Accepted Manuscript

Title: Hydromagnetic dynamos in rotating non-uniformly stratified spherical fluid shells in dependence on the Rayleigh number

Authors: Ján Šimkanin, Pavel Hejda, Dana Saxonbergová

PII: S0031-9201(11)00027-6
DOI: doi:10.1016/j.pepi.2011.02.002
Reference: PEPI 5378

To appear in: *Physics of the Earth and Planetary Interiors*

Received date: 11-3-2010
Revised date: 25-1-2011
Accepted date: 5-2-2011

Please cite this article as: Šimkanin, J., Hejda, P., Saxonbergová, D., Hydromagnetic dynamos in rotating non-uniformly stratified spherical fluid shells in dependence on the Rayleigh number, *Physics of the Earth and Planetary Interiors* (2008), doi:10.1016/j.pepi.2011.02.002

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

We present an investigation of dynamo in non-uniformly stratified spherical shells dependent on the Rayleigh number and thickness of a stable stratified sub-shell with emphasis on the case characterized by the identical thicknesses of both sub-shells. Our previous study of rotating convection showed that this case of stratification is a typical extreme case. However, the dynamo action in the shell characterized by the identical thicknesses of both sub-shells is not the typical extreme case. The generated magnetic field is close to the cases of uniform stratification and non-uniform one if the thickness of a stable stratified sub-shell is smaller than the thickness of an unstable stratified sub-shell. Similarly as in the case of rotating convection also in the case of dynamo action the multilayer convection ("teleconvection") is not developed in such a case because of the significant amount of stable stratification.

Hydromagnetic dynamos in rotating non-uniformly stratified spherical fluid shells in dependence on the Rayleigh number

Ján Šimkanin^{*,a}, Pavel Hejda^a, Dana Saxonbergová^b

^a*Institute of Geophysics, Academy of Sciences of the Czech Republic, Prague, Czech Republic*

^b*Institute of Atmospheric Physics, Academy of Sciences of the Czech Republic, Prague, Czech Republic*

Abstract

We present an investigation of dynamo in non-uniformly stratified spherical shells dependent on the Rayleigh number and thickness of a stable stratified sub-shell with emphasis on the case characterized by the identical thicknesses of both sub-shells. Our previous study of rotating convection showed that this case of stratification is a typical extreme case. However, the dynamo action in the shell characterized by the identical thicknesses of both sub-shells is not the typical extreme case. The generated magnetic field is close to the cases of uniform stratification and non-uniform one if the thickness of a stable stratified sub-shell is smaller than the thickness of an unstable stratified sub-shell. Similarly as in the case of rotating convection also in the case of dynamo action the multilayer convection (“teleconvection”) is not developed in such a case because of the significant amount of stable stratification.

Key words:

Hydromagnetic dynamo, Non-uniform stratification, Penetrative convection, Rayleigh number

*Corresponding author

Email address: jano@ig.cas.cz (Ján Šimkanin)

1. Introduction

The dynamo theory, which is a significant part of cosmic magnetohydrodynamics, explains the generation mechanism and origin of the Earth's and planetary magnetic fields and their spatial and temporal evolution and changes (Roberts and Glatzmaier, 2000; Christensen and Wicht, 2007). Complicated processes going on in the Earth's and planetary fluid interiors, e.g., a chemical homogenisation, gravitational differentiation, solidification processes acting on the inner core boundary constitute the driving mechanism of dynamos, i.e. they are the fundamental source of convection or magnetoconvection (Jones, 2000). Non-uniform stratifications of the outer liquid Earth's core and the liquid interiors of Giant planets are a consequence of these processes (Fearn and Loper, 1981; Zhang and Schubert, 2000; Takehiro and Lister, 2001).

