogue, 1983). Unlike in earlier studies, the learning in production and growth in demand functions are validated in this paper using available empirical data collected from an electronics manufacturing plant (Badiru, 1995).

The remainder of this paper is organised as follows. Section 2 is for empirical validation of the production learning and demand functions. Section 3 incorporates these validated functions into the economic lot size problem. Section 4 is for numerical studies, and section 5 contains a summary and concluding remarks.

2. Empirical validation of the production and demand functions

In this section, we will use the empirical data from Badiru (1995) to validate the learning in production and demand functions. The data from Badiru (1995; Table 1, p. 786) represents a record of a particularly troublesome production line collected over a period of four years. The production line was a new addition to an electronics manufacturing plant and it was subject to significant learning effects. The production line was subject to many quality problems and frequent interruptions of production. It was the practice of the company to temporarily stop production if significant quality problems were encountered. Production would then resume after the cause of the quality problem had been identified and rectified. Using the empirical data in Table 1 from Badiru (1995, p. 786), we validate the learning in production and demand functions in the subsequent sections.

<< INSERT TABLE 1 ABOUT HERE >>

2.1. Validation of the production function

Learning curves are considered to be an effective planning and control aid in the aircraft, appliance, electronics, shipbuilding, machine shop and home construction industries (Andress, 1954; Steven, 1999). Although Wright's original learning curve model has remained the most popular due to its simplicity, a number of other geometric versions of the curve have been developed, as it was found that the application of the Wright's learning curve to certain learning situations resulted in discrepancies. Using learning data from a research project into the cost effectiveness of training in telephone exchanges in England, Hackett (1983) compared the efficiency of a number of learning curve models. The method of comparison was based on an extension of an iterative two-parameter curve-fitting algorithm. From a selection of 18 models, many were rejected initially, including the Wright learning curve model. From Hackett's analysis, it was found that the Time Constant Model was the most practical model to use for two reasons: (1) it nicely fits the learning data observed, and (2) the three parameters may be easily defined in terms which are acceptable to those members in the industry concerned with manpower planning, delivery date estimation and the setting of performance and training standards. In addition, using the output per unit time as the dependent variable, and time on the job as the independent variable of the learning curve has a number of advantages when the curve is used as a management tool (Towill and Kaloo, 1978). Thus, the Time Constant Model is tested in this paper and is given as

$$P(t) = p_0 + (p - p_0)\left(1 - e^{-t/\tau}\right) = p - (p - p_0)e^{-t/\tau} = p - \pi e^{-t/\tau} \tag{1}$$

where $P(t)$ is the time dependent production rate (units/worker/unit time), p_0 is the initial production rate with no previous experience, p is the maximum production capacity that could be attained, t is time, and τ is the time constant. The time constant is a direct measure of rate of improvement with experience (Hackett, 1983). The larger the time constant, the slower will be the rate of improvement. It can be observed that if τ was to approach infinity, the maximum rate of production would be equal to p_0.

Figure 1 presents the productivity per worker (units/number of workers) computed from Table 1 (Column "units"/Column "Number of workers"). Equation (1) was found to fit the data better than the logarithmic function $y(t) = \alpha \ln(t) + \beta$. Two criteria were used, the MSE and the balance of the model (ratio of above- to under-predictions). The values of the time constant model in (1) that minimises the MSE are p_0 = 18.622, p = 30.561, and τ = 16.159. For these parameter values, the MSE = 2.61 and the balance 50:50. Whereas for the logarithmic function parameters α = 3.353 and β = 17.052 correspond to a MSE = 2.71 and a balance of 50:50. Thus, we conclude that the learning function conforms to the equation described in (1).

<< INSERT FIGURE 1 ABOUT HERE >>

The data in Figure 1 shows that the variation in performance reduces significantly as experience is gained. This is attributed to reduction in breakdown time and rework time per worker, as illustrated in Figure 2.

<< INSERT FIGURE 2 ABOUT HERE >>

2.2. Validation of the demand function

The data in Table 1 does not indicate the demand, but rather the production level. However, it is reasonable to assume that a production facility will not produce more than what is demanded. Thus, we assume that the production level is equal to the demand in a given month. Figure 3 illustrates the behaviour of demand over the four years, which resembles that of an exponential function of the form $y(t) = \alpha e^{\beta t}$. However, the growth in demand tends to stabilise towards the end of the 4-year period. This indicates that it might follow the logistic curve (S-curve; see Carr, 1946), which is of the form:

$$D(t) = \frac{c}{1+ae^{-bt}} \quad (2)$$

where c, a, and b are parameters. Thus, two demand functions are fitted for the data in Figure 3. For the exponential function the MSE = 2,475,551 (MSE = Mean Square Error) and the balance is 54:46, for α = 1282.1 and β = 0.08. For the logistic curve described in (2), the MSE = 2,076,908 and the balance is 50:50, for a = 262.595, b = 0.091, and c = 259720.851. Thus, we conclude that the demand function conforms to the equation described in (2).

<< INSERT FIGURE 3 ABOUT HERE >>

3. Mathematical model

Consider a situation where learning is evident in production with $P(t)$ describing the production rate as in (1), and where the demand rate increases over time according to $D(t)$ as described in (2). Three principal production-consumption patterns may occur in this case (see Figure 4). If the production rate follows alternative a), the production rate is always higher than the demand rate, wherefore no shortages occur. If production pattern b) is prevalent, the production rate is higher than the demand rate at the beginning of the cycle and lower at the end. However, as enough inventory is built up at the beginning of the cycle, running the facility with a lower production rate at the end of the cycle reduces inventory in the system, but does also not lead to shortages. If the production rate follows pattern c), however, insufficient inventory is build up at the beginning of the production cycle, which leads to shortages in the second half of the cycle. It is clear that shortages may disturb the introduction of a product to the market significantly, wherefore it is assumed in the following that shortages have to be avoided in the system. To balance production and demand, we assume that the number of workers is varied over time to assure that the inventory position never drops below zero. Since $P(t)$ represents the production output per worker in time t, the production output of the system in time t equals $W(t)P(t)$ where $W(t)$ is the number of workers required in time t. We note that it is not required that the production rate of the system is always higher than the demand rate, as long as a non-negative inventory is maintained at all time. The assumption that shortages are avoided is motivated by many practical examples, such as the automotive industry, where the penalty for short shipments is severe. For instance, Saturn levies fines of \$500 per minute to suppliers who cause production line stoppages (Frame, 1992), and Chrysler fines suppliers \$32,000 per hour when an order is late (Russell and Taylor, 1998). Define H as the length of the ramp-up period where $t \in (0,H)$. It is clear that the production rate of the system may not be lower than the demand rate at the beginning of the planning period (where there is no inventory in the system), wherefore we can define

$$W(0) = \frac{D(0)}{P(0)} = \frac{c}{p_0(1+a)} \tag{3}$$

<< INSERT FIGURE 4 ABOUT HERE >>

Assume an intermittent production situation with its parameters defined as: K is the setup cost which occurs whenever the setup of the production line is changed (for example by implementing a new work schedule), h is the holding cost per unit per unit of time, and L is the labour cost per worker per unit of time. The inventory level at time t in cycle i can be described as

