Dynamic oligopoly with partial cooperation and antitrust threshold

Akio Matsumoto, Ugo Merlone, Ferenc Szidarovszky

Title: Dynamic oligopoly with partial cooperation and antitrust threshold

Authors: Akio Matsumoto, Ugo Merlone, Ferenc Szidarovszky

PII: S0167-2681(09)00213-3
DOI: doi:10.1016/j.jebo.2009.08.014
Reference: JEBO 2444

To appear in: *Journal of Economic Behavior & Organization*

Received date: 23-7-2007
Revised date: 24-8-2009
Accepted date: 29-8-2009

Please cite this article as: Matsumoto, A., Merlone, U., Szidarovszky, F., Dynamic oligopoly with partial cooperation and antitrust threshold, *Journal of Economic Behavior and Organization* (2008), doi:10.1016/j.jebo.2009.08.014

Dynamic oligopoly with partial cooperation and antitrust threshold

Akio Matsumoto [a], Ugo Merlone [b,*], Ferenc Szidarovszky [c]

[a] *Department of Economics, Chuo University, 742-1, Higashi-Nakano, Hachioji, Tokyo, 192-0393, Japan*

[b] *Department of Statistics and Applied Mathematics, University of Torino, Piazza Arbarello 8, Torino I-10122, Italy, phone +39 011 6705753, fax +39 011 6705783*

[c] *Systems and Industrial Engineering Department, University of Arizona, Tucson, AZ 85721-0020, U.S.A.*

Abstract

A general framework of partial cooperation and shareholding interlocks in oligopolies is first introduced, and then the best responses of the firms are determined. The monotonic dependence of the equilibrium industry output on the cooperation levels of the firms is proved. Conditions are given for the local asymptotic stability of the equilibrium which require sufficiently small speed of adjustments. Antitrust thresholds are then introduced into the model which may result in the loss of equilibrium or in the presence of multiple equilibria. The dynamic behavior of the associated dynamic models with adaptive output adjustments also becomes more complex: period-2 cycles may emerge and coexist with stationary states.

JEL code: C71

Key words: oligopolies, partial cooperation, shareholding interlocks, antitrust threshold

1 Introduction

Cournot oligopolies are the most frequently discussed economic models in the literature of mathematical economics. This research area was originated by Cournot (1838), and based on his pioneering work, many scientists have introduced and examined the different variants of the classical Cournot model. A

* Corresponding author.
Email address: merlone@econ.unito.it (Ugo Merlone).

Preprint submitted to JEBO 24 August 2009

comprehensive summary of earlier works is given in Okuguchi (1976) and their multiproduct extensions and some applications are presented in Okuguchi and Szidarovszky (1999). The most frequently discussed extensions are single-product models with product differentiation, multi-product oligopolies, labor-managed and rent-seeking games. The existence and uniqueness of the equilibria was the central research issue in the earlier years, and later the attention of scientists has been turned to the dynamic variants of these models. The asymptotic behavior of dynamic oligopolies became the main focus, first in the linear case and later nonlinearities have been introduced into the models. In most studies the total profits or the profits per labor units were the payoff functions of the firms, and no cooperation among the firms was assumed. In the noncooperative case the Nash-equilibrium is the solution of the static game and in most cases it is the steady state of the dynamic extensions. The equilibrium is a strategy vector such that no firm can improve its own payoff by unilaterally diverting from the equilibrium. However, the firms might be able to increase their payoffs by simultaneously moving away from the equilibrium, as it is well known in the case of prisoners' dilemma. Any simultaneous move of the firms requires some level of coordination, so some kind of cooperative effort has to be assumed. The cooperation of the firms may take many different forms, including information sharing, side payments, profit sharing, shareholding interlocks to mention a few. Cyert and DeGroot (1973) introduced the concept of partial cooperation, when each firm's payoff function is the sum of its own profit and certain proportions of the profits of its competitors. Chiarella and Szidarovszky (2005) examined dynamic oligopolies with partially cooperating firms under continuous time scales, their main focus was the loss of stability in the case of information delay. One common way to achieve partial cooperation is by cross shareholding.

The literature has considered the cross shareholding from different points of view. While Berglof and Perotti (1994) and Arikawa and Kato (2004) consider cross shareholding effect in terms of corporate governance, Flath (1991, 1992) and Merlone (2001) proved some results in terms of cartelizing effects.
Different profit formulations are considered when studying the collusive effects of cross-shareholding, see for instance Reynolds and Snapp (1986) and Flath (1992). Merlone (2007) introduces a different profit formulation and gives a common form to the different profit formulations which is similar to the partial cooperation described in Bischi et al. (2008). In this paper we will introduce a general framework that includes partial cooperation and different models of shareholding interlock. We will examine the dependence of the equilibrium on model parameters and the asymptotic properties of the dynamic extensions under discrete time scales.
The structure of the paper is the following. The general model will be introduced in Section 2 and the best responses of the firms will be determined. In Section 3 we will investigate the dependence of the equilibrium on model parameters, and the local dynamic behavior of the corresponding dynamic ex-

tensions will be discussed and illustrated in Section 4. Models with antitrust threshold will be introduced in Section 5 and global asymptotics will be investigated in Section 6. The last section concludes the paper.

2 The General Mathematical Model

Consider an n-firm single-product oligopoly without product differentiation. Oligopolies with product differentiation and with multi-product firms can be discussed in a similar way. Let x_k be the output of firm k, $p(\sum_{l=1}^{n} x_l)$ the inverse demand function and $C_k(x_k)$ the cost function of firm k. Then the profit of this firm can be given as the difference of its revenue and cost:

$$\varphi_k(x_1, \dots, x_n) = x_k p(\sum_{l=1}^{n} x_l) - C_k(x_k). \tag{1}$$

Assume first that the firms *partially cooperate* in the sense of Cyert and DeGroot (1973) and that γ_{kl} denotes the cooperation level of firm k toward any other firm l. Then it is assumed that the payoff of firm k is given as

$$\Pi_k = \varphi_k + \sum_{l \neq k} \gamma_{kl} \varphi_l, \tag{2}$$

where in addition to its own profit, firm k takes certain proportions of the profits of its competitors into account. It is usually assumed that $\gamma_{kl} \geq 0$ for all k and l, and $\sum_{l \neq k} \gamma_{kl} \leq 1$. However there are other cases when these conditions are violated (e.g. overhelping or damaging other firms).

Following the formulations used in the literature, we can formulate four different models for describing shareholding interlocks. Reynolds and Snapp (1982, 1986) propose and analyze two profit formulations. In the first case *joint ventures* are considered and the payoff of firm k is given as

$$\Pi_k = (1 - \sum_{l \neq k} \delta_{lk}) \varphi_k + \sum_{l \neq k} \delta_{kl} \varphi_l, \tag{3}$$

where δ_{kl} is the ownership interest of firm k in firm l. The ownership interest of firm l in firm k is represented in the first term by δ_{lk}. We also notice that $\sum_k \Pi_k = \sum_k \varphi_k$, so this formulation well represents profit sharing among the firms. Clearly, the multiplier of φ_k is positive, so maximizing (3) is equivalent to maximizing function (2) with

$$\gamma_{kl} = \frac{\delta_{kl}}{1 - \sum_{l \neq k} \delta_{lk}}. \tag{4}$$

The other formulation they propose assumes *partial equity interests*, then the payoff of firm k has the form

$$\Pi_k = \varphi_k + \sum_{l \neq k} \delta_{kl} \varphi_l, \tag{5}$$

which is the same as (2) with $\gamma_{kl} = \delta_{kl}$. In their analysis Reynolds and Snapp (1982) compare models (3) and (5) and claim that in the second formulation the manager of each firm considers its own interests in the other firms, even if they "are blind to the ownership by rivals". They validate this assumption for two reasons. First, they assume that these investments are too small to convey control and so the rivals have very small or no control on the business decisions. Second, since the firms are unaware of the interdependent nature of their output decisions, they are also unaware of the effects on the rivals of their claims on other firms profits.