It is assumed that the upper part of the outer liquid Earth's core (close to the core-mantle boundary¹) is stably stratified (subadiabatic radial temperature gradient) and the lower part (towards the inner core boundary²) unstably stratified (superadiabatic radial temperature gradient). Models of a non-uniformly stratified fluid shell present an acceptable simplification of the real Earth-like conditions. In the Earth's core the stably stratified sublayer is probably very thin (the outer Earth's core is almost unstably stratified, see Braginsky, 1964; Fearn and Loper, 1981; Zhang and Schubert, 2000; Šimkanin et al., 2003, 2006; Stanley and Mohammadi, 2008). A non-uniform stratification is probably typical for the terrestrial planets (Zhang and Schubert, 2000; Stanley and Bloxham, 2006; Stanley and Mohammadi, 2008). However, in the other planets the ratio of the thickness of the appropriate sublayers (e.g., of the stably stratified to unstably stratified sublayers) and their geometric configuration vary. This is noticeable especially with the Giant planets (Stanley and Bloxham, 2004, 2006; Zhang and Schubert, 2000; Stanley and Mohammadi, 2008). Let us briefly describe some examples where the geometric configuration plays an important role. Mercury is characterized by a weak magnetic field. A possible explanation could be given by a hydromagnetic dynamo working in the similar geometric configuration as supposed for the terrestrial planets (stable/unstable) but in this case a larger fraction of spherical shell is stably stratified (Christensen, 2006). In such a case the dynamo

¹hereinafter referred to as CMB

²hereinafter referred to as ICB

35 runs in the small unstably stratified sublayer (close to ICB). Such a weak
 36 dynamo action and skin-effect (the magnetic field generated in the unstably
 37 stratified sublayer permeates through the stably stratified sublayer where it
 38 is damped due to skin-effect) lead to the weak magnetic field observed on the
 39 surface of Mercury (Christensen, 2006). For a different geometric configura-
 40 tion the influence of a non-uniform stratification is fundamental. Stanley and
 41 Bloxham (2004, 2006) assumed reverse stratification, i.e. the stably strati-
 42 fied sublayer is surrounded by the unstably stratified one. This configuration
 43 leads to non-dipolar and non-axisymmetric magnetic fields, which are typical
 44 e.g., for Uranus and Neptune (Stanley and Bloxham, 2004, 2006).

45 Non-uniform stratification can be simulated thermodynamically also in
 46 the Boussinesq models by means of internal heat sources (Zhang and Schu-
 47 bert, 2000; Šimkanin et al., 2003, 2006, 2010). If the stably stratified sublayer
 48 is very thin (for a geometric configuration stable/unstable), then behaviour
 49 is close to the case of uniform stratification when the whole layer is unsta-
 50 bly stratified. Likewise, if the unstably stratified sublayer is very thin, then
 51 behaviour is close to the case of uniform stratification when the whole layer
 52 is stably stratified. Thus, the effects of non-uniform stratification are no-
 53 ticeable if the thicknesses of the stably and unstably stratified sublayers are
 54 comparable (Zhang and Schubert, 2000; Šimkanin et al., 2003, 2006, 2010).

55 In Šimkanin et al. (2010) we studied a convection in rotating non-uniformly
 56 stratified spherical fluid shells in dependence on the Ekman and Prandtl num-
 57 bers, and the change of the sign of temperature gradient was located to the
 58 middle of the convective shell. The investigation provided the following main
 59 results. The multilayer convection (“teleconvection”) did not develop in our
 60 case of non-uniform stratification because of the significant amount of stable
 61 stratification. The case characterized by the identical thicknesses of both
 62 sublayers presents a typical extreme case. It is dissimilar to the case where
 63 the stably stratified sublayer is thinner than the unstably stratified one and
 64 also dissimilar to the case where the unstably stratified sublayer is thinner
 65 than the stably stratified one. In the present paper we study the influence of
 66 magnetic field to our model, i.e. a dynamo action in the non-uniformly strati-
 67 fied spherical fluid shell in dependence on the Rayleigh number and thickness
 68 of a stable stratified sub-shell with emphasis on the case characterized by the
 69 identical thicknesses of both sublayers.

70 The model and governing equations are given in Section 2. The numerical
 71 results are presented in Section 3. Finally, Section 4 provides the conclusions.