(4) $$I_i(t) = \begin{cases} Q_i(t) - C_i(t) & \text{if} \quad T_{i-1} \leq t \leq T_{i-1} + t_{1i} \\ Z(t_{1i}) - C_i(t) & \text{if} \quad T_{i-1} + t_{1i} < t \leq T_i \end{cases}$$

where $Q_i(t)$ is the production quantity in cycle i by time t, $C_i(t)$ is the consumption quantity in cycle i by time t, t_{1i} is the production time in cycle i, and t_{2i} is the process idle time during which a maximum inventory of level $Z(t = t_{1i})$ is consumed, where $T_i = \sum_{j=1}^{i}(t_{1j} + t_{2j})$ is the sum of all production cycles prior to and including i and thus defines the end of production cycle i and the beginning of production cycle i+1. Figure 5 illustrates the resulting inventory time plots. To assure that demand can be met without interruption, the number of workers is varied over time. At the beginning of cycle i, the demand rate is $D(T_{i-1})$ and the production rate is $P(k_{i-1})$ where k_i is the cumulative production time of cycles 1,...,i with $k_i = \sum_{j=1}^{i} t_{1j}$ and $k_0 = 0$. Note that the demand rate depends on T_i, since products are consumed during both the production and consumption phase (i.e. during t_{1i} and t_{2i}), whereas the production rate depends on k_i, since only the time the system produces influences learning (i.e. only t_{1i}). Let $W(t) = W(k_i) + \lambda_i t$, assumed of a linear form for simplicity, where $t \in [k_{i-1},k_i]$, λ_i is the incremental increase/decrease in the number of workers per unit time, and $W(k_i)$ is the initial number of workers at beginning of cycle i. Thus, we have $W(k_i) = W(k_{i-1}) + \lambda_{i-1} t_{1i-1}$.

<< INSERT FIGURE 5 ABOUT HERE >>

$Q_i(t)$ can now be formulated as

(5)

$$Q_i(t) = \int_{k_{i-1}}^{k_{i-1}+t} W(y)P(y)\mathrm{d}y = \int_{k_{i-1}}^{k_{i-1}+t} (W(T_{i-1}) + \lambda_i y)\left(p - \pi e^{-y/\tau}\right)\mathrm{d}y$$

$$= \frac{1}{2} e^{-\frac{k_{i-1}+t}{\tau}} \left(e^{\frac{k_{i-1}+t}{\tau}} pt(2W(T_{i-1}) + (2k_{i-1} + t)\lambda_i) \right.$$

$$\left. + 2\pi\tau\left(\left(1 - e^{\frac{t}{\tau}}\right) W(T_{i-1}) + \left(t + \tau + k_{i-1} - e^{\frac{t}{\tau}}(\tau + k_{i-1})\right)\lambda_i\right)\right)$$

$C_i(t)$, in turn, is given as

(6) $$C_i(t) = \int_{T_{i-1}}^{T_{i-1}+t} D(y)\mathrm{d}y = \int_{T_{i-1}}^{T_{i-1}+t} \frac{c}{1+ae^{-by}}\mathrm{d}y = \frac{c}{b}\mathrm{Ln}\left(\frac{a+e^{b(T_{i-1}+t)}}{a+e^{bT_{i-1}}}\right)$$

The maximum inventory attained at t_{1i} in cycle i, $Z(t_{1i})$, is given from (5) and (6) as

(7)

$$Z(t_{1i}) = Q_i(t_{1i}) - C_i(t_{1i})$$

$$= \frac{1}{2} e^{-\frac{k_{i-1}+t_{1i}}{\tau}} \left(e^{\frac{k_{i-1}+t_{1i}}{\tau}} pt_{1i}(2W(T_{i-1}) + (2k_{i-1} + t_{1i})\lambda_i) \right.$$

$$+ 2\pi\tau\left(\left(1 - e^{\frac{t_{1i}}{\tau}}\right) W(T_{i-1})\right.$$

$$\left.\left. + \left(t_{1i} + \tau + k_{i-1} - e^{\frac{t_{1i}}{\tau}}(\tau + k_{i-1})\right)\lambda_i\right)\right) - \frac{c}{b}\mathrm{Ln}\left(\frac{a + e^{b(T_{i-1}+t_{1i})}}{a + e^{bT_{i-1}}}\right)$$

In any cycle, there are two costs that are accounted for, the procurement costs $PC(t_{1i})$ and the holding costs $HC(t_{1i},t_{2i})$. The procurement cost in cycle i is the sum of the set-up cost K and the labour cost, and it is computed as:

(8) $$PC(t_{1i}) = K + L\int_{k_{i-1}}^{k_{i-1}+t_{1i}} W(y)dy = K + L\int_{k_{i-1}}^{k_{i-1}+t_{1i}} (W(T_{i-1}) + \lambda_i y)\, dy = K + \frac{L}{2}t_{1i}(2W(T_{i-1}) + (2k_{i-1} + t_{1i})\lambda_i)$$

The holding cost in cycle *i* is computed from (4), (5), (6) and (7) as

(9)

$$HC(t_{1i},t_{2i}) = h\left(\int_0^{t_{1i}} I(y)dy + \int_{t_{1i}}^{t_{1i}+t_{2i}} I(y)dy\right)$$

$$= h\left(\int_0^{t_{1i}} Q_i(y)\mathrm{d}y - \int_0^{t_{1i}} C_i(y)\mathrm{d}y + \int_{t_{1i}}^{t_{1i}+t_{2i}} Z(t_{1i})dy\right.$$

$$\left.- \int_{t_{1i}}^{t_{1i}+t_{2i}} C_i(y)\mathrm{d}y\right)$$

$$= \frac{1}{6b^2} e^{-\frac{k_{i-1}+t_{1i}}{\tau}}\left(6b^2\pi\tau\left(W(T_{i-1})(t_{2i}-\tau)\right.\right.$$

$$\left. + \lambda_i\left((\tau+k_{i-1}+t_{1i})t_{2i} - \tau(2\tau+k_{i-1}+t_{1i})\right)\right)$$

$$+ e^{\frac{t_{1i}}{\tau}}\left(e^{\frac{k_{i-1}}{\tau}}\left(6c\left(\mathrm{Li}_2\left(-\frac{e^{b(T_{i-1}+t_{1i}+t_{2i})}}{a}\right) - \mathrm{Li}_2\left(-\frac{e^{bT_{i-1}}}{a}\right)\right)\right.\right.$$

$$+ bt_{1i}\left(6c\mathrm{Ln}\left(\frac{a+e^{bT_{i-1}}}{a}\right) + bpt_{1i}\left(3W(T_{i-1}) + \lambda_i(3k_{i-1}+t_{1i})\right)\right)$$

$$+ 3b\left(2c\mathrm{Ln}\left(\frac{(a+e^{bT_{i-1}})^2}{a(a+e^{b(T_{i-1}+t_{1i})})}\right)\right.$$

$$\left.\left. + bpt_{1i}\left(2W(T_{i-1}) + \lambda_i(2k_{i-1}+t_{1i})\right)\right)t_{2i}\right)$$

$$+ 6b^2\pi\tau\left(W(T_{i-1})(\tau - t_{1i} - t_{2i})\right.$$

$$\left.\left.\left. + \lambda_i\left(2\tau^2 + k_{i-1}(\tau - t_{1i} - t_{2i}) - \tau(t_{1i}+t_{2i})\right)\right)\right)\right)$$

where $T_i = T_{i-1} + t_{1i} + t_{2i}$ and $\mathrm{Li}_s(z)$ is the polylogarithm. Therefore the total cost in cycle i is the sum of (8) and (9); i.e., $TC(t_{1i},t_{2i}) = PC(t_{1i}) + HC(t_{1i},t_{2i})$. There are n cycles in the planning horizon H, and the mathematical programming problem can be written as

(10a) Minimize $\psi(n, \lambda_i, t_{1i}, t_{2i} | i = 1, \dots, n) = \sum_{i=1}^{n} TC(t_{1i}, t_{2i})$

Subject to:

(10b) $\sum_{i=1}^{n}(t_{1i} + t_{2i}) = H$

(10c) $Q_i(t) - D(t) \geq 0$; $\forall t \in [k_{i-1},k_i]$; $i = 1,\dots,n$

(10d) $Z(t_{1i}) - D(T_i) = 0$; $i = 1,\dots,n$

(10e) $n \geq 1$, where n is an integer

(10f) $t_{1i}, t_{2i} \geq 0$; $i = 1,\dots,n$

Note that in the optimisation problem given in (10a) to (10f), the production and process idle times t_{1i} and t_{2i} and the number of setups n are decision variables.