If *indirect shareholding* (Flath 1991, 1992) is assumed, then the payoff of firm k has the form

$$\Pi_k = \varphi_k + \sum_{l \neq k} \delta_{kl} \Pi_l, \tag{6}$$

in which it is assumed that firm k maximizes its profit that includes its operating earnings φ_k and its return on equity holding in the other firms. We can easily rewrite equation (6) in the special form of (2) by introducing matrix $\mathbf{D} = (\delta_{kl})$ with $\delta_{kk} = 0$ for all k, and vectors $\boldsymbol{\varphi} = (\varphi_k)$ and $\boldsymbol{\Pi} = (\Pi_k)$. Equation (6) can be written in vector-matrix form as

$$\boldsymbol{\Pi} = \boldsymbol{\varphi} + \mathbf{D}\boldsymbol{\Pi}$$

from which we conclude that

$$\boldsymbol{\Pi} = (\mathbf{I} - \mathbf{D})^{-1} \boldsymbol{\varphi}. \tag{7}$$

Since $\sum_{l \neq k} \delta_{kl} < 1$ for all k, matrix $\mathbf{I} - \mathbf{D}$ has unit diagonal elements, nonpositive off-diagonal elements, and it is an M-matrix (see for example, Szidarovszky and Bahill, 1998). It is also well known that

$$(\mathbf{I} - \mathbf{D})^{-1} = \mathbf{I} + \mathbf{D} + \mathbf{D}^2 + \dots$$

So the diagonal elements of $(\mathbf{I} - \mathbf{D})^{-1}$ are greater than or equal to unity. Therefore the inverse matrix exists and is nonnegative. If b_{kl} denotes the (k, l)

element of $(\mathbf{I}-\mathbf{D})^{-1}$, then (7) has the form

$$\Pi_k = \sum_{l=1}^{n} b_{kl}\varphi_l.$$

Maximizing this function is equivalent to maximizing (2) with

$$\gamma_{kl} = \frac{b_{kl}}{b_{kk}} \tag{8}$$

for all k and l.

More recently, Merlone (2007), introduced a model considering *net indirect shareholding*, in which the profit of firm k is

$$\Pi_k = (1 - \sum_{l\neq k}\delta_{lk})(\varphi_k + \sum_{l\neq k}\delta_{kl}\Pi_l^G) \tag{9}$$

where the gross profit Π_l^G of the firms are defined implicitly by relations

$$\Pi_k^G = \varphi_k + \sum_{l\neq k}\delta_{kl}\Pi_l^G. \tag{10}$$

In this case both operating earnings and equity holdings are netted. We can show that this model is also equivalent to (2). By introducing vector $\mathbf{\Pi}^G = (\Pi_k^G)$, similarly to equation (6) we have

$$\mathbf{\Pi}^G = (\mathbf{I}-\mathbf{D})^{-\mathbf{1}}\boldsymbol{\varphi}, \tag{11}$$

so from (9) we get expression

$$\mathbf{\Pi} = \mathbf{\Delta}(\boldsymbol{\varphi} + \mathbf{D}\mathbf{\Pi}^G) \tag{12}$$

where

$$\mathbf{\Delta} = \mathrm{diag}(1 - \sum_{l\neq 1}\delta_{l1}, 1 - \sum_{l\neq 2}\delta_{l2}, \ldots, 1 - \sum_{l\neq n}\delta_{ln}).$$

Simple algebra shows that

$$\mathbf{\Pi} = \mathbf{\Delta}((\mathbf{I}-\mathbf{D})(\mathbf{I}-\mathbf{D})^{-1}\boldsymbol{\varphi} + \mathbf{D}(\mathbf{I}-\mathbf{D})^{-1}\boldsymbol{\varphi} = \mathbf{\Delta}(\mathbf{I}-\mathbf{D})^{-1}\boldsymbol{\varphi}, \tag{13}$$

that is, for all firms k,

$$\Pi_k = (1 - \sum_{l \neq k} \delta_{lk}) \sum_{l=1}^{n} b_{kl} \varphi_l. \tag{14}$$

Maximizing this function is equivalent to maximizing (2) with γ_{kl} given by (8).
As shown in Merlone (2007), it is possible to summarize the different profit formulations considered in the literature as follows:

	operating earning	operating earning and equity holding
net	*joint ventures* (3)	*net indirect shareholding* (9)
gross	*partial equity interests* (5)	*indirect shareholding* (6)

Nevertheless, we have proved that with all above formulations firm k maximizes function (2) with certain coefficients γ_{kl}, which are either the cooperation levels of the firms toward their competitors, or the joint ownership interests, or they are given by equation (8) using the elements of the inverse matrix $(\mathbf{I} - \mathbf{D})^{-1}$. We note that in the case of partial cooperation the firms consider their payoffs rather than their profits. In fact, their payoffs (2) represent their attitude of each firm towards its competitors by considering proportions of their interests in its own payoff. So in this case it is not necessary to require that $\sum_k \Pi_k = \sum_k \varphi_k$.

In the case of an oligopoly market the payoff of firm k has the form

$$\begin{aligned} \Pi_k &= x_k p(x_k + Q_k) - C_k(x_k) + \textstyle\sum_{l \neq k} \gamma_{kl}(x_l p(x_l + Q_l) - C_l(x_l)) \\ &= (x_k + S_k) p(x_k + Q_k) - C_k(x_k) - \textstyle\sum_{l \neq k} \gamma_{kl} C_l(x_l) \end{aligned} \tag{15}$$

where

$$Q_k = \sum_{l \neq k} x_l$$

and

$$S_k = \sum_{l \neq k} \gamma_{kl} x_l.$$

As it is usual in oligopoly theory, we assume that functions p and C_k ($k = 1, 2, \ldots, n$) are twice continuously differentiable, and that

(A) $p' < 0$;

(B) $p' + (x_k + S_k)p'' \leq 0$;

(C) $p' - C_k'' < 0$

for all k and feasible values of the relevant variables. These are the standard assumptions in the theory of concave oligopolies (Bischi et al., 2008). Under these conditions

$$\frac{\partial \Pi_k}{\partial x_k} = p + (x_k + S_k)p' - C_k' \tag{16}$$

and

$$\frac{\partial^2 \Pi_k}{\partial x_k^2} = 2p' + (x_k + S_k)p'' - C_k'' < 0, \tag{17}$$

so Π_k is strictly concave in x_k. If firm k has a finite capacity limit L_k, then with any fixed values of Q_k and S_k there is a unique *best response* of firm k which is denoted by $R_k(Q_k, S_k)$, and clearly

$$R_k(Q_k, S_k) = \begin{cases} 0, & \text{if} \quad p(Q_k) + S_k p'(Q_k) - C_k'(0) \leq 0 \\ L_k, & \text{if} \quad p(L_k + Q_k) + (L_k + S_k)p'(L_k + Q_k) - C_k'(L_k) \geq 0 \\ \overline{x}_k, & \text{otherwise,} \end{cases} \tag{18}$$

where $\overline{x}_k$ is the unique solution of the monotonic equation

$$p(x_k + Q_k) + (x_k + S_k)p'(x_k + Q_k) - C_k'(x_k) = 0 \tag{19}$$

inside interval $(0, L_k)$. Assuming interior best response, implicit differentiation shows that

$$R'_{kQ_k} = -\frac{p' + (x_k + S_k)p''}{2p' + (x_k + S_k)p'' - C_k''} \in (-1, 0] \tag{20}$$

and

$$R'_{kS_k} = -\frac{p'}{2p' + (x_k + S_k)p'' - C_k''} < 0. \tag{21}$$

A strategy vector (x_k^*) is an *equilibrium* if and only if for all k,

$$0 \leq x_k^* \leq L_k;$$

and

$$x_k^* = R_k(\sum_{l \neq k} x_l^*, \sum_{l \neq k} \gamma_{kl} x_l^*).$$

The existence of an equilibrium is a simple consequence of the Nikaido-Isoda theorem (see for example, Forgo et al., 1999). The uniqueness of the equilibrium however is not guaranteed in general. Later in Example 3 we will show a case with multiple equilibria. It is well-known that the noncooperative equilibrium is always unique (Bischi et al., 2008). In order to develop a practical method to determine equilibria we will rewrite the best response functions as functions of the total output of the industry and S_k. From (18) we have with the notation $Q = \sum_{l=1}^n x_l$,

$$\overline{R_k}(Q, S_k) = \begin{cases} 0, & \text{if} \quad p(Q) + S_k p'(Q) - C_k'(0) \leq 0 \\ L_k, & \text{if} \quad p(Q) + (L_k + S_k)p'(Q) - C_k'(L_k) \geq 0 \\ \overline{\overline{x_k}}, & \text{otherwise,} \end{cases} \tag{22}$$

where $\overline{\overline{x}}_k$ is the unique solution of equation

$$p(Q) + (x_k + S_k)p'(Q) - C_k'(x_k) = 0 \tag{23}$$

inside interval $(0, L_k)$. The derivative of the left hand side with respect to x_k is $p'(Q) - C_k''(x_k) < 0$, so it is strictly decreasing. By assuming interior optimum, implicit differentiation shows, that

$$\overline{R}'_{kQ} = -\frac{p' + (x_k + S_k)p''}{p' - C_k''} \leq 0 \tag{24}$$

and

$$\overline{R}'_{kS_k} = -\frac{p'}{p' - C_k''} < 0. \tag{25}$$

In the first two segments $\overline{R_k}$ is constant and is continuous everywhere, so it is also decreasing in both variables Q and S_k everywhere.