2. Governing equations and model

Here we consider dynamo action due to thermal convection of an electrically conducting incompressible fluid in the Boussinesq approximation in a non-uniformly stratified spherical shell ($r_i < r < r_o$) rotating with angular velocity Ω . The evolution of the magnetic field $\mathbf{B}$, the velocity $\mathbf{V}$ and the temperature T is described by the following system of dimensionless equations:

$$E \left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} - \nabla^2 \mathbf{V} \right) + 2\mathbf{1}_z \times \mathbf{V} + \nabla P = R_a \frac{\mathbf{r}}{r_o} T + \frac{1}{P_m} (\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \frac{1}{P_m} \nabla^2 \mathbf{B}, \quad (2)$$

$$\frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla) T = \frac{1}{P_r} \nabla^2 T + H, \quad (3)$$

$$\nabla \cdot \mathbf{V} = 0, \quad \nabla \cdot \mathbf{B} = 0. \quad (4)$$

The radius of the outer sphere L , is the typical length scale, which makes the dimensionless radius $r_o = 1$; the inner core radius r_i is, similar to that of the Earth, equal to 0.35. (r, θ, φ) is the spherical system of coordinates, $\mathbf{1}_z$ is the unit vector. The typical time, t , is measured in the unit of L^2/ν , typical velocity, $\mathbf{V}$, in ν/L , typical magnetic induction, $\mathbf{B}$, in $(\rho\mu\eta\Omega)^{1/2}$, typical temperature, T , in ΔT , and pressure, P , in $\rho\nu^2/L^2$. The dimensionless parameters appearing in Eqs (1)-(4) are the Prandtl number, $P_r = \nu/\kappa$, the magnetic Prandtl number, $P_m = \nu/\eta$, the Ekman number, $E = \nu/\Omega L^2$ and the modified Rayleigh number $R_a = \alpha g_0 \Delta T L / \nu \Omega$, where κ is the thermal diffusivity, ν is the kinematic viscosity, μ is the magnetic permeability, η is the magnetic diffusivity, ρ is the density, α is the coefficient of thermal expansion, ΔT is the drop of temperature through the shell and g_0 is the gravity acceleration at $r = r_o$.

Eqs (1)-(4) are closed by the non-penetrating and no-slip boundary conditions for the velocity field at the rigid surfaces and fixed temperature boundary conditions. The regions outside the fluid shell are electrical insulators (the magnetic field on the boundaries matches with appropriate potential fields in the exterior that imply no external sources of the field).

The last term in Eq. (3), H , constitutes the internal heat sources (it is the non-dimensional volumetric heat source strength), which enables simulating

thermodynamically the various stratification of the spherical shells also in the Boussinesq models. The outer sphere was assumed to be stratified non-uniformly (it is divided into stably and unstably stratified sub-shells) with constant temperature $T_i = 1$ and $T_o = 0$ at the inner and outer boundaries of the shell, respectively. The width of both sub-shells was considered to be the same (we denote this case NON-UNI 50), i.e. $\partial T/\partial r$ changes its sign in the middle of the convective shell, $r_m = (r_i + r_o)/2$. It is accomplished if $H = -23P_r^{-1}$. Let us describe why we used this value. The non-uniform stratification is considered due to internal heat sources and in the basic state Eq.(3) gets the form

$$\frac{1}{P_r} \nabla^2 T + H = 0. \quad (5)$$

91 If we solve Eq. (5) considering $T_i = 1$, $T_o = 0$ and $\partial T/\partial r = 0$ at $r = r_m$,
 92 we obtain $H = -23P_r^{-1}$. It is necessary to remark that for the uniform
 93 stratification $H = 0$. In order to compare the case NON-UNI 50 to cases in
 94 which is the ratio of stable to unstable stratified sub-shells different, we study
 95 gradually cases where 5% (denoted NON-UNI 5), 25% (denoted NON-UNI
 96 25), 75% (denoted NON-UNI 75) and 95% (denoted NON-UNI 95) of the
 97 shell is stably stratified.

98 3. Numerical results

99 Eqs (1)-(4) were solved using the MAG dynamo code (Olson and Glatzmaier, 1995, 1996; Olson et al., 1999; Christensen et al., 1999, 2001; Olson
 100 and Glatzmaier, 2005; Christensen and Aubert, 2006). It is a serial version of
 101 Gary Glatzmaier's rotating spherical convection/magnetoconvection/dynamo
 102 code, modified by Ulrich Christensen and Peter Olson. The code solves the
 103 non-dimensional Boussinesq equations for time-dependent thermal convec-
 104 tion in a rotating spherical shell filled with an electrically conducting fluid.
 105 MAG uses toroidal-poloidal decomposition for velocity and magnetic field
 106 with explicit time steps. Linear terms are evaluated spectrally (spherical
 107 harmonics plus Chebyshev polynomials in radius) and nonlinear terms are
 108 evaluated on a spherical grid (for more details, see Olson and Glatzmaier,
 109 1995, 1996; Olson et al., 1999; Christensen et al., 1999, 2001; Olson and
 110 Glatzmaier, 2005). The computations were performed on the Sun Grid En-
 111 gine (LUNA) at the Institute of Physics, Academy of Sciences of CR, Prague,
 112

113 and the Nemo cluster (SGI) at the Institute of Geophysics, Academy of Sci-
114 ences of CR, Prague.