4. Numerical example

To illustrate the behaviour of the model developed in Section 3, we solved a set of test problems whose input parameters are shown in Table 2. Apart from the data given in Table 2, we assumed the following: $p = 500$, $p_0 = 200$, $h = 0.5$, $a = 250$, $c = 5000$ and $K = 250$. The results are summarised in Table 3. Note that S_P and S_D describe the time when the production and demand processes have reached 98% of the maximum production or demand rate, respectively.

Due to the complexity of the objective function (10a), we adopted a steepest descent-algorithm to find a solution (see Gill et al., 1981). The convexity of the model was tested for different values of the input parameters using simulation, where the results showed that for all 5000 runs the objective function has a single minimum. So, it is reasonable to conjuncture that the objective function is unimodal and holds a unique minimum. The solution procedure works as follows: First, for a given number of batches, t_{11} was set equal to $H - \sum_{i=2}^{n}(t_{1i} + t_{2i}) - t_{21}$ and $t_{12},...,t_{1n}$ and $t_{21},...,t_{2n}$ were set equal to zero (note that in some cases, it was necessary to select a value greater than zero for t_{21} to assure that conditions (10c) and (10d) could be met). After the total costs had been calculated, $t_{12},...,t_{1n}$ and $t_{21},...,t_{2n}$ were successively increased by 0.01 and the total costs were re-calculated. In case any of the $2n-1$ alternatives led to a decrease in total costs, the alternative which led to the highest cost decrease was adopted. Thus, t_{11} was successively reduced and $t_{12},...,t_{1n}$ and $t_{21},...,t_{2n}$ were successively increased until a (locally) optimal solution had been found. For given t_{1i}- and t_{2i}-values, a solution for the λ_i-variables could easily be found with the help of the constraints (10c) and (10d). A solution for the number of batches could finally be found by increasing n stepwise from 1 until the total costs started to increase. In this case, the optimal number of batches was given as $n-1$.

<< INSERT TABLE 2 ABOUT HERE >>

<< INSERT TABLE 3 ABOUT HERE >>

Considering problem #1 first, it can be seen that producing without interruption (i.e. with $t_{2i} = 0\ \forall i$) led to the lowest total cost. This suggests that it is beneficial to produce with the lowest possible production rate for the entire consumption time, which reduces the build-up of inventory in the system. This also supports the findings of Baloff (1970), who recommended not interrupting the production process during the ramp-up phase to avoid a loss of ramp-up momentum and control. The effect of increasing the number of batches is illustrated in Figure 6. As can also be seen, with an increasing number of batches, the production line is restructured more and more frequently, which gives the production planner the opportunity to adjust the workforce at the production line in such a way that the production rate approaches the demand rate more and more. Graphically, the area between the production function $P(t)$ and the demand function $D(t)$ equals excessive production (which leads to inventory build-up, cf. area A in the upper left part of Figure 6) and excessive consumption (which reduces inventory, cf. area B in the upper left part of Figure 6). Thus, by increasing n, this area (and therewith inventory in the system) is reduced. For the hypothetical case where $n \to \infty$, production and consumption would be perfectly synchronised and no inventory would occur in the system. Since an increase in n leads to higher setup costs due to more frequent changes in the organisation of the production line, inventory carrying costs, production costs and setup costs have to be balanced, which occurs at $n = 4$ for problem # 1.

<< INSERT FIGURE 6 ABOUT HERE >>

Problems #2 to #5 illustrate the effect of changes in the time constant τ. As explained above, τ measures the rate of improvement in production, where high values of τ describe production processes where the performance of the workers improves slowly and vice versa. Table 3 illustrates that a higher learning effect (i.e. a faster improvement in production) reduces the total costs of the system. To avoid that too much inventory is build up, the production line is restructured more frequently, which brings the production rate closer to the demand rate. Further, as workers learn faster, fewer workers need to be added to the production line over time, which leads to lower values for the λ_i-variables. In contrast, if workers learn less fast (which corresponds to higher τ-values, cf. problems #4 and #5), more workers are needed at the production line, which results in higher values for the λ_i-variables. It is obvious that if learning in production decreases, total costs increase.

The effect of varying parameter b is illustrated in examples #6 to #9. Similarly to the time constant τ, b measures the rate at which demand increases, with high values of b defining demand processes that ramp up quickly. If demand increases at higher rates and learning in production remains constant, the number of workers at the production line has to be increased stronger to assure that demand can be satisfied without interruption. Thus, comparing problems #6 and #7 with problem #1, higher b-values are associated with higher values for the λ_i-variables for i = 1,...,n-1. However, considering the value of λ_n for problems #6 and #7, it becomes obvious that it may also be beneficial to reduce the number of workers or to keep it virtually constant in certain cases. This aspect is illustrated in Figure 7. In this example, the demand process is ramped up to 98% of the maximum demand rate at time T_I, but the planning horizon continues until time H. If production is ramped up to 98% at a similar time than the demand rate, which is the case for problem #6, the production and demand rate only change insignificantly after time T_I, wherefore the number of workers at the production line can be kept constant between time T_I and time H (cf. case a) in Figure 7). If, in contrast, production is ramped up at a later time than T_I, which is the case for problem #7, the output per worker continues to increase after the demand rate has reached a steady state. In this case, workers can be removed from the production line to synchronise production and demand (cf. case b) in Figure 7). Looking at the t_{2i}-values, it becomes clear that demand processes which ramp up faster than production processes may necessitate inserting idle times in the production process to reduce excessive inventory that has been built up before. Idle times, however, decrease as the number of batches adopts higher values since a higher batch frequency enables the production planner to adjust the workforce more frequently and to avoid excessive inventory. As to the cost values given for problems #6 to #9, we note that comparing these values is not possible. If the demand process is characterised by a low *b*-value, which leads to a long duration of the start up-process, fewer products are produced and consumed in *H* time units than in case *b* adopts a high value. It is clear that in this case production and inventory cost have to be lower as compared to the case where a high volume of goods is produced and consumed.

<< INSERT FIGURE 7 ABOUT HERE >>

Problems #10 to #13 illustrate the impact of the planning horizon *H* on the behaviour of the model. While *H* was assumed to be constant and equal to 12.5 for problems #1 to #12, *H* was set equal to the ramp-up time of the demand process S_D for problems #10 to #13. Thus, the case where demand adopts a steady level prior to *H* (as illustrated in Figure 7) was avoided. The rationale is that as increasing the planning horizon leads to a higher number of batches, which reduces inventory in the system. Further, it becomes clearer that in case where only the ramp-up process is considered (and not the steady state phase which succeeds the ramp-up phase), it is neither necessary to allow for idle times in the production process nor to keep the output rate of the system constant towards the end of the planning period. Since the ramp-up phase can only occupy a small share of the overall life cycle of a product, and since several authors have noted that learning curves may only be valid for the ramp-up phase of a product (see, e.g., Baloff, 1970; Pogue, 1983), this brings us to the conclusion that our model should be used to coordinate the ramp-up phase of a product, and that classical planning tools (such as the EOQ-model or models that consider lot streaming) should be applied for planning the remaining time of the product life cycle.