In order to compute the equilibrium we have to solve the nonlinear system of algebraic equations

$$\begin{aligned}&\textstyle\sum_{l\neq k}\gamma_{kl}\overline{R}_l(Q,S_l)=S_k \quad (1\le k\le n),\\ &\textstyle\sum_{l=1}^{n}\overline{R}_l(Q,S_l)=Q\end{aligned} \tag{26}$$

for unknowns $S_1, S_2, \ldots, S_n$ and Q, and then the equilibrium outputs are $x_k^* = \overline{R}_k(Q^*, S_k^*)$, where $S_1^*, S_2^*, \ldots, S_n^*, Q^*$ are solutions of system (26). By using the monotonicity of the left hand side of equation (26) a monotonic iteration method can be used to solve these equations (see for example, Argyros and Szidarovszky, 1993).

3 Dependence of the Equilibrium on Model Parameters

The noncooperative equilibrium is obtained by selecting $\gamma_{kl} = 0$ for all k and l, and the totally cooperative equilibrium with $\gamma_{kl} = 1$ for all k and l. In the noncooperative case $S_k = 0$ for all firms. Let Q and Q^* denote the industry outputs in the noncooperative case and any partial cooperative case, respectively. We will first show that $Q^* \le Q$, that is, partial cooperation or shareholding interlocks have a decreasing effect on the equilibrium industry output. In contrary, assume that $Q^* > Q$, then

$$Q^* = \sum_{k=1}^{n}\overline{R}_k(Q^*, S_k^*) \le \sum_{k=1}^{n}\overline{R}_k(Q^*, 0) \le \sum_{k=1}^{n}\overline{R}_k(Q, 0) = Q,$$

which is an obvious contradiction.

In order to compare industry equilibrium outputs with different cooperation levels we make the simplifying assumption that $\gamma_{kl} \equiv \gamma_k$, that is, each firm treats all of its competitors equally. In this special case we can rewrite the best response expression (22) as follows:

$$\overline{R_k}(Q,\gamma_k) = \begin{cases} 0, & \text{if} \quad p(Q) + \gamma_k Q p'(Q) - C_k'(0) \le 0 \\ L_k, & \text{if} \quad p(Q) + (\gamma_k Q + (1-\gamma_k)L_k)p'(Q) - C_k'(L_k) \ge 0 \\ \overline{\overline{x_k^*}}, & \text{otherwise,} \end{cases} \tag{27}$$

where $\overline{\overline{x}}_k^*$ is the solution of equation

$$p(Q) + (\gamma_k Q + (1-\gamma_k)x_k)p'(Q) - C_k'(x_k) = 0 \tag{28}$$

in interval $(0, L_k)$. The left hand side of this equation is positive at $x_k = 0$, negative at $x_k = L_k$, and is strictly decreasing if we assume the following condition:

$$(C^{'}) \quad (1-\gamma_k)p^{'} - C_k^{"} < 0$$

for all k and feasible values of the relevant variables. Notice that $(C^{'})$ is slightly more restrictive than (C).
By implicit differentiation,

$$R_{kQ}^{'} = -\frac{(1+\gamma_k)p^{'} + (\gamma_k Q + (1-\gamma_k)x_k)p^{"}}{(1-\gamma_k)p^{'} - C_k^{"}} \leq 0$$

if we assume in addition to $(C^{'})$ a weaker version of condition (B), namely

$$(B^{'}) \quad (1+\gamma_k)p^{'} + (\gamma_k Q + (1-\gamma_k)x_k)p^{"} \leq 0.$$

Furthermore

$$R_{k\gamma_k}^{'} = -\frac{(Q-x_k)p^{'}}{(1-\gamma_k)p^{'} - C_k^{"}} \leq 0.$$

In the first two segments of (27) $\overline{R}_k$ is constant, and it is continuous in its entire domain, therefore it is (not necessarily strictly) decreasing in both variables Q and γ_k. We can now show the following result.

Theorem 1 *Assume that for all $k, \gamma_k^{(1)} \leq \gamma_k^{(2)}$, and let $Q^{(1)}$ and $Q^{(2)}$ be the industry outputs at the equilibrium with cooperation levels $\gamma_k^{(1)}$ and $\gamma_k^{(2)}$, respectively. If conditions $(A), (B^{'})$ and $(C^{'})$ hold in both cases, then $Q^{(1)} \geq Q^{(2)}$.*

Proof. In contrary, assume that $Q^{(1)} < Q^{(2)}$, then

$$\begin{aligned} Q^{(2)} &= \textstyle\sum_{k=1}^n \overline{R}_k(Q^{(2)}, \gamma_k^{(2)}) \leq \sum_{k=1}^n \overline{R}_k(Q^{(2)}, \gamma_k^{(1)}) \\ &\leq \textstyle\sum_{k=1}^n \overline{R}_k(Q^{(1)}, \gamma_k^{(1)}) = Q^{(1)}, \end{aligned}$$

which is a contradiction.

■

This result can be interpreted as any increase in the cooperation levels of the firms has a decreasing effect on the industry equilibrium output.

4 Local Stability Analysis

Assuming discrete time scales and adaptive output adjustments by the firms, the firms are assumed to adjust their outputs according to the dynamic rule

$$x_k(t+1) = x_k(t) + K_k[R_k(\sum_{l \neq k} x_l(t), \sum_{l \neq k} \gamma_{kl} x_l(t)) - x_k(t)] \tag{29}$$

for $k = 1, 2, \ldots, n$, where $0 < K_k \leq 1$ is the speed of adjustment of firm k. This scheme is known as partial adjustment towards best responses.

The local asymptotic behavior of this system depends on the eigenvalues of the Jacobian, which can be written as follows:

$$\mathbf{J} = \begin{pmatrix} 1 - K_1 & K_1(R'_{1Q_1} + \gamma_{12} R'_{1S_1}) & \ldots & K_1(R'_{1Q_1} + \gamma_{1n} R'_{1S_1}) \\ K_2(R'_{2Q_2} + \gamma_{21} R'_{2S_2}) & 1 - K_2 & \ldots & K_2(R'_{2Q_2} + \gamma_{2n} R'_{2S_2}) \\ \vdots & \vdots & & \vdots \\ K_n(R'_{nQ_n} + \gamma_{n1} R'_{nS_n}) & K_n(R'_{nQ_n} + \gamma_{n2} R'_{nS_n}) & \ldots & 1 - K_n \end{pmatrix} \tag{30}$$

where we assume that the equilibrium is not on the boundary between the cases of (18). All derivatives are taken at the equilibrium.

By using a slight modification of the proof presented in Chiarella and Szidarovszky (2005) we can show the following result.

Theorem 2 *Assume conditions* $(A), (B')$ *and* (C') *hold, and* $\gamma_{kl} \equiv \gamma_k$ *for all firms. Then the equilibrium is locally asymptotically stable if*

$$(i) \quad K_k < \frac{2}{1 + R'_{kQ_k} + \gamma_k R'_{kS_k}} \quad \textit{for all} \quad k$$

and

$$(ii) \quad \sum_{k=1}^{n} \frac{K_k(R'_{kQ_k} + \gamma_k R'_{kS_k})}{2 - K_k(1 + R'_{kQ_k} + \gamma_k R'_{kS_k})} > -1.$$

Remark. From equations (20) and (21) we see that $R'_{kQ_k} + \gamma_k R'_{kS_k} \in (-1, 0]$, so the denominator of condition (i) is always positive. Therefore conditions (i) and (ii) hold if the speeds of adjustments are sufficiently small. The right hand side of condition (i) is always at least two, so under realistic assumption on the K_k values, (i) always holds.