115 The dependence of solutions on various values of the Rayleigh number, R_a ,
116 for the given Prandtl number, P_r , magnetic Prandtl number, P_m , and Ekman
117 number, E , was investigated. Computations started from zero initial velocity
118 and strong dipole-dominated field with $B \sim \mathcal{O}(1)$, and were performed for
119 $P_r = 1$, $E = 10^{-4}$, $P_m = 2$, and $R_a = 550, 650, 800, 900, 1000$. In the case
120 of non-uniform stratification $H = -2.41$ (NON-UNI 5), $H = -4.60$ (NON-
121 UNI 25), $H = -23$ (NON-UNI 50), $H = 15.9$ (NON-UNI 75) and $H = 8.96$
122 (NON-UNI 95) as $P_r = 1$ and in the case of uniform one, $H = 0$ (in this case
123 the whole shell is unstably stratified).

124 For $P_r = 1$, the characteristic thermal diffusion time is equal to the char-
125 acteristic viscous diffusion time ($\tau_\kappa = \tau_\nu$), i.e. thermal and viscous diffusion
126 processes affect equally the dynamics of convection and dynamo. The de-
127 pendences of the mean kinetic energy, E_k , and the mean magnetic energy,
128 E_m , on R_a and the type of stratification (uniform and non-uniform) are given
129 in Fig. 1. E_k increases for given E , P_r and P_m with increasing of R_a and
130 for a given R_a it is greater in the cases of non-uniform stratification than in
131 the case of uniform one. E_k increases for a given R_a also with increasing of
132 the thickness of a stable stratified sub-shell, except the case NON-UNI 95
133 when it decreases comparing to the case NON-UNI 75 (see Fig. 1). E_m also
134 increases for a given E , P_r and P_m with increasing of R_a , and for a given
135 R_a , it is greater in the case of uniform stratification than in the cases of non-
136 uniform one. E_m decreases for a given R_a with increasing of the thickness of
137 a stable stratified sub-shell, even in the case NON-UNI 95 the dynamo fails
138 for $t > 10$ (see Fig. 1). Thus, the dependences of E_k and E_m on the type of
139 stratification are strong.

140 The typical space distributions of the radial magnetic field component,
141 B_r are presented in Fig. 2, the radial velocity field component, V_r in Fig.
142 3, and, finally, the typical space distributions of the complete magnetic field
143 $\mathbf{B}$ at CMB are presented in Fig. 4. All the figures are snapshots done at
144 time $t = 10$ (10 time units). Fig. 2 and 3 were done using the value of
145 $r = 0.65$, which lies in the case NON-UNI 50 in the unstable region but close
146 to the interface $r = r_m$ ($r_m = 0.675$). We used this value in order to compare
147 magnetic and velocity fields in the same region for different thicknesses of
148 stable stratified sub-shells. However, Fig. 4 was done using $r = r_o$ (at CMB).

149 The generated magnetic field is dipolar in all investigated cases except
150 the cases NON-UNI 75 and NON-UNI 95 when it is non-dipolar, even for

Figure 1: Dependences of the mean kinetic energy, E_k , (top) and the mean magnetic energy, E_m , (bottom) on the Rayleigh number, R_a , and the type of stratification (uniform - UNI and non-uniform - NON-UNI 5,25,50,75,95) for $E = 10^{-4}$, $P_r = 1$ and $P_m = 2$.

