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The *RGM* data structure: a nominal interpretation of an acquired high point density model for automatic tolerance inspection

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In a previous paper (Di Angelo et al. 2010) we proposed an original methodology for the automation of the geometric inspection, starting from an acquired high-density surface. That approach performed a recognition process on the acquired data aiming at the identification of some *Intrinsic Nominal References*. An *Intrinsic Nominal Reference* was detected when a geometric property was recognised to be common to a set of adjacent points in the 3D data set representing the acquired object. The recognition of these properties was carried out based on some rules. Starting from these concepts, a new specification language was defined, which is based on recognisable geometric entities. This paper expands the category of *Intrinsic Nominal References* to include new mutual intrinsic orientation, location and dimensional properties pertaining to 3D features. This approach involves the automatic construction of a geometric reference model for a scanned workpiece, called *Recognised Geometric Model (RGM)*. The domain of the representable entities within the RGM strictly depends on the rules used for the recognition of the intrinsic properties. In particular, this paper focuses on the rules for the recognition of the orientation and location properties between non-ideal features. When using the *RGM*, tolerances are specified according to the set of available and recognisable intrinsic nominal references. Based on the geometric product specification, the *RGM* data structure can be queried to capture some quantitative information concerning special intrinsic geometric parameters and/or non-idealities.

Keywords: ISO Tolerancing, Three dimensional Metrology, Triangular Meshes, Shape recognition, Automated inspection

1. Introduction

Geometric inspection has actually become critical in the last few years due, on the one hand, to the increase in geometric complexity and product variety, and, on the other hand, to the evermore higher demands for geometric accuracy. Furthermore, and as far as some applications are concerned, such as, for instance, the aerospace engineering, each and every manufactured part, and not just a sample of them, needs inspecting. Consequently, a reliable, effective, and automated inspection system will improve the industrial manufacturing responsiveness so as to increase product competition. Most of these requirements (for example, the requirements for assemblability) can be expressed by using the geometric product specifications defined by the International Organization for Standardization (ISO) or by the ANSI/ASME. Some years ago, the ISO proposed a new language to express the geometric product specifications. This language, which is based on mathematical principles, is usually acronymed GPS (Geometric Product Specification).

Any language intended for tolerance specification must strictly be linked to the instruments available for the inspection. The GPS normative is mainly based on

the inspection capabilities of CMM (Coordinates Measurement Machine), gauges, dial gauges, etc. The concepts of tolerance and dimension also depend on the scheme of representation adopted to specify the geometric entities. Currently, in the GPS standards, these concepts refer to a 2D representation obtained by projecting the objects orthogonally onto a plane.

Nowadays, with the advent of high-resolution optical digitisers, new prospects are offered for real automatic geometric inspection (Gao et al. 2006) and, by extension, for tolerance specification (Srinivasan, 2001). The measurement process carried out by these high-resolution digitisers allows of a 3D acquisition of the real object. Many data points taken from surfaces of the inspected object (with a resolution up to 10 μm) allow us to define both new tolerance categories and new procedures in order to verify the traditional ones.

The development of a new methodology for automatic tolerance inspection starting from an acquired high-density 3D model is an ongoing activity by our research group. To that end, the new approach carries out a recognition process on the acquired data which serves to identify some *intrinsic nominal references* (Di Angelo et al. 2010). Taking these references as a starting point, new and old categories of form error are defined and some procedures are proposed for a more robust verification of traditional tolerance categories. There exists an *Intrinsic Nominal Reference* when a geometric property is recognised to be common to a set of adjacent points from the 3D data set representing the acquired object.

In this paper, the category of *intrinsic nominal references* is widened to include new mutual intrinsic orientation, location and dimensional properties pertaining to 3D features. This approach involves the automatic construction of a geometric reference model for the scanned workpiece, called *Recognised Geometric Model (RGM)*. In order to evaluate the *non-idealities*¹ of the acquired object, the unstructured 3D data points in the *RGM* are clustered around some recognised *nominal categories*. The recognition of these categories is carried out based on some *rules*, which therefore play a leading role in the definition of the domain of the representable entities within the *RGM*. In particular, this paper focuses on the rules concerning the recognition of intrinsic properties in the orientation and location relationships between *non-ideal features*. Those rules intended for geometric form recognition have already been presented in (Di Angelo et. al. 2010).

2. Literature review

For the last few years, and as far as research is concerned, a considerable amount of effort has been directed toward the development of methodologies aiming at performing an effective and reliable automated geometric tolerance verification (Prieto et al. 2002, Li and Gu 2005, Gao et al. 2006, Wong et al. 2006). A great number of these methodologies require knowing the CAD model for the workpiece under inspection. The CAD model provides us with the nominal references in the form of parameterised equations describing the surfaces of the geometric model (Prieto et al. 2002, Li and Gu 2004, Li and Gu 2005, Gao et al. 2006) and it often contains an explicit coding of the orientation and location relationships between the surfaces. Since tolerance specifications usually refer to some features of the

¹ We prefer the term 'non-ideality' over 'error'. Whereas the error can be corrected, it is not possible to correct the non-ideality since it is inevitably associated with real objects to which the concept of perfection or ideality cannot be applied.

workpiece, the mapping between one surface (or feature) of the CAD model and the corresponding scanned point sub-cloud needs to be performed. Tolerance specifications can be either included in the CAD model as textual information, or interactively defined by the user. However, no standard language capable of specifying tolerances and suited to automatic verification has yet been defined. Prieto et al. (2002) proposed and implemented a methodology for automated inspection of manufactured parts. Their methodology first registers the experimental point cloud with the corresponding CAD model of the workpiece by using the *Iterative Closest Point (ICP)* algorithm, and then segments the 3D point cloud by associating the points matching the same local geometric properties with the nearest CAD surface. This methodology is able to verify both dimensional and geometric tolerances.