Example 1. Assume linear inverse demand function $p(Q) = A - BQ$ and linear cost functions, $C_k(x_k) = c_k x_k + d_k$. The payoff of firm k is given as

$$\begin{aligned}\Pi_k = & x_k(A - Bx_k - BQ_k) - (c_k x_k + d_k) + \\ & + \sum_{l \neq k} \gamma_k \left[x_l(A - Bx_k - BQ_k) - (c_l x_l + d_l) \right].\end{aligned} \tag{31}$$

Assuming interior optimum, the first order conditions imply that

$$A - 2Bx_k - BQ_k - c_k - BS_k = 0,$$

so

$$R_k(Q_k, S_k) = -\frac{1}{2}Q_k - \frac{1}{2}S_k + \frac{A - c_k}{2B}$$

with derivatives

$$R'_{kQ_k} = R'_{kS_k} = -\frac{1}{2}.$$

The conditions of Theorem 2 have now the following special forms:

$$(i) \quad K_k < \frac{2}{1 - \frac{1}{2}(1 + \gamma_k)} = \frac{4}{1 - \gamma_k} \tag{32}$$

and

$$(ii) - 1 < \sum_{k=1}^{n} \frac{-\frac{K_k}{2}(1 + \gamma_k)}{2 - K_k(1 - \frac{1}{2}(1 + \gamma_k))}$$

which can be rewritten as

$$\sum_{k=1}^{n} \frac{K_k(1 + \gamma_k)}{4 - K_k(1 - \gamma_k)} < 1. \tag{33}$$

The right hand side of (i) is at least 4, so it is always satisfied under realistic conditions. Condition (ii) holds if the K_k values are sufficiently small. As a special case assume symmetry, when $\gamma_k \equiv \gamma$ and $K_k \equiv K$. Then condition (ii) becomes

$$\frac{nK(1 + \gamma)}{4 - K(1 - \gamma)} < 1,$$

that is,

$$K < \frac{4}{n(1+\gamma)+(1-\gamma)}. \tag{34}$$

In the case of duopoly $n = 2$, so the right hand side equals $\frac{4}{3+\gamma}$, which is larger than 1 if $\gamma < 1$, so in this special case duopolies are always locally asymptotically stable.

▼

Example 2. Consider next the case of best response dynamics, when $K_k = 1$ for all firms. In this case condition (i) of Theorem 2 is always satisfied, and by introducing the notation $\alpha_k = R'_{kQ_k} + \gamma_k R'_{kS_k}$ condition (ii) simplifies as

$$\sum_{k=1}^{n} \frac{\alpha_k}{1-\alpha_k} > -1. \tag{35}$$

If we let $a_k = -\alpha_k \in [0, 1)$, then this inequality can be rewritten as

$$\sum_{k=1}^{n} \frac{a_k}{1+a_k} < 1. \tag{36}$$

Notice that for all k, $\frac{a_k}{1+a_k} < \frac{1}{2}$, so in the case of duopolies, the equilibrium is always locally asymptotically stable.

▼

5 Modified Model with Antitrust Threshold

Cartelizing effects of the financial interlocks have been proved in the literature (see, for instance Reynolds and Snapp, 1986, Flath, 1992 and Merlone, 2001) On the other hand this issue is not ignored by antitrust regulation as Clayton Act 7 forbids the acquisition of the “whole or any part” of the stocks or assets of a corporation where the effect may be substantially to lessen competition.

While a complete analysis of Partial Stock Acquisition can be found in ¶1203 of the multivolume analysis of antitrust principles by Areeda and Hovenkamp (2002) and Hovenkamp (2005), in the following we are interested in what can be the consequences on the dynamics when Antitrust’s possible actions are taken into account.

In our approach we assume that firms are concerned that aggregate quantity reduction could attract Antitrust attention. Therefore, in our model we assume that there exists a threshold $\bar{Q}$ under which firms are believed to act noncooperatively, so the authorities will not take action against them.

In our model the $\bar{Q}$ is assumed exogenous and common knowledge for the firms.

These assumptions are modeled by the following dynamic system:

$$x_k(t+1) = \begin{cases} x_k(t) + K_k[R_k(\sum_{l\neq k} x_l(t), 0) - x_k(t)] & \text{if} \quad \sum_{l=1}^n x_l(t) \leq \overline{Q} \\ x_k(t) + K_k[R_k(\sum_{l\neq k} x_l(t), \sum_{l\neq k} \gamma_{kl} x_l(t)) - x_k(t)] & \text{otherwise.} \end{cases} \tag{37}$$

Assume that $\mathbf{x}^* = (x_1^*, \ldots, x_n^*)$ is a steady state of this system. If $\sum_{l=1}^n x_l^* \leq \overline{Q}$, then $\mathbf{x}^*$ has to be the noncooperative equilibrium, and if $\sum_{l=1}^n x_l^* > \overline{Q}$, then $\mathbf{x}^*$ is a partially cooperative equilibrium. Based on this observation we have the following result.

Theorem 3 *Let $\mathbf{x}^{**} = (x_1^{**}, \ldots, x_n^{**})$ be the noncooperative equilibrium. If $\sum_{l=1}^n x_l^{**} \leq \overline{Q}$, then $\mathbf{x}^{**}$ is the unique steady state of system (20). Otherwise any partially cooperative equilibrium with total industry output greater than $\overline{Q}$ is a steady state. All steady states of system (20) can be obtained in this way.*

Example 3. Assume duopoly, $n = 2$. Let x and y be the outputs of the two firms which are not bounded above. The price function is $p(x+y) = A - B(x+y)$, the cost functions are $C_1(x) = c_1 x + d_1$ and $C_2(y) = c_2 y + d_2$. The profit of firm 1 without cooperation is given as

$$\Pi_1 = x(A - Bx - By) - (c_1 x + d_1) \tag{38}$$

with derivative

$$\frac{\partial \Pi_1}{\partial x} = A - 2Bx - By - c_1,$$

so the best response of firm 1 is as follows:

$$R_1(y, 0) = \begin{cases} 0 & \text{if } A - By - c_1 \leq 0 \\ -\frac{y}{2} + \frac{A - c_1}{2B} & \text{otherwise.} \end{cases} \tag{39}$$

Similarly, the best response of firm 2 is

$$R_2(x,0) = \begin{cases} 0 & \text{if } A - Bx - c_2 \leq 0 \\ -\frac{x}{2} + \frac{A-c_2}{2B} & \text{otherwise.} \end{cases} \tag{40}$$

Assume next partial cooperation among the firms with cooperation levels γ_1 and γ_2, respectively. Then the payoff of firm 1 is as follows:

$$\Pi_1 = x(A - Bx - By) - (c_1 x + d_1) + \gamma_1[y(A - Bx - By) - (c_2 y + d_2)] \tag{41}$$

with derivative

$$\frac{\partial \Pi_1}{\partial x} = A - 2Bx - By - c_1 - B\gamma_1 y = A - 2Bx - B(1+\gamma_1)y - c_1.$$

So the best response of firm 1 is given as

$$R_1(y,\gamma_1 y) = \begin{cases} 0 & \text{if } \quad A - B(1+\gamma_1)y - c_1 \leq 0 \\ -\frac{(1+\gamma_1)y}{2} + \frac{A-c_1}{2B} & \text{otherwise} \end{cases} \tag{42}$$

and similarly,

$$R_2(x,\gamma_2 x) = \begin{cases} 0 & \text{if } \quad A - B(1+\gamma_2)x - c_2 \leq 0 \\ -\frac{(1+\gamma_2)x}{2} + \frac{A-c_2}{2B} & \text{otherwise.} \end{cases} \tag{43}$$

The best response functions are illustrated in Figure 1 if both γ_1 and γ_2 are below unity. The noncooperative best response functions are obtained by selecting $\gamma_1 = \gamma_2 = 0$, so they have similar graphs. If γ_1 or γ_2 is zero, then the best response of the firm with zero cooperation level is the same as the best response function without cooperation. If $0 < \gamma_k \leq 1$ with some k, then the threshold, where the best response becomes zero, decreases, so the best response function in the positive domain also decreases. If at least one of γ_1 and γ_2 is below unity, then there is a unique intercept which is smaller in both coordinates than the noncooperative equilibrium. This gives an illustration of the general result proved in Section 3. Let us investigate the special case of $\gamma_1 = \gamma_2 = 1$ in more detail. Then the slope of the best response function in the positive domain is -1 for both firms, so these lines are either parallel or coincide. If they are parallel, then there is a unique boundary equilibrium (x or y equals zero). If they coincide, then there are infinitely many equilibria.