Figure 2: Space distributions of radial magnetic field components B_r at $r = 0.65$ for $R_a = 550, 650, 800, 900$ (from left to right), for the cases of uniform stratification, NON-UNI 5, NON-UNI 25, NON-UNI 50 and NON-UNI 75 (from top to bottom), and for $E = 10^{-4}$, $P_r = 1$ and $P_m = 2$. Red (blue) colours indicate positive (negative) values. Minimal and maximal values for the first row are $(-4.9, 4.9)$, $(-8.6, 8.6)$, $(-10, 10)$, $(-12, 12)$, for the second row $(-5.7, 5.7)$, $(-8.3, 8.3)$, $(-11, 11)$, $(-8.0, 8.0)$, for the third row $(-6.9, 6.9)$, $(-5.7, 5.7)$, $(-9.1, 9.1)$, $(-10, 10)$, for the fourth row $(-5.1, 5.1)$, $(-9.2, 9.2)$, $(-8.8, 8.8)$, $(-12, 12)$ and for the fifth row $(-3.0, 3.0)$, $(-3.1, 3.1)$, $(-8.5, 8.5)$, $(-6.2, 6.2)$. Snapshots at $t = 10$.

Figure 3: Space distributions of radial velocity field components V_r at $r = 0.65$ for $R_a = 550, 650, 800, 900$ (from left to right), for the cases of uniform stratification, NON-UNI 5, NON-UNI 25, NON-UNI 50 and NON-UNI 75 (from top to bottom), and for $E = 10^{-4}$, $P_r = 1$ and $P_m = 2$. Red (blue) colours indicate positive (negative) values. Minimal and maximal values for the first row are $(-165, 165)$, $(-123, 123)$, $(-291, 291)$, $(-276, 276)$, for the second row $(-185, 185)$, $(-128, 128)$, $(-171, 171)$, $(-232, 232)$, for the third row $(-158, 158)$, $(-119, 119)$, $(-251, 251)$, $(-228, 228)$, for the fourth row $(-323, 323)$, $(-250, 250)$, $(-197, 197)$, $(-276, 276)$ and for the fifth row $(-286, 286)$, $(-397, 397)$, $(-366, 366)$, $(-387, 387)$. Snapshots at $t = 10$.

Figure 4: The complete magnetic field at the outer surface of the shell (at $r = r_o$) for $R_a = 550$ (left column) and $R_a = 900$ (right column), for the cases of uniform stratification, NON-UNI 5, NON-UNI 25, NON-UNI 50 and NON-UNI 75 (from top to bottom), and for $E = 10^{-4}$, $P_r = 1$ and $P_m = 2$. Red (blue) colours indicate positive (negative) values. Minimal and maximal values for the first row are $(-7.3, 7.3)$, $(-17, 17)$, for the second row $(-8.5, 8.5)$, $(-13, 13)$, for the third row $(-11, 11)$, $(-16, 16)$, for the fourth row $(-7.6, 7.6)$, $(-17, 17)$ and for the fifth row $(-4.4, 4.4)$, $(-9.3, 9.3)$. Snapshots at $t = 10$.

151 the NON-UNI 95 a dynamo fails (see Figs. (2,4) but the case NON-UNI
 152 95 is not presented because of failed dynamo). The strong antisymmetric
 153 character of magnetic field about the equator (a typical feature of the dipole
 154 dominated magnetic fields) is visible in the case of uniform stratification and
 155 if the thickness of a stable stratified sub-shell is smaller/equal than/to the
 156 thickness of an unstable stratified sub-shell (see Figs. (2,4)). Increasing the
 157 value of R_a small-scale structures are more visible.

158 The symmetric character of the velocity field about the equator and the
 159 spiralling cross section of the columns are typical features for both cases
 160 of stratification (see Fig. 3). For low values of R_a in the case of uniform
 161 stratification, a large-scale columnar convection is developed in the whole
 162 volume, while in the case of non-uniform stratification it is shifted out of CMB
 163 (suppressed to the unstably stratified region) but penetrates to the stably
 164 stratified one. A measure of such a penetration is governed by the value of the
 165 Brunt-Väisälä frequency (e.g. Takehiro and Lister, 2001, 2002). Increasing
 166 the thickness of a stable stratified sub-shell such a shifting becomes stronger
 167 (the convection is more and more suppressed to the unstably stratified region
 168 and less and less penetrates to the stably stratified region). In addition,
 169 increasing the thickness of a stable stratified sub-shell it is possible to observe
 170 a rise in the breakdown of the columnar character of the convection (see Fig.
 171 3). A columnar structure of the convection is believed to be important in the
 172 generation of strong dipolar magnetic fields (Olson et al., 1999; Christensen
 173 and Aubert, 2006; Christensen and Wicht, 2007). Increasing R_a the small
 174 scales become important but the convection is still columnar. It is possible
 175 to observe in the stably stratified sub-shell a weak tendency of flow to be
 176 more toroidal.