Problems #14 to #17 finally illustrate the impact of changes in the labour costs on the behaviour of the system. As can be seen, increasing *L* leads to higher total costs and vice versa, but that the production policy remains almost unchanged for alternative *L*-values. This can be explained by the fact that for a given demand that has to be produced in *H* time units, and for a given improvement rate in production, the number of worker hours that are required for producing the demanded products are given and may not be influenced by the production policy. This is a result of the assumptions made in this paper, which presumed that products that have been completed may be directly consumed and that forgetting is not prevalent in the production process. If, in contrast, a scenario is considered where only complete lots may be forwarded to the next stage (or the customer) and be consumed there, and where interruptions in the production process lead to forgetting, changes in *L* affect the production policy as well. The impact of the other problem parameters on the behaviour of the model will not be further investigated here, as their effect on the behaviour of lot size-models is well known from the literature.

5. Summary and Conclusions

This paper developed a production-planning model for a manufacturing process that undergoes a ramp-up period with growth in demand and production. The labour production and demand functions assumed in this paper were validated using available empirical data, and a mathematical programming model was developed and numerical examples were presented. The results of the paper indicate that in the case where finished products can immediately be transferred to the customer, production planners should try to synchronise production and demand in the ramp-up phase to avoid unnecessary inventory accumulations. If the learning rate of the workers cannot be controlled, a synchronised production-demand process can be achieved by assigning additional workers to the production line or by removing them if necessary. Since the general behaviour of the model developed in the paper was shown to be dependent on the planning horizon (and the relation of the planning horizon to the ramp-up time of the production and demand process), we recommend using our model as a heuristic planning tool during the ramp-up phase of a product and employe classical planning tools during the steady-state that follows, such as the EOQ model or models which consider lot streaming, for the remaining part of the product life cycle.

One limitation of the paper is that it considers a production process where finished products can immediately be consumed by the customer. Although production processes exist in practice where this assumption is valid, companies may decide to send only complete batch shipments to customers (instead of each individual unit) especially if the geographical distance between manufacturer and customer is significant. In such a case, higher inventory occurs in the system, and synchronising production and demand does not reduce inventory to zero. It is therefore clear that studying our model in such an environment may lead to different results, wherefore we recommend extending our model in this direction. Another interesting extension would be to integrate forgetting in the learning process of the employees. If workers are removed from the production line and assigned to it at a later time (or if production is interrupted), forgetting may become an issue and re-structuring the production line may disrupt and slow down the production process. Finally, it would be interesting to study how the manufacturer can influence the learning rates of the employees or the demands of customers (for example by offering discounts).

Acknowledgements

The first author wishes to thank the in-kind support provided to him, as a visiting researcher, by Ryerson University, Toronto, Canada, and the Natural Science and Engineering Research Council (NSERC) of Canada for their partial financial support. This work was further supported by a fellowship within the Postdoc-Programme of the German Academic Exchange Service (DAAD). The second and third authors thank NSERC for their financial support. Finally, the authors thank the anonymous reviewers for their positive and constructive reviews.

References

Almgren, H., 1999a. Towards a framework for analyzing efficiency during start-up: An empirical investigation of a Swedish auto manufacturer. International Journal of Production Economics, 60-61 (1), 79-86.

Almgren, H., 1999b. Start-up of advanced manufacturing systems-a case study. Integrated Manufacturing Systems, 10 (3-4), 126-135.

Almgren, H., 2000. Pilot production and manufacturing start-up: The case of Volvo S80. International Journal of Production Research, 38 (17), 4577-4588.

Andress, F., 1954. The learning curve as a production tool. Harvard Business Review, 32, 87-89.

Badiru, A.B., 1995. Multivariate analysis of the effect of learning and forgetting on product quality. International Journal of Production Research, 33 (3), 777-794.

Baloff, N., 1970. Startup Management. IEEE Transactions on Engineering Management, 17 (4), 132-141.

Bayus, B.L., 1997. Speed-to-market and new product performance trade-offs. The Journal of Product Innovation Management, 14 (6), 485–497.

Cantemessa, M., Valentini, C., 2000. Planning and managing manufacturing capacity when demand is subject to diffusion effects. International Journal of Production Economics, 66 (3), 227-240.

Carr, W., 1946. Peacetime cost estimation requires new learning curves. Aviation, 45 (April).

Cohen, M.A., Eliashberg, J., Ho, T., 1996. New product development: The performance and time-to-market tradeoff. Management Science, 42 (2), 173–186.

Fjällström, S., Säfsten, K., Harlin, U., Stahre, J., 2009. Information enabling production ramp-up. Journal of Manufacturing Technology Management, 20 (2), 178-196.

Frame, P., 1992. Saturn to fine suppliers $500/minute for delays, Automotive News 21, 36.

Gill, P.E., Murray, W., Wright, M.H., 1981. Practical Optimization, London et al.: Academic Press.

Gross, U., Renner, T., 2010. Coordination and Cooperation during Production Ramp-up: An Empirical Study of the European Manufacturing Industry. POMS 21st Annual Conference, Vancouver, Canada.

Hackett, E. A., 1983. Application of a set of learning curve models to repetitive tasks. Radio and Electronic Engineer, 53 (1), 25-32.

House, C., Price, R., 1991. The return map: Tracking product teams. Harvard Business Review, 69 (1), 92-100.

Jaber, M.Y., 2006. Learning and Forgetting Models and Their Applications. In: Badiru, A.B. (Ed.): Handbook of Industrial Systems Engineering, Boca Raton: CRC/Taylor & Francis, 30.1-30.27.

Jaber, M.Y., Bonney, M., 1999. The economic manufacture/order quantity (EMQ/EOQ) and the learning curve: past, present, and future. International Journal of Production Economics, 59 (1-3), 93-102.

Jaber, M.Y., Bonney, M., 2011. The lot sizing problem and the learning curve: A review. In: Jaber, M.Y. (Ed.), Learning Curves: Theory, Models, and Applications, CRC Press (Taylor and Francis Group), Chapter 14, pp. 267-293, FL: Baco Raton.

Jørgensen, S., Kort, P.M., Zaccour, G., 1999. Production, inventory, and pricing under cost and demand learning effects. European Journal of Operational Research, 117 (2), 382-395.

Matta, A., Tomasella, M., Valente, A., 2007. Impact of ramp-up on the optimal capacity-related configuration policy. International Journal of Flexible Manufacturing Systems, 19 (3), 173-194.

Naim, M., 1993. Learning curve models for predicting performance of industrial systems. *PhD Thesis*, School of Electrical and Systems Engineering, University of Wales, Cardiff, U.K.

Nembhard, H.B., Birge, J.R., 1998. A startup procedure for process industries using a multiple objective nonlinear program. IEE Transactions, 30 (4), 291-300.

Ngwenyama, O.K., Guergachi, A., McLaren, T., 2007. Using the learning curve to maximize IT productivity: A decision analysis model for timing software upgrades. International Journal of Production Economics, 105 (2), 524-535.

Plaza, M., Ngwenyama, O.K., Rohlf, K., 2010. A comparative analysis of learning curves: Implications for new technology implementation management. European Journal of Operational Research, 200 (2), 518-528.