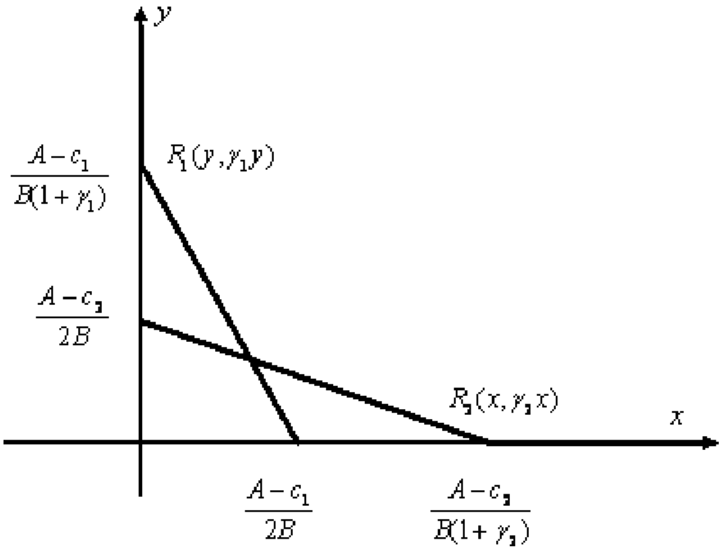


Fig. 1. Equilibrium with partial cooperation

This is the case when $c_1 = c_2$, that is, if the marginal costs of the firms are equal. In this case both firms maximize the common payoff function.

$$
\begin{aligned}
&x(A - Bx - By) - (cx + d_1) + y(A - Bx - By) - (cy + d_2) \\
&= (x + y)(A - B(x + y)) - c(x + y) - d_1 - d_2
\end{aligned}
$$

which is a concave parabola in $x + y$. This function has its maximum under nonnegativity assumption if

$$
x + y = \begin{cases} 0 & \text{if} \quad A - c \leq 0 \\ \frac{A-c}{2B} & \text{otherwise} \end{cases} .
$$

So in the case of $A \leq c, x^* = y^* = 0$ is the only equilibrium, and if $A > c$, then there are infinitely many equilibria which form the set

$$
\{(x^*, y^*) | 0 \leq x^* \leq \frac{A - c}{2B}, y^* = \frac{A - c}{2B} - x^*\}.
$$

If we drop the assumption that $\gamma_k \leq 1$, we may have multiple and finitely many equilibria. A such case with sufficiently large values of γ_1 and γ_2 is shown in Figure 2 with three equilibria. Assume now that $0 < \gamma_1 < 1$ and $0 < \gamma_2 < 1$, furthermore

$$
0 < \frac{A - c_2}{2} < A - c_1 < 2(A - c_2).
$$

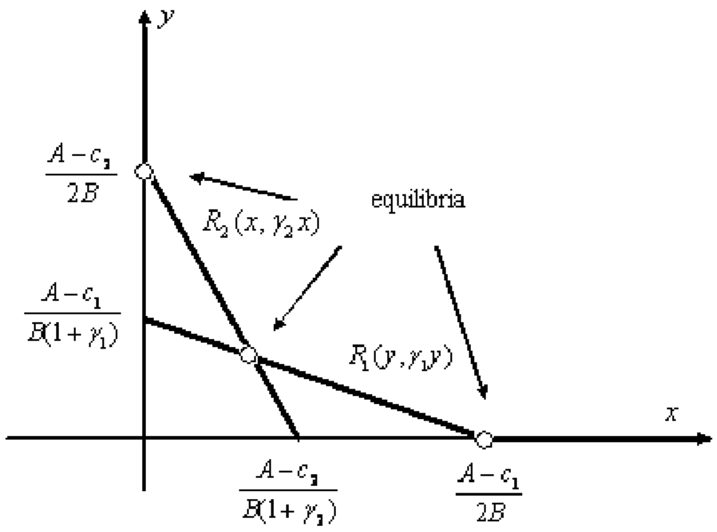


Fig. 2. The case of three equilibria

The noncooperative equilibrium is the intercept of the lines

$$x = -\frac{y}{2} + \frac{A - c_1}{2B} \quad \text{and} \quad y = -\frac{x}{2} + \frac{A - c_2}{2B},$$

which is

$$x^{**} = \frac{A + c_2 - 2c_1}{3B}, y^{**} = \frac{A + c_1 - 2c_2}{3B} \tag{44}$$

with total industry output

$$Q^{**} = \frac{2A - c_1 - c_2}{3B}. \tag{45}$$

The partially cooperative equilibrium is the intercept of the lines

$$x = -\frac{(1+\gamma_1)y}{2} + \frac{A - c_1}{2B} \quad \text{and} \quad y = -\frac{(1+\gamma_2)x}{2} + \frac{A - c_2}{2B}$$

which is

$$x^* = \frac{(1-\gamma_1)A - 2c_1 + (1+\gamma_1)c_2}{B(4 - (1+\gamma_1)(1+\gamma_2))}, \quad y^* = \frac{(1-\gamma_2)A - 2c_2 + (1+\gamma_2)c_1}{B(4 - (1+\gamma_1)(1+\gamma_2))} \tag{46}$$

with total industry output

$$Q^* = \frac{A(2 - \gamma_1 - \gamma_2) - (1-\gamma_2)c_1 - (1-\gamma_1)c_2}{B(4 - (1+\gamma_1)(1+\gamma_2))}. \tag{47}$$

A simple calculation shows that $Q^* < Q^{**}$. Now we have three possibilities. The first one occurs if $\overline{Q} < Q^*$, then system (41) has a unique steady state (x^*, y^*), while if $Q^* \leq \overline{Q} < Q^{**}$, then there is no steady state. Otherwise, that is, when $\overline{Q} \geq Q^{**}$, the unique steady state is (x^{**}, y^{**}).

6 Global Asymptotic Behavior

We now illustrate the global dynamics of the equilibrium in the special case of duopolies with linear price and cost functions, which was discussed earlier in Example 3. For the sake of simplicity the best responses in the noncooperative case are denoted by $R_1^N(y)$ and $R_2^N(x)$ (equation (43) and (44)), and in the partially cooperative case by $R_1^C(y)$ and $R_2^C(x)$ (equations (46) and (47)). The corresponding equilibrium coordinates are denoted by x^N, y^N and x^C, y^C (equations (48) and (50)).

Let us denote the nonnegative region of (x, y) as Ω and its two subregions Ω^N and Ω^C, each of which is defined as follows:

$$\Omega^N = \{(x, y) \in \Omega \;\Big|\; x + y \leq \bar{Q}\},$$

and

$$\Omega^C = \{(x, y) \in \Omega \;\Big|\; x + y > \bar{Q}\}.$$

The dynamic model with antitrust threshold $\bar{Q}$, that we call the hybrid system, is defined as follows:

$$T^N(x, y) = \{R_1^N(y),\ R_2^N(x)\} : \Omega^N \to \Omega, \tag{48}$$

and

$$T^C(x, y) = \{R_1^C(y),\ R_2^C(x)\} : \Omega^C \to \Omega. \tag{49}$$

The stationary point (x^N, y^N) is stable with respect to $T^N(x, y)$ and so is (x^C, y^C) with respect to $T^C(x, y)$. For the sake of analytical simplicity, we make the following assumption:

(D) $\gamma = \gamma_1 = \gamma_2$ and $0 < \gamma < 1$.

Define two constants Q^N and Q^C as sums of equilibrium outputs under non-cooperation and partial cooperation, respectively. From (49) and (51) we know that

$$Q^N = \frac{2A - (c_1 + c_1)}{3B} \left(\equiv x^N + y^N\right),$$

and

$$Q^C = \frac{2A - (c_1 + c_1)}{B(3+\gamma)} \left(\equiv x^C + y^C\right).$$

Then we have the following results on dynamics.