177 The radial stratification causes the toroidal motions in the outermost part
 178 of the shell (“teleconvection”). It cannot take place without such a radial
 179 stratification and is possible if the unstable stratification dominates over the
 180 stable one (Zhang and Schubert, 2000; Busse and Simitev, 2005; Šimkanin et
 181 al., 2010). The multilayer convection (“teleconvection”) is developed only in
 182 the cases NON-UNI 5 and NON-UNI 25 and is more visible for larger values
 183 of R_a (see Fig. 3, especially the case NON-UNI 25 for $R_a = 900$). However,
 184 the multilayer convection (“teleconvection”) is not developed in the cases
 185 NON-UNI 50, NON-UNI 75 and NON-UNI 95 because of the significant
 186 amount of stable stratification (see Fig. 3).

187 4. Conclusions

188 Density stratification plays a significant role in the astrophysical and geo-
 189 physical processes. It is an important dynamic factor in the processes acting
 190 in planetary atmospheres (e.g., Heimpel et al., 2005), planetary interiors
 191 (e.g., Braginsky, 1964; Stanley and Bloxham, 2004, 2006; Zhang and Schu-
 192 bert, 2000; Stanley and Mohammadi, 2008; Christensen, 2006; Šimkanin et
 193 al., 2003, 2006), etc. Systematical parameter studies of rotating convection
 194 and dynamo in uniformly and non-uniformly stratified spherical shells were
 195 done in many previous studies (e.g., Zhang and Schubert, 2000; Takehiro and
 196 Lister, 2001, 2002; Stanley and Bloxham, 2004, 2006; Stanley and Moham-
 197 madi, 2008; Christensen, 2006; Christensen and Wicht, 2008). However, our
 198 attention is focused on the case when the thickness of both sublayers (stable
 199 and unstable) is the same. It is true that neither planet nor satellite with this
 200 geometric configuration has been observed. Nevertheless, it is interesting to
 201 investigate such a case in order to answer a question whether its behaviour
 202 is closer to the case of a stable stratification or an unstable one.

203 We investigated in Šimkanin et al. (2010) a convection in rotating non-
 204 uniformly stratified spherical fluid shells in dependence on the Ekman and
 205 Prandtl numbers, and the change of the sign of temperature gradient was
 206 located to the middle of the convective shell. The investigations provided
 207 the following main results. The multilayer convection (“teleconvection”) is
 208 not developed in our case of non-uniform stratification, because of the signif-
 209 icant amount of stable stratification. The case characterized by the identical
 210 thicknesses of both sublayers is a typical extreme case. It is dissimilar to
 211 the case where the stably stratified sublayer is thinner than the unstably
 212 stratified one (although the convection penetrates to the stably stratified
 213 sublayer, it is more suppressed to the unstably stratified sublayer than in
 214 the case of thinner stably stratified sublayer) and also dissimilar to the case
 215 where the unstably stratified sublayer is thinner than the stably stratified one
 216 (although the convection is significantly suppressed to the unstably stratified
 217 sublayer, still penetrates to the stably stratified sublayer more than in the
 218 case of thinner unstably stratified sublayer). The Prandtl number was the
 219 most significant dynamic factor (for more details, see Šimkanin et al., 2010).