In (Gao et al. 2006) the authors develop an automated geometric inspection system within a commercial RE (Reverse Engineering) software. A *Nominal Inspection Frame (NIF)* for a CAD model or a digitised reference model (Master Model) is defined for which every dimensional and geometric tolerance specification may be defined and interactively specified by the user. The GD&T items, which can be defined by the *NIF*, pertain to:

- flatness;
- datum (point, line, axis or plane);
- parallelism;
- perpendicularity;
- true position.

Once the *NIF* is created, the scanned data are then aligned with the nominal model; GD&T items on the measured parts are automatically computed and extracted from the *NIF* previously defined by means of the software functions and macro programs.

In (Wong et al. 2006), the authors develop an algorithmic approach for inspection feature recognition directly from a CAD model. Several types of inspection features (external, internal, offset and free form), usually found in industrial practice, can be extracted. In order to facilitate the extraction and recognition of an inspection feature, a multi-attribute spatial graph is proposed where both the nodes and the edges may have specified attributes. The node attributes specify the geometric type of a CAD face (planar, cylindrical or free form). The edge attributes identify some spatial relationships between pairs of adjacent and non-adjacent faces. This graph is capable of supporting the extraction of inspection features for linear as well as angular dimensional measurement. The syntax proposed by (Wong et al. 2006) for the formal representation of inspection features provides us with a basis for automating inspection process planning by using conventional metrological equipment (comparator, dial gauge, etc.). Some specific inspection equipment as well as a particular application method can be recommended for each dimensional inspection feature. Moreover, the *boundary face list* provides the corresponding constraints on the access, location, alignment and handling of the measuring face.

3. The RGM operations

The *Recognised Geometric Model (RGM)* is a special kind of geometry representation of a measured object deriving from the recognition of some ideal properties of the object. Figure 1 shows the flowchart for the process of derivation and query of the *RGM*. This representation is drawn from a high density point cloud, which reproduces the real object being acquired. The recognition of these properties

(form, orientation and location) is carried out based on some *rules*, which play a leading role in the *RGM* construction.

Figure 1: Flowchart for the process of derivation and query of the *RGM*

3.1 The non-ideal feature segmentation

RGM construction involves a complex segmentation process directed toward the identification of the *non-ideal features* and the associated category of recognisable ideal properties. We define a *non-ideal feature* as a set of adjacent points that are recognised to be smooth, of the same type (flat, umbilical, ruled and generic) and to pertain to a unique regular surface. The type of point is deduced by evaluating some differential geometric properties. This definition completes the vagueness of the ISO standard (ISO/TS 17450-1: 2005) which defines the *non-ideal feature* as the “*imperfect feature fully dependent on the non-ideal surface model (skin model)*”. The ISO definition seems to refer to a property of a non-ideal feature (the imperfection) rather than to its description. According to the concept of *non-ideal feature*, the acquired data in the *RGM* are segmented in a way that is consistent with recognisable patches of the object’s surface. The set of *non-ideal features* represents the object with the exception of its non-regular parts (ridges and singularities). These non-regular parts do not follow a recognisable rule and are therefore excluded from further elaborations in *RGM* (Di Angelo and Di Stefano, 2010). This work makes use of a surface hybrid segmentation approach (Di Angelo et al. 2007), based on fuzzy logic. The non ideal feature segmentation gives results in any case. In the case of an object embodied by one and only free form surface, the feature segmentation returns an only entity.

3.2 The ideal feature recognition

According to GPS standards, the feature operation called *association* makes it possible “to fit an ideal feature to a non ideal feature based on a criterion” (ISO/TS 17450-1: 2005). These criteria (herein referred to as *recognition rules*) are based on the evaluation of some local and global differential properties of the segmented model. The present work classifies the *ideal features* into two main categories: *analytical* and *non-analytical features*. *Analytical features* are those recognised to pertain to an analytical geometric surface (plane, sphere, cylinder, cone, etc.). For these *ideal features*, an analytical type of geometric surface is automatically recognised and associated with the *non-ideal feature*. Those *ideal features* which are not analytical are defined to be *non-analytical features*. *Non-analytical features* also include surfaces characterised by some specific and recognisable geometric properties. Based on the recurrence of specific differential geometric properties among the points pertaining to the *non-ideal feature*, they can be classified as: generic extruded, generic cone, generic axially-symmetric. Henceforth we will be referring to them as *generic ruled (GR)* and *generic axially-symmetric (GA)*. The *non-analytical features* which cannot be included within any of the previous categories are *free form features*. In any case, for any non-analytical feature a parameterised equation can be associated with a CAD model by a registration process. In the *RGM* data structure the concept of *ideal feature* has to do with the qualitative property (or attribute) of a

corresponding *non-ideal feature*: its geometric type (*analytical*, *GA*, *GR*, *free form*). The *ideal feature* itself does not identify quantitative elements, although some measurable (dimensionable) intrinsic characteristics (diameter, apex angle, etc.) and situation features (centre, axis, etc.) can be identified. Quantitative elements are identified during the querying phase of the