Pogue, G. A., 1983. The learning curve and unit costs. Management Accounting, 61 (4), 6-37.

Russell, R.S., Taylor B., 1998. Operations Management: Focusing on Quality and Competitiveness. 2nd edition. Upper Saddle River: Prentice-Hall.

Steven, G. J., 1999. The learning curve: from aircraft to spacecraft? Management Accounting, 77 (5), 64-65.

Terwiesch, C., Bohn, R.E., 2001. Learning process improvement during production ramp-up. International Journal of Production Economics, 70 (1), 1-19.

Terwiesch, C., Xu, Y., 2004. The Copy-Exactly Ramp-Up Strategy: Trading-Off Learning with Process Change. IEEE Transactions on Engineering Management, 51 (1), 70-84.

Towill, D. R., Kaloo, U., 1978. Productivity drift in extended learning curves. Omega, 6 (4), 295-305.

Winkler, H., Slamanig, M., 2008. Konzeption eines aktivitätsorientierten Instruments zur Anlaufkostenplanung. Zeitschrift für Planung und Unternehmenssteuerung, 19 (1), 85-106.

Xu, D., Albin, S.L., 2002. Manufacturing start-up problem solved by mixed-integer quadratic programming and multivariate statistical modelling. International Journal of Production Research, 40 (3), 625-640.

Yelle, L.E., 1980. Industrial life cycles and learning curves: Interaction of marketing and production. Industrial Marketing Management, 9 (4), 311-318.

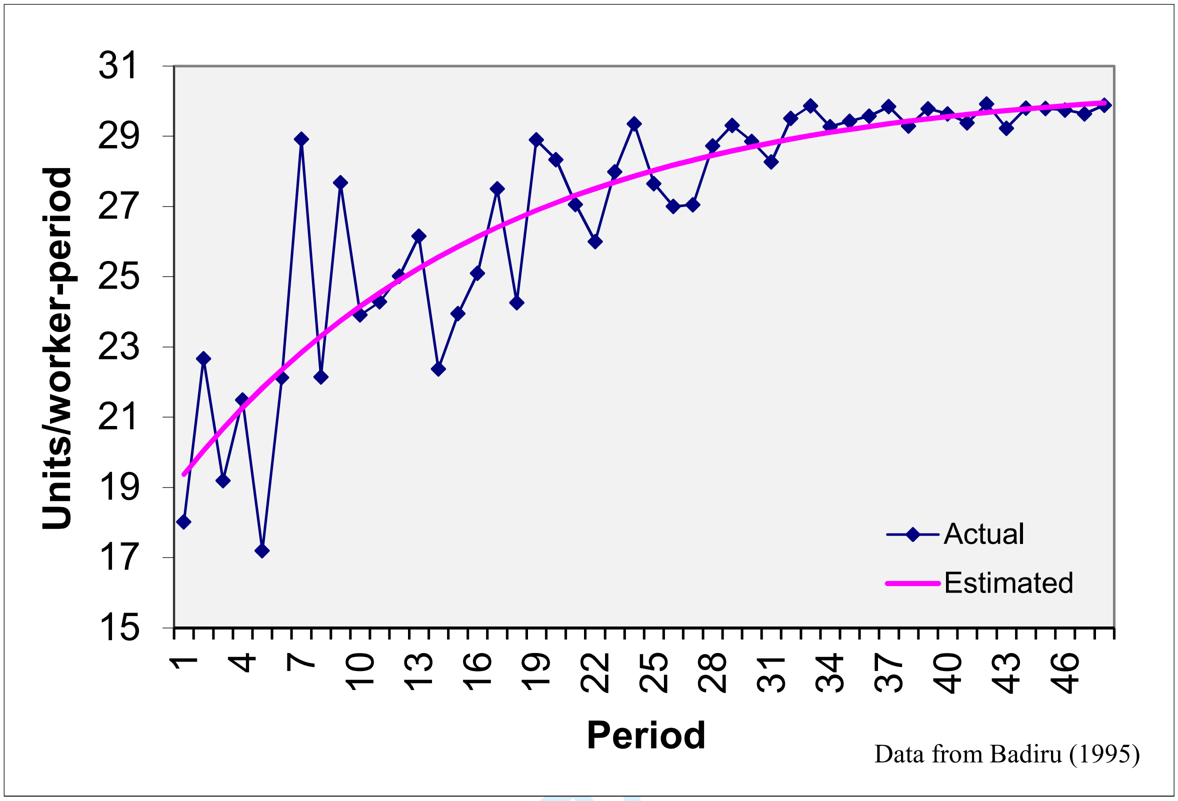

Figure 1: Productivity per worker, computed from the empirical data in Table 1

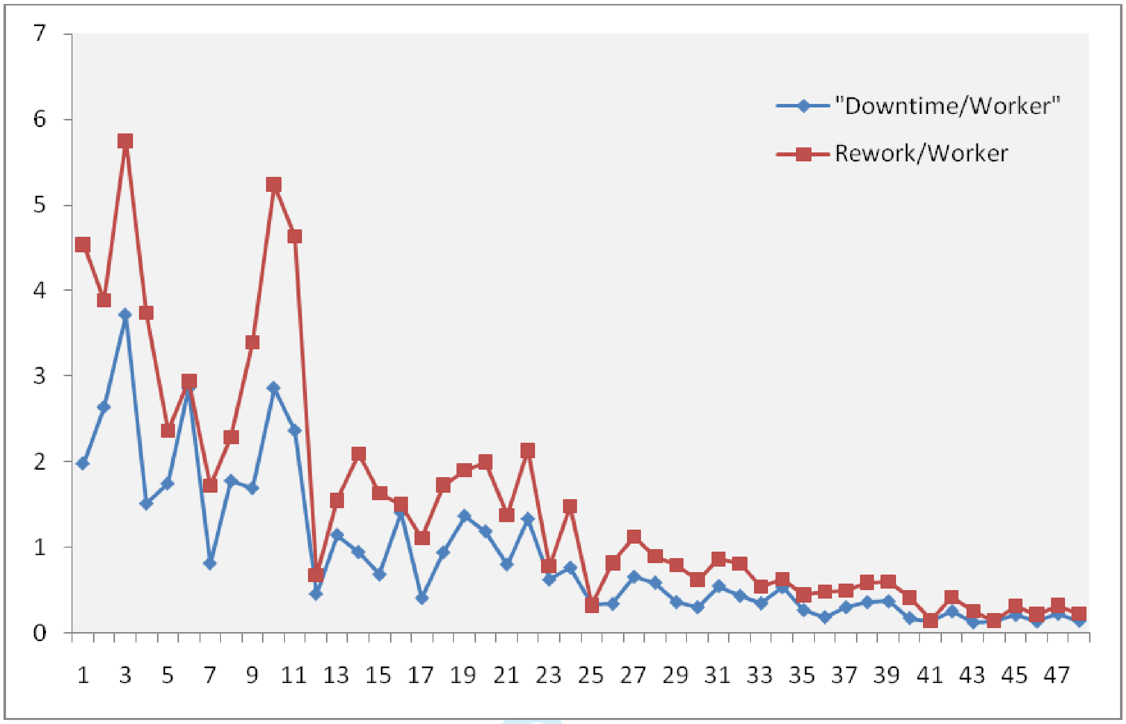

Figure 2: Downtime and rework time per worker, computed from the empirical data in Table 1