220 In the present paper we continue with a study of a dynamo action. In
 221 order to compare the case NON-UNI 50 to cases in which is the ratio of stable
 222 to unstable stratified sub-shells different, we gradually studied the case of
 223 uniform stratification and then cases of non-uniform stratification increasing

the thickness of a stable stratified sub-shell in dependence on the Rayleigh number. The results showed that generated magnetic fields are in cases UNI, NON-UNI 5, NON-UNI 25 and NON-UNI 50 dipolar, in the case NON-UNI 75 non-dipolar and in the case NON-UNI 95 the dynamo failed. Magnetic fields become weak with increase of the thickness of a stable stratified sub-shell (the dipolar magnetic field in the case NON-UNI 50 is weaker than in the case NON-UNI 5 but it is stronger than the non-dipolar magnetic field in the case NON-UNI 75) because the stably stratified sublayer acts to destabilize the dynamo (Stanley and Mohammadi, 2008). Let us compare the results of Christensen (2006); Christensen and Wicht (2008) with our results. In their models 60% of the shell is stably stratified (it could be the case “NON-UNI 60” in our denotation). In such a case the dynamo action is strongly suppressed in the upper stably stratified sublayer, i.e. the dynamo runs in the small unstably stratified sublayer (close to ICB). Such weak dynamo action and skin-effect (the magnetic field generated in the unstably stratified sublayer permeates through the stably stratified sublayer where it is damped due to skin-effect) lead to the weak magnetic field observed on the surface of Mercury (Christensen, 2006). A large-scale columnar convection is in the case of non-uniform stratification shifted out of CMB (suppressed to the unstably stratified region) but penetrates to the stably stratified one. The measure of such a penetration is governed by the value of the Brunt-Väisälä frequency (e.g. Takehiro and Lister, 2001, 2002). From this point of view a stratified layer acts as a filter of the convection. Increasing the thickness of a stable stratified sub-shell such a shifting becomes strong. Increasing R_a the small scales become important but the convection is still columnar. It is possible to observe in the stably stratified sub-shell a weak tendency of flow to be more toroidal. The multilayer convection (“teleconvection”) is developed only in the cases NON-UNI 5 and NON-UNI 25 but it is not developed in the cases NON-UNI 50, NON-UNI 75 and NON-UNI 95 because of the significant amount of stable stratification.

Our results support previous analyses done in the uniformly or non-uniformly stratified spherical shells (e.g., Zhang and Schubert, 2000; Takehiro and Lister, 2001, 2002; Šimkanin et al., 2003, 2006; Busse and Simitev, 2005; Christensen, 2006; Christensen and Wicht, 2007, 2008; Stanley and Bloxham, 2004, 2006; Stanley and Mohammadi, 2008). In Šimkanin et al. (2010), the rotating convection, the results showed that the case characterized by the identical thicknesses of both sublayers is a typical extreme case. However, the dynamo action in the shell characterized by the identical thicknesses of

both sublayers (NON-UNI 50) is not the typical extreme case. The generated magnetic field is close to the cases of uniform stratification and non-uniform one if the thickness of a stable stratified sub-shell is smaller than the thickness of an unstable stratified sub-shell. From this point of view, the case “NON-UNI 60” described in Christensen (2006); Christensen and Wicht (2008) could be an extreme case because it separates two different dynamos (dipolar (NON-UNI 50) / non-dipolar (NON-UNI 75)).

Acknowledgements

This research has been supported by the Grant Agency of the Academy of Sciences of the Czech Republic (Grant No. IAA300120704). We would like to thank G. Glatzmaier, U. Christensen, P. Olson, and CIG for MAG dynamo code, and the Institute of Physics of Academy of Sciences of CR, Prague for CPU time on the Sun Grid Engine (the Luna project).

References

- Braginsky, S., 1964. Magnetohydrodynamics of the Earth’s Core. *Geomagn. Aeron.* 4, 898–916 (Engl. Transl. 698–712).
- Busse, F.H., Simitev, R., 2005. Convection in rotating spherical fluid shells and its dynamo states. In: Soward, A.M., Jones, C.A., Hughes, D.W., Weiss, N.O. (Eds.), *Fluid Dynamics and Dynamos in Astrophysics and Geophysics*, 359–392, CRC Press, New York, USA.
- Christensen, U.R., Olson, P., Glatzmaier, G.A., 1999. Numerical modelling of the geodynamo: a systematic parameter study. *Gephys. J. Int.* 138, 393–409.
- Christensen, U.R., Aubert, J., Cardin, P., Dormy, E., Gibbons, S., Glatzmaier, G.A., Grote, E., Honkura, Y., Jones, C., Kono, M., Matsushima, M., Sakuraba, A., Takahashi, F., Tilgner, A., Wicht, J., Zhang, K., 2001. A numerical dynamo benchmark. *Phys. Earth Planet. Inter.* 128, 25–34.
- Christensen, U.R., 2006. A deep dynamo generating Mercury’s magnetic field. *Nature* 444, 1056–1058. doi:10.1038/nature05342