Theorem 4 *The hybrid dynamic system (52) and (53) has three distinctive dynamics, depending on the threshold value $\bar{Q}$: (i) no stationary state exists but a period-2 cycle emerges if $Q^C \leq \bar{Q} \leq Q^N$; (ii) a stable stationary state coexists with a period-2 cycle if $Q^N < \bar{Q} < \tilde{Q}^N$ or $\tilde{Q}^C < \bar{Q} < Q^C$ and (iii) a unique stable stationary state emerges if $\bar{Q} > \tilde{Q}^N$ or $\bar{Q} < \tilde{Q}^C$ where*

$$\tilde{Q}^N = \frac{2A - (c_1 + c_2)}{B(3-\gamma)} \quad \text{and} \quad \tilde{Q}^C = (1-\gamma)\frac{2A - (c_1 + c_2)}{B(3-\gamma)}.$$

Before proving the theorem some observations are in order. We denote the second equations of (43) and (44) by $f_1^N(y)$ and $f_2^N(x)$ and also the second equations of (46) and (47) by $f_1^C(y)$ and $f_2^C(x)$, respectively. Suppose first that $Q^N < \bar{Q}$. We can define the locus of (x, y) such that $f_1^N(y) + f_2^N(x) = \bar{Q}$ which is written as

$$x + y = q^N(\bar{Q})$$

where

$$q^N(\bar{Q}) = \frac{2A - (c_1 + c_2) - 2B\bar{Q}}{B}.$$

By the definition of the locus, it is clear that if

$$q^N(\bar{Q}) \lesseqqgtr x + y, \text{ then } f_1^N(y) + f_2^N(x) \lesseqqgtr \bar{Q}. \tag{50}$$

Then we can define the locus of (x, y) such that $f_1^C(y) + f_2^C(x) = q^N(\bar{Q})$ which can be written as

$$x + y = q^C(\bar{Q})$$

where

$$q^C(\bar{Q}) = \frac{-2A + (c_1 + c_2) + 4B\bar{Q}}{(3+\gamma)B}.$$

It is also clear that if

$$q^C(\bar{Q}) \lesseqgtr x + y, \text{ then } f_1^C(y) + f_2^C(x) \lesseqgtr q^N(\bar{Q}). \tag{51}$$

By (54) and (55), it can be seen that periodic points of a period-2 cycle, (x_I, y_I) and (x_{II}, y_{II}), must satisfy the following four conditions:

(a) $f_1^N(y_I) = x_{II}$ and $f_2^N(x_I) = y_{II}$,
(b) $f_2^C(y_{II}) = x_I$ and $f_2^C(x_{II}) = y_I$,
(c) $x_I + y_I < q^N(\bar{Q})$,
(d) $x_{II} + y_{II} > \bar{Q}$,

where $(x_I, y_I) < (x_{II}, y_{II})$ is assumed. The first two conditions imply that the periodic points are fixed points of $f_1^C(f_2^N(x)) = x$ and $f_2^C(f_1^N(y)) = y$:

$$x_I = x_{II} - \frac{\gamma(A - c_2)}{B(3-\gamma)}, \quad x_{II} = \frac{A + c_2 - 2c_1}{B(3-\gamma)}$$

and

$$y_I = y_{II} - \frac{\gamma(A - c_1)}{B(3-\gamma)}, \quad y_{II} = \frac{A + c_1 - 2c_2}{B(3-\gamma)}.$$

The last two conditions of (56) imply that (x_I, y_I) is mapped by the non-cooperative system, $T^N(x, y)$, since $(x_I, y_I) \in \Omega^N$, and (x_{II}, y_{II}) by the partially cooperative system, $T^C(x, y)$, since $(x_{II}, y_{II}) \in \Omega^C$. These conditions can be reduced to

$$\tilde{Q}^N > \bar{Q}, \tag{52}$$

where $\tilde{Q}^N$ is the fixed point of $q^C(Q) = Q$.

Proof of Theorem 4.

Case (iii) with $\tilde{Q}^N \leq \bar{Q}$. Conditions (c) and (d) above are not satisfied. Thus the dynamic system does not produce a period-2 cycle. Any trajectory is conveyed to region Ω^N where the stationary point (x^N, y^N) exists and $T^N(x, y)$ is the dynamic system. Therefore any trajectory converges to (x^N, y^N).

Case (ii) with $Q^N < \bar{Q} < \tilde{Q}^N$. Since $q^N(\bar{Q}) < \bar{Q}$, any point (x, y) such that $x+y > \bar{Q}$ is mapped to the regions where $x+y < q^N(\bar{Q})$ and thus the point is bounced back to the region where $x + y > \bar{Q}$. Such a point converges to one of two periodic points. If an initial condition is selected from the region bounded by $x + y < \bar{Q}$ and $x + y > q^N(\bar{Q})$, then points are mapped into the same region where the stationary state (x^N, y^N) exists and $T^N(x, y)$ is the dynamic system. Therefore any trajectory converges to the stationary state. This case has multistability.

Case (i) with $Q^C \leq \bar{Q} \leq Q^N$. (x^N, y^N) is in the region where $T^C(x, y)$ is the dynamic system while (x^C, y^C) is in the region where $T^N(x, y)$ is the dynamic system. Thus no convergence occurs. It can be confirmed that the above four conditions (a) to (d) are satisfied. So a period-2 cycle exists.

In the case of $\bar{Q} < Q^C$, Cases (ii) and (iii) can be proved in a similar way as above in which we define the locus of (x, y) as $f_1^C(y) + f_2^C(x) = \bar{Q}$ and the locus of (x, y) as $f_1^N(y) + f_2^N(x) = k^N(\bar{Q})$ with

$$k^N(\bar{Q}) = \frac{2A - (c_1 + c_2) - 2B}{B(1 + \gamma)}.$$

In this case the stationary point with partial cooperation (x^C, y^C) becomes the stable point.

■

In the following figures we illustrate some of the possible dynamics of system (41) in the case of duopoly with inverse demand function $p(x + y) = 10 - (x + y)$, cost functions $c_1(x) = 5x$, $c_2(y) = 5y$ and different cooperation level values. In Figure 3 cooperation values are $\gamma_1 = \gamma_2 = 0.5$ and the two downward sloping straight lines are the reaction functions in the noncooperative case. Antitrust threshold is $\overline{Q} = 3.9$. In this case we can observe the coexistence of a stationary state and a period two cycle from the initial states $(4.5, 0.2)$ $(1.0, 4.5)$, respectively. The light area is the basin of the period-2 cycle, and the dark area is the basin of the stationary state.

On the contrary, in Figure 4 all values but the antitrust threshold are the same. In particular, with $\overline{Q} = 3.2$, no steady state exists, there is a unique period-2 cycle, and we can see that from the same initial conditions only the period-2 cycle occurs. The two straight lines are $y = Q^C - x$ and $y = Q^N - x$ which are passing through the stationary points $\left(x^C, y^C\right)$ and $\left(x^N, y^N\right)$, respectively. In this case we can observe the stable period-2 cycle. The value k^C is defined similarly as k^N for the partially cooperative case.

Finally Figure 5 illustrates the special case of $\gamma_1 = \gamma_2 = 1$, where infinitely many equilibria coexist: the steady states are depicted on the down sloping

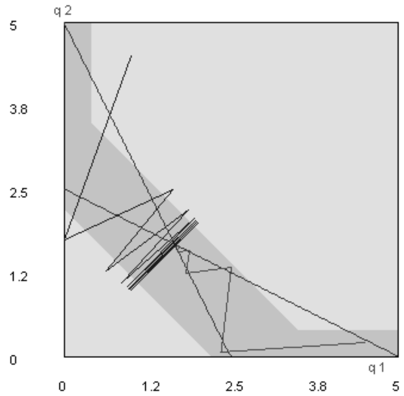


Fig. 3. Possible dynamics for $\bar{Q} < \tilde{Q}^N = 4$.

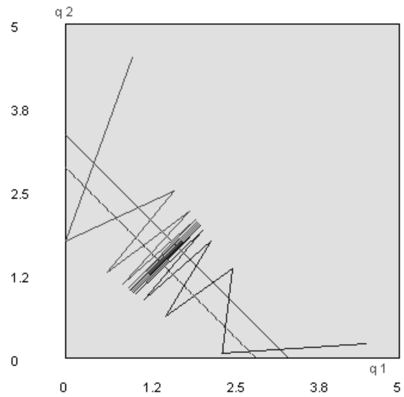


Fig. 4. Possible dynamics for $20/7 = k^C \leq \bar{Q} \leq k^N = 10/3$.

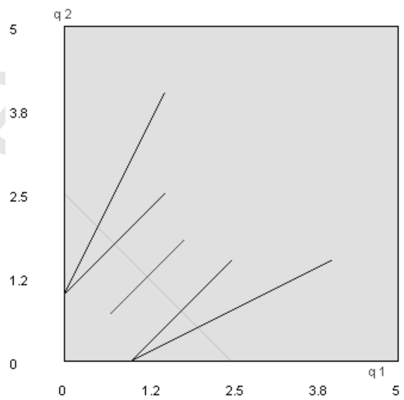


Fig. 5. Possible dynamics for different steady states.

straight line. There are infinitely many period-2 cycles, and trajectories always converge to one of them depending on the selection of the initial state.

7 Conclusion

A general framework of partial cooperation and shareholding interlocks were first introduced resulting in a special payoff structure in which the payoff of each firm is a sum of its profit and a linear combination of the profits of the competitors. The best response functions were then determined and the existence of the equilibrium proved. A simple example illustrates the possibility of multiple equilibria. Conditions were derived for the local asymptotical stability of the equilibria requiring that the speeds of adjustments of the firms be sufficiently small. These results are very similar to those known from the literature for concave, classical Cournot oligopolies.