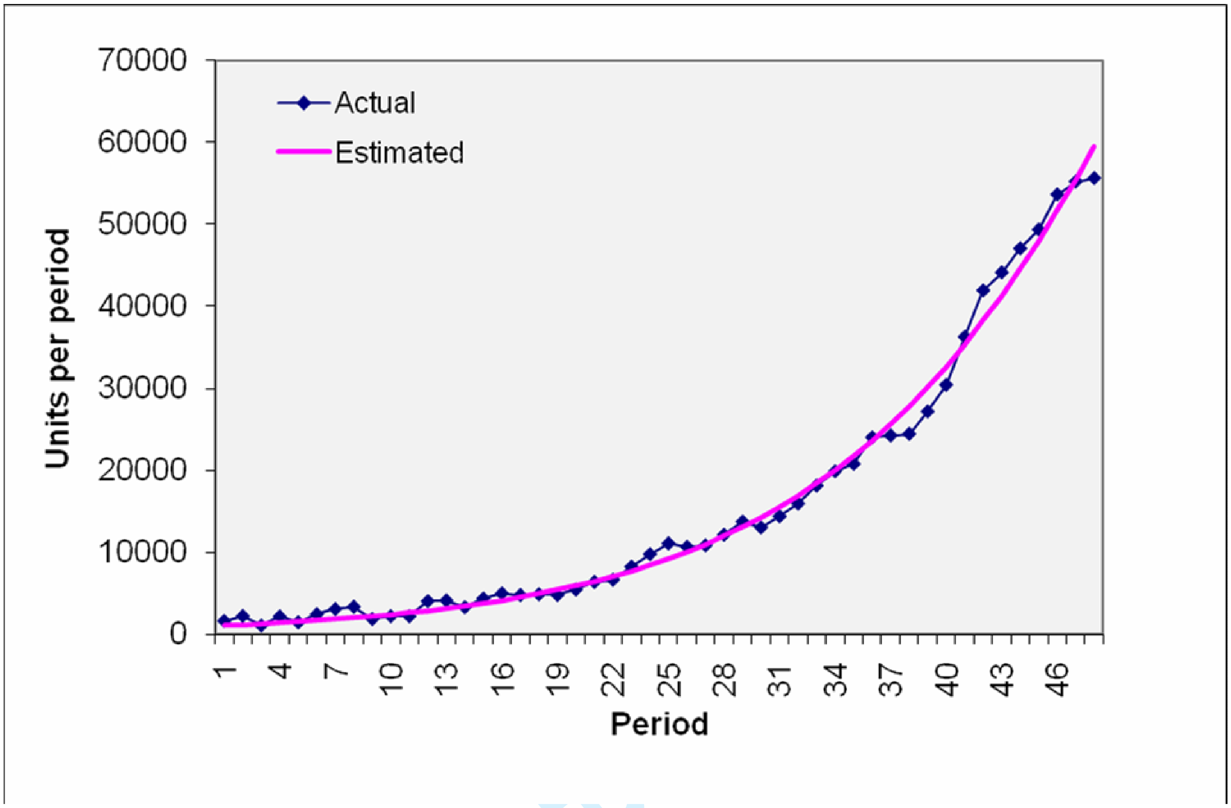

Figure 3: Development of demand, computed from the empirical data in Table 1

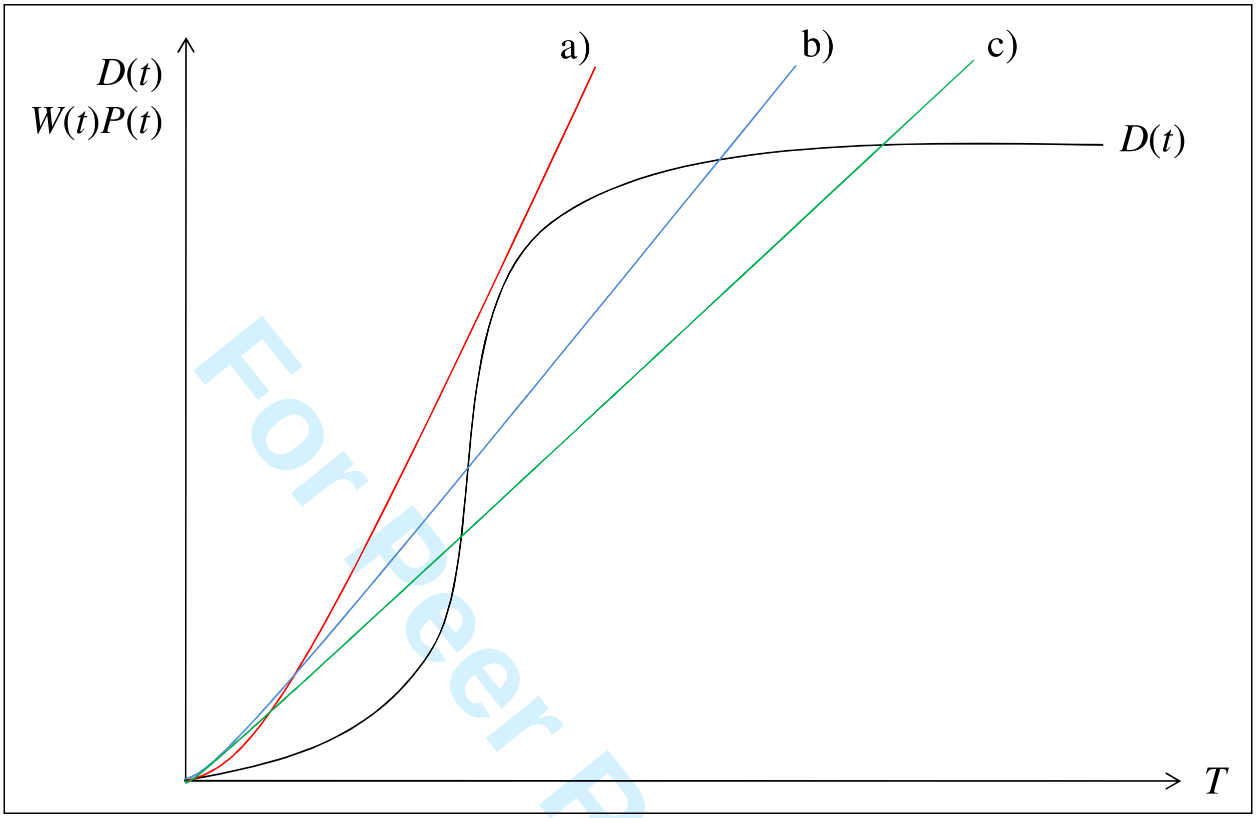

Figure 4: Alternative production-consumption patterns for the model developed in this paper