- Christensen, U.R., Aubert, J., 2006. Scaling properties of convection driven dynamos in rotating spherical shells and application to planetary magnetic fields. *Geophys. J. Int.* 166, 97–114.
- Christensen, U.R., Wicht J., 2007. Numerical dynamo simulations. In: Kono, M. (Ed.), Volume 8 - Core Dynamics, 245–282, Schubert, G. (Ed.-in-Chief), *Treatise on Geophysics*, Elsevier, Amsterdam, The Netherlands.
- Christensen, U.R., Wicht J., 2008. Models of magnetic field generation in partly stable planetary cores: Applications to Mercury and Saturn. *Icarus* 196, 16–34.
- Fearn, D.R., Loper, D.E., 1981. Compositional convection and stratification of the Earth's core. *Nature* 289, 393–394.
- Heimpel, M., Aurnou, J., Wicht, J., 2005. Simulation of equatorial and high-latitude jets on Jupiter in a deep convection model. *Nature* 438, 193–196. doi:10.1038/nature04208
- Jones, C.A., 2000. Convection-driven geodynamo models. *Phil. Trans. R. Soc. Lond. A* 358, 873–897.
- Olson, P., Glatzmaier, G.A., 1995. Magnetoconvection in a rotating spherical shell: structure of flow in the outer core. *Phys. Earth Planet Inter.* 92, 109–118.
- Olson, P., Glatzmaier, G.A., 1996. Magnetoconvection and thermal coupling of the Earth's core and mantle. *Phil. Trans. R. Soc. Lond. A* 354, 1413–1424.
- Olson, P., Christensen, U.R., Glatzmaier, G.A., 1999. Numerical modeling of the geodynamo: mechanisms of field generation and equilibration. *J. Geophys. Res.* 104(10), 383–404.
- Olson, P., Glatzmaier, G.A., 2005. Probing the geodynamo. *Scientific American* 292(2), 29–35.
- Roberts, P.H., Glatzmaier, G.A., 2000. Geodynamo theory and simulations. *Rev. Mod. Phys.* 72 (4), 1081–1123.
- Stanley, S., Bloxham, J., 2004. Convective-region geometry as the cause of Uranus' and Neptune's unusual magnetic fields. *Nature* 428, 151–153.

- 322 Stanley, S., Bloxham, J., 2006. Numerical dynamo models of Uranus' and
323 Neptune's magnetic fields. *Icarus* 184, 556–572.
- 324 Stanley, S., Mohammadi, A., 2008. Effects of an outer thin stably stratified
325 layer on planetary dynamos. *Phys. Earth Planet. Inter.* 168, 179–190.
- 326 Šimkanin, J., Brestenský, J., Ševčík, S., 2003. Problem of the rotating mag-
327 netoconvection in variously stratified fluid layer revisited. *Stud. Geophys.*
328 *Geod.* 47, 827–845.
- 329 Šimkanin, J., Brestenský, J., Ševčík, S., 2006. On hydromagnetic instabilities
330 and the mean electromotive force in a non-uniformly stratified Earth's core
331 affected by viscosity. *Stud. Geophys. Geod.* 50, 645–661.
- 332 Šimkanin, J., Hejda, P., Saxonbergová-Jankovičová, D., 2010. Convection in
333 rotating non-uniformly stratified spherical fluid shells in dependence on
334 Ekman and Prandtl numbers. *Phys. Earth Planet. Inter.* 178, 39–47.
- 335 Takehiro, S., Lister, J.R., 2001. Penetration of columnar convection into an
336 outer stably stratified layer in rapidly rotating spherical fluid shells. *Earth*
337 *Planet. Sci. Lett.*, 187, 357–366.
- 338 Takehiro, S., Lister, J.R., 2002. Surface zonal flows induced by thermal con-
339 vection trapped below a stably stratified layer in a rapidly rotating spher-
340 ical shell. *Geophys. Res. Lett.*, 29, 50–1–4.
- 341 Zhang, K., Schubert, G., 2000. Teleconvection: remotely driven thermal con-
342 vection in rotating stratified spherical layers. *Science* 290, 1944–1947.