The introduction of antitrust thresholds create a new situation: the possibility of the loss of equilibrium, and the presence of multiple equilibria. A complete description of the existence and the number of equilibria is presented. The associated dynamic models also show more complex asymptotic behavior. In the case of linear price and cost functions period-2 cycles emerge and they can coexist with stationary states.

The introduction of nonlinear price and cost functions into these models and their analysis is the subject of our future research.

References

Areeda, P.E., Hovenkamp, H., 2002. Antitrust Law. Vol. V (2nd Edition), New York, NY: Aspen Law and Business.

Argyros, I.K., F. Szidarovszky, F., 1993. The Theory and Applications of Interation Methods. Boca Raton/London: CRC Press.

Arikawa, Y., Kato, A., 2004. Cross shareholding and initiative effects. RIETI Discussion Paper Series, 04-E-017.

Berglof, E., Perotti, E., 1994. The governance structure of the Japanese financial keiretsu. Journal of Financial Economics 36, 259-284.

Bischi, G-I., Chiarella,C., Kopel, M., Szidarovszky, F., 2009. Nonlinear Oligopolies: Stability and Bifurcations. Amsterdam: Elsevier.

Chiarella, C., Szidarovszky, F., 2005. The complex asymptotic behavior of dynamic oligopolies with partially cooperating firms. Pure Mathematics and Applications 16(4), 365-375.

Cournot, A., 1838. Recherches sur les Principes Mathematiques de la Theorie des Richesses. Paris: Hachette (English translation (1960). Researches into the

Mathematical Principles of the Theory of Wealth. New York, NY: Kelly.

Cyert, R.M., DeGroot, M. H., 1973. An analysis of cooperation and learning in a duopoly context. American Economic Review 63(1), 24-37.

Dietzenbacher, E., Smid, B., Volkering, B., 2000. Horizontal intergration in the Dutch financial sector. International Journal of Industrial Organization 18, 1223-1242.

Flath, D., 1991. When is it rational for firms to acquire silent interests in rivals? International Journal of Industrial Organization 9, 573-583.

Flath, D., 1992. Horizontal shareholding interlocks. Managerial and Decision Economics 13, 75-77.

Forgo, F., Szep, J., Szidarovszky F., 1999. Introduction to the Theory of Games. Dordrecht: Kluwer Academic Publishers.

Hovenkamp, H., 2005. The Antitrust Enterprise. Cambridge, MA: Harvard University Press.

Merlone, U., 2001. Cartelizing effects of horizontal shareholding interlocks. Managerial and Decision Economics 22, 333-337.

Merlone, U., 2007. Shareholding interlocks: profit formulations and cartelizing effects. Journal of Interdisciplinary Mathematics 10(1), 99-117.

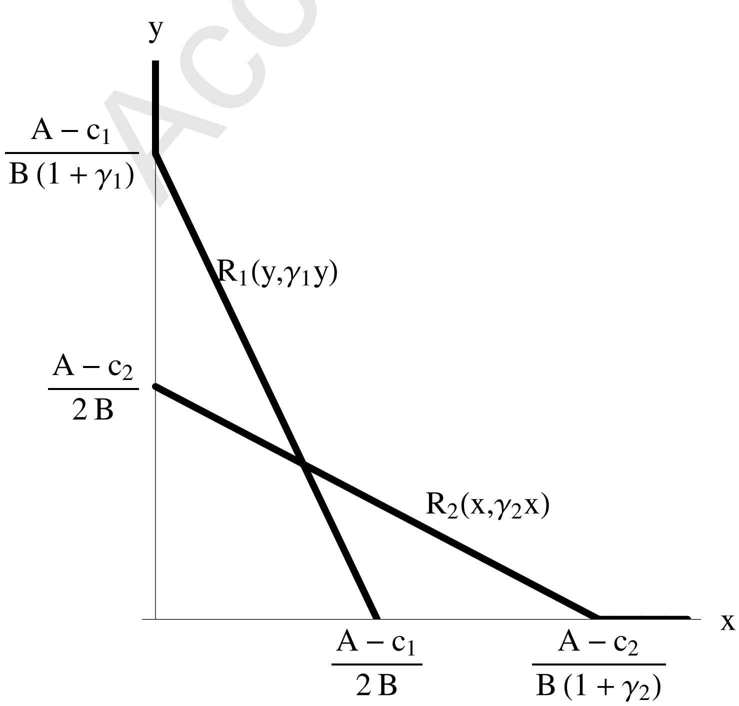
Okuguchi, K., 1976. Expectations and Stability in Oligopoly Models. Berlin/Heidelberg/New York: Springer-Verlag.

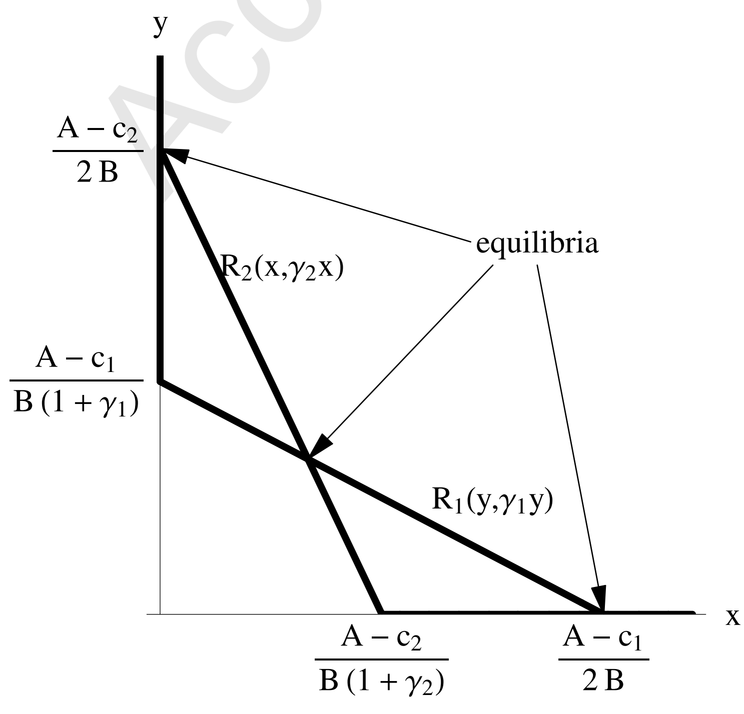
Okuguchi, K., Szidarovszky, F., 1999. The Theory of Oligopoly with Multi-product Firms (2nd Edition). Berlin/Heidelberg/New York: Springer-Verlag.

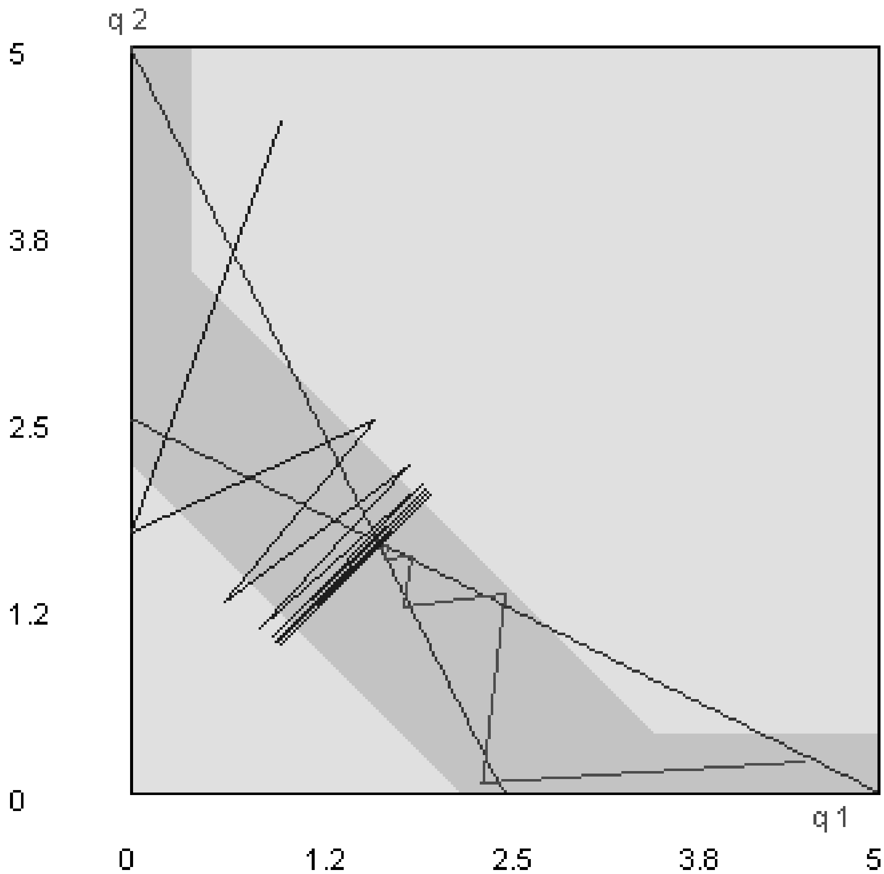
Reynolds, R.J., Snapp, B.R., 1982. The economic effects of partial equity interests and joint ventures. US Department of Justice, Antitrust Division, Economic Policy Office Discussion Paper #3.