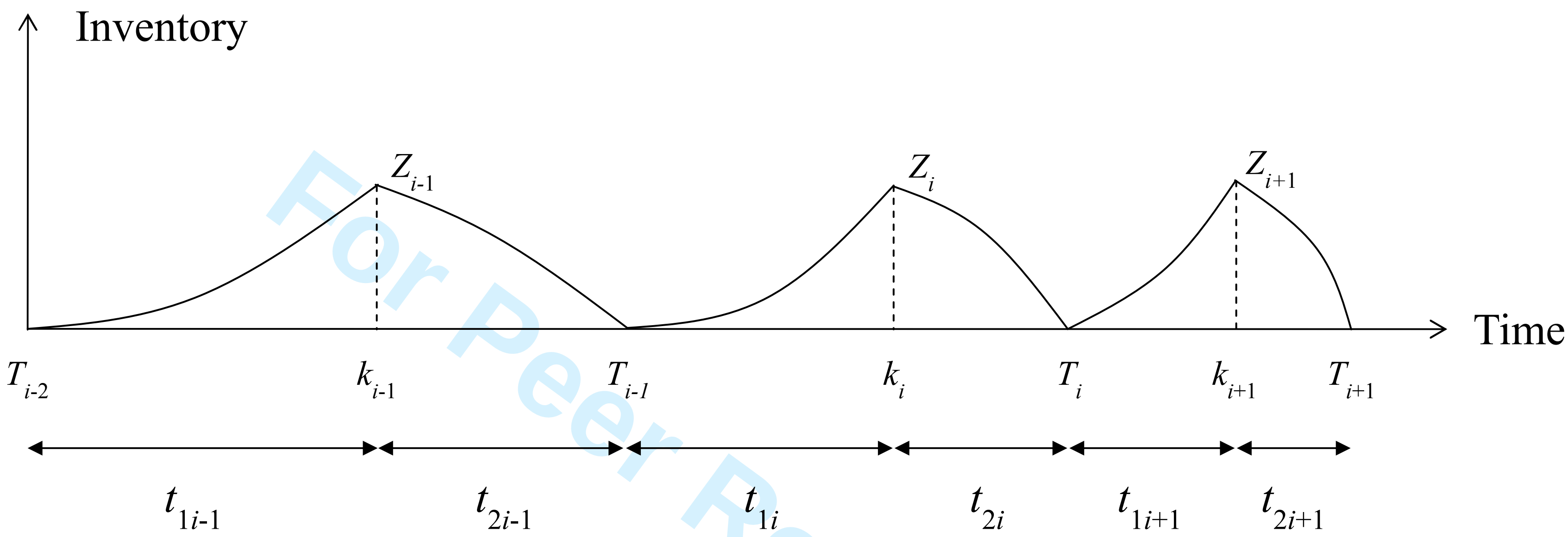

Figure 5: Inventory time plots for three successive batches

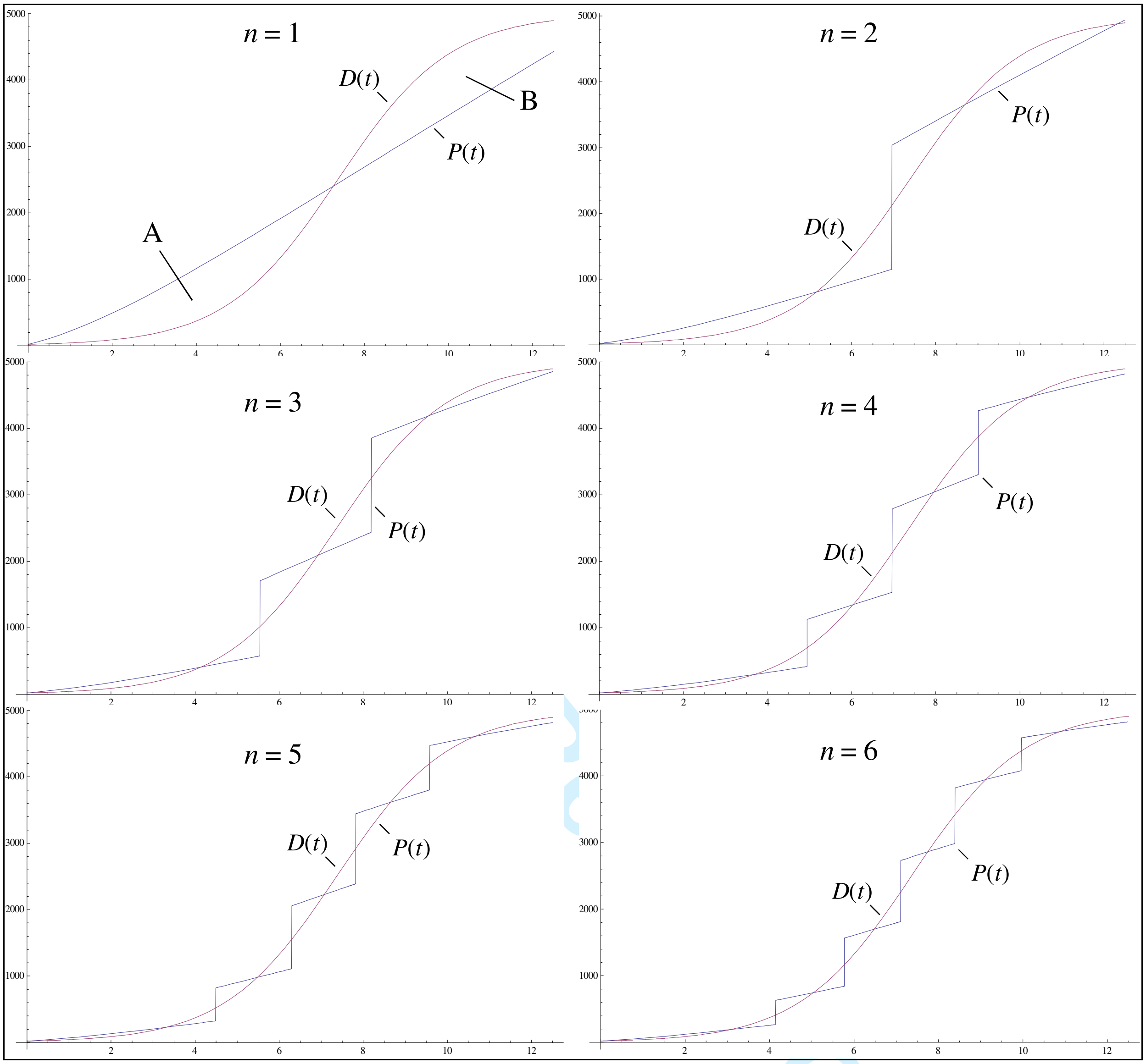

Figure 6: Alternative production-consumption patterns for problem #1

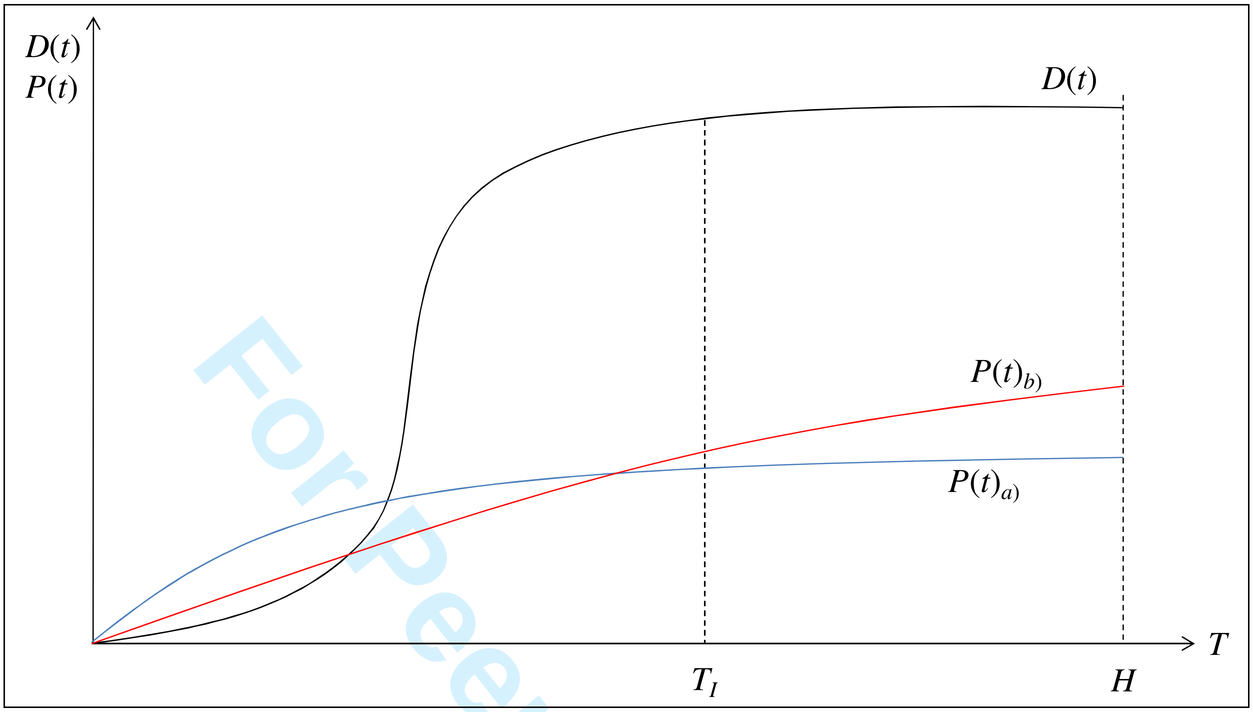

Figure 7: Comparison of alternative ramp-up-scenarios for the production and demand rate

Month	Actual Cx	units	Number of Workers	Production Downtime	Production Rework	Total cost
1	103	1640	91	180	413	168920
2	77	2244	99	261	385	172788
3	116	1075	56	208	322	124700
4	77	2192	102	154	381	168784
5	76	1479	86	150	203	112404
6	68	2456	111	317	327	167008
7	41	3094	107	87	184	126854
8	59	3367	152	270	348	198653
9	58	1882	68	115	231	109156
10	88	2224	93	266	487	195712
11	80	2210	91	215	422	176800
12	35	4052	162	74	109	141820
1	43	4132	158	181	245	177676
2	55	3312	148	140	310	182160
3	47	4383	183	125	298	206001
4	45	4995	199	280	299	224775
5	36	4785	174	71	193	172260
6	48	4877	201	189	347	234096
7	43	4797	166	227	316	206271
8	44	5469	193	229	385	240636
9	40	6386	236	189	326	255440
10	49	6657	256	341	546	326193
11	33	8257	295	184	229	272481
12	38	9776	333	254	494	371488
1	29	11089	401	131	131	321581
2	34	10666	395	134	323	362644
3	37	10848	401	263	451	401376
4	34	12149	423	249	381	413066
5	32	13746	469	169	371	439872
6	31	13014	451	136	280	403434
7	34	14391	509	279	439	489294
8	32	15933	540	234	437	509856
9	29	18128	607	210	328	525712
10	31	19877	679	363	431	616187
11	29	20776	706	188	315	602504
12	29	24013	812	151	388	696377
1	29	24232	812	244	404	702728
2	30	24427	834	299	494	732810
3	30	27163	912	337	549	814890
4	28	30409	1026	179	425	851452
5	26	36260	1234	174	181	942760
6	28	41911	1401	353	589	1173508
7	27	44102	1509	187	389	1190754
8	26	47019	1578	217	225	1222494
9	27	49328	1656	348	530	1331856
10	26	53605	1802	241	392	1393730
11	27	55186	1862	415	599	1490022
12	26	55581	1860	265	418	1445106

Table 1: Empirical data of the ramp-up-phase of a production process

#	τ	L	b	H	S_P	S_D
1	4	25	0.75	12.5	9.94	12.55
2	3	25	0.75	12.5	7.45	12.55
3	2	25	0.75	12.5	4.97	12.55
4	5	25	0.75	12.5	12.42	12.55
5	6	25	0.75	12.5	14.91	12.55
6	4	25	1.00	12.5	9.94	9.41
7	4	25	1.25	12.5	9.94	7.53
8	4	25	0.50	12.5	9.94	18.83
9	4	25	0.25	12.5	9.94	37.65
10	4	25	1.00	9.41	9.94	9.41
11	4	25	1.25	7.53	9.94	7.53
12	4	25	0.50	18.83	9.94	18.83
13	4	25	0.25	37.65	9.94	37.65
14	4	15	0.75	12.5	9.94	12.55
15	4	5	0.75	12.5	9.94	12.55
16	4	35	0.75	12.5	9.94	12.55
17	4	45	0.75	12.5	9.94	12.55

Table 2: Test problems used for numerical experimentation

#	n	TC	$\{t_{11},\ldots,t_{1n}\}$	$\{t_{21},\ldots,t_{2n}\}$	$\{\lambda_1,\ldots,\lambda_n\}$
1	4	3182.03	{4.94, 2.02, 2.05, 3.49}	{0, 0, 0, 0}	{0.18, 0.35, 0.40, 0.23}
2	5	3164.71	{4.53, 1.82, 1.54, 1.77, 2.84}	{0, 0, 0, 0, 0}	{0.15, 0.27, 0.34, 0.29, 0.15}
3	5	3164.14	{4.52, 1.84, 1.56, 1.79, 2.80}	{0, 0, 0, 0, 0}	{0.13, 0.26, 0.33, 0.29, 0.15}
4	4	3199.89	{4.90, 2.01, 2.03, 3.57}	{0, 0, 0, 0}	{0.20, 0.36, 0.42, 0.24}
5	4	3227.86	{4.86, 1.99, 2.02, 3.64}	{0, 0, 0, 0}	{0.21, 0.37, 0.43, 0.25}
6	4	3585.06	{4.13, 1.95, 3.61, 2.80}	{0, 0, 0, 0.01}	{0.33, 0.67, 0.52, -0.02}
7	3	4010.69	{4.15, 4.34, 3.91}	{0, 0.09, 0.01}	{0.68, 1.09, -0.16}
8	4	2171.17	{6.19, 2.47, 1.97, 1.87}	{0, 0, 0, 0}	{0.08, 0.14, 0.18, 0.18}
9	2	826.62	{8.71, 3.79}	{0, 0}	{0.02, 0.03}
10	3	2517.85	{4.10, 1.98, 3.33}	{0, 0, 0}	{0.32, 0.67, 0.52}
11	3	2084.76	{3.24, 1.58, 2.71}	{0, 0, 0}	{0.42, 0.87, 0.69}
12	6	4470.95	{6.26, 2.51, 2.02, 1.97, 2.39, 3.68}	{0, 0, 0, 0, 0, 0}	{0.08, 0.14, 0.19, 0.19, 0.14, 0.07}
13	11	8041.31	{8.95, 3.91, 2.95, 2.49, 2.27, 2.18, 2.21, 2.37, 2.68, 3.29, 4.35}	{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}	{0.02, 0.03, 0.04, 0.05, 0.06, 0.06, 0.05, 0.05, 0.03, 0.02, 0.01}
14	4	2624.59	{4.95, 2.02, 2.04, 3.49}	{0, 0, 0, 0}	{0.19, 0.35, 0.40, 0.23}
15	4	2067.14	{4.95, 2.02, 2.04, 3.49}	{0, 0, 0, 0}	{0.19, 0.35, 0.40, 0.23}
16	4	3739.47	{4.94, 2.02, 2.05, 3.49}	{0, 0, 0, 0}	{0.18, 0.35, 0.40, 0.23}
17	4	4296.91	{4.93, 2.03, 2.05, 3.49}	{0, 0, 0, 0}	{0.18, 0.35, 0.40, 0.22}

Table 3: Results for the test problems