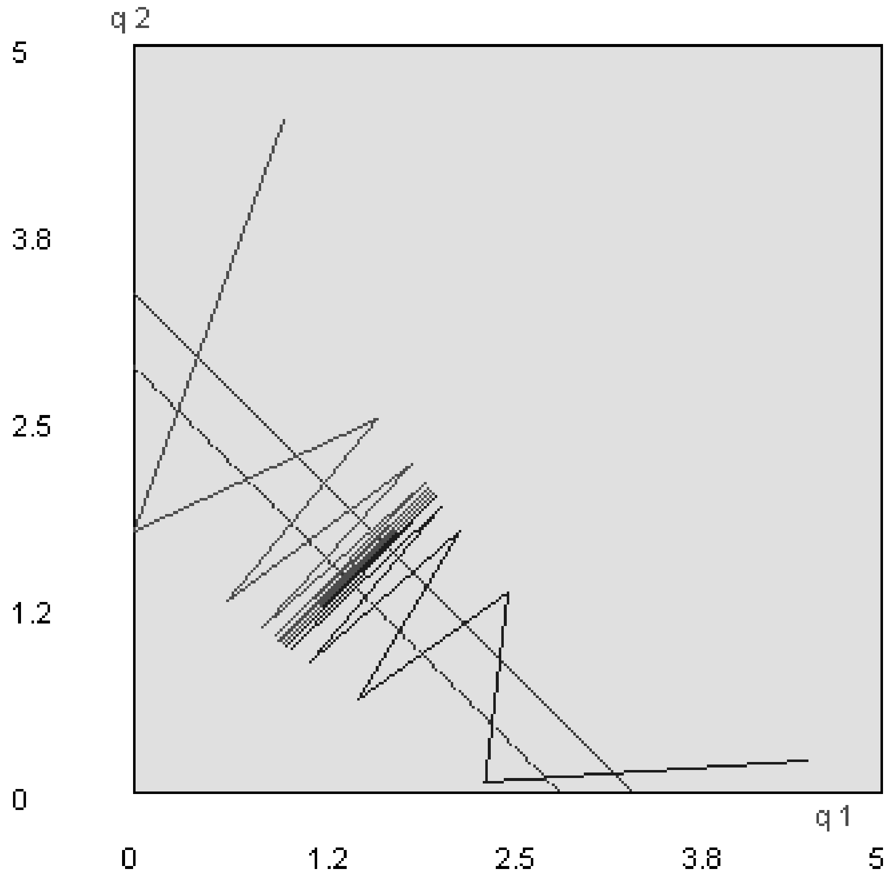
Reynolds, R.J., Snapp, B.R., 1986. The competitive effects of partial equity interest and joint ventures. International Journal of Industrial Organization 4, 141-153.

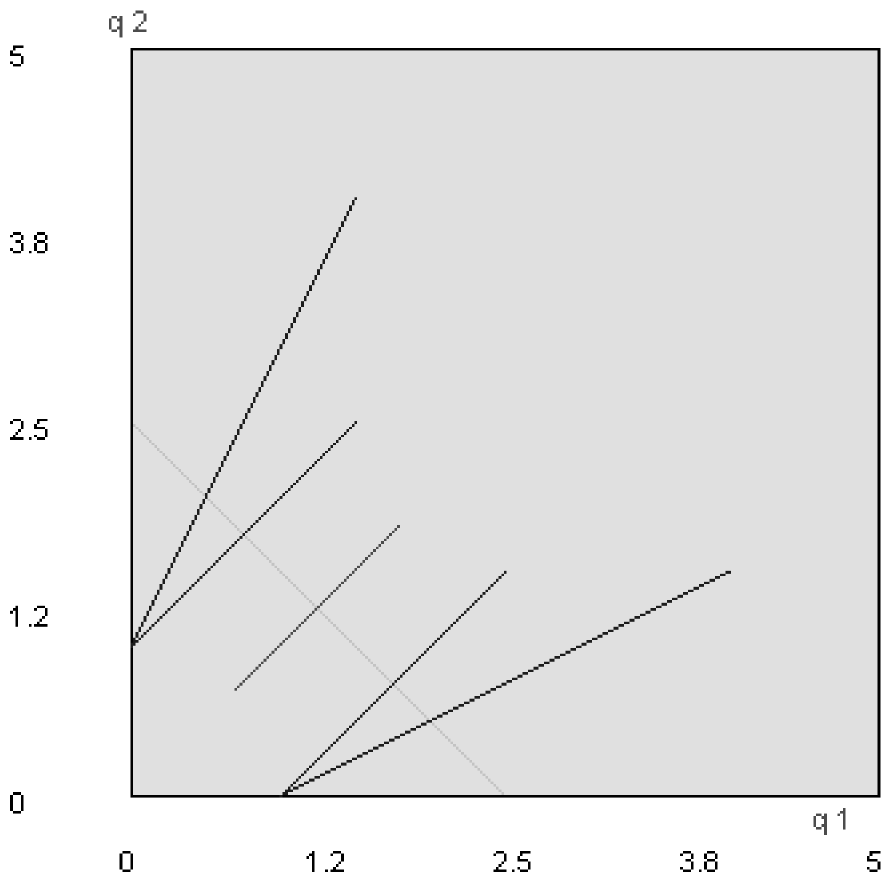
Szidarovszky, F., Bahill, T., 1998. Linear Systems Theory. Boca Raton/London: CRC Press.


y
$\frac{A - c_1}{B(1+\gamma_1)}$
$R_1(y, \gamma_1 y)$
$\frac{A - c_2}{2B}$
$R_2(x, \gamma_2 x)$
x
$\frac{A - c_1}{2B}$
$\frac{A - c_2}{B(1+\gamma_2)}$


y
A − c2
2 B
equilibria
R2(x,γ2x)
A − c1
B (1 + γ1)
R1(y,γ1y)
x
A − c2
B (1 + γ2)
A − c1
2 B


q 2
5
3.8
2.5
1.2
0
q 1
0
1.2
2.5
3.8
5


q 2
5
3.8
2.5
1.2
0
q 1
0
1.2
2.5
3.8
5


q 2
5
3.8
2.5
1.2
0
q 1
0
1.2
2.5
3.8
5

Dynamic oligopoly with partial cooperation and antitrust threshold

Akio Matsumoto [a], Ugo Merlone [b,*], Ferenc Szidarovszky [c]

[a] *Department of Economics, Chuo University, 742-1, Higashi-Nakano, Hachioji, Tokyo, 192-0393, Japan*

[b] *Department of Statistics and Applied Mathematics, University of Torino, Piazza Arbarello 8, Torino I-10122, Italy, phone +39 011 6705753, fax +39 011 6705783*

[c] *Systems and Industrial Engineering Department, University of Arizona, Tucson, AZ 85721-0020, U.S.A.*

Abstract

A general framework of partial cooperation and shareholding interlocks in oligopolies is first introduced, and then the best responses of the firms are determined. The monotonic dependence of the equilibrium industry output on the cooperation levels of the firms is proved. Conditions are given for the local asymptotic stability of the equilibrium which require sufficiently small speed of adjustments. Antitrust thresholds are then introduced into the model which may result in the loss of equilibrium or in the presence of multiple equilibria. The dynamic behavior of the associated dynamic models with adaptive output adjustments also becomes more complex: period-2 cycles may emerge and coexist with stationary states.

JEL code: C71

Key words: oligopolies, partial cooperation, shareholding interlocks, antitrust threshold

* Corresponding author.
Email address: `merlone@econ.unito.it` (Ugo Merlone).

Preprint submitted to JEBO *27 February 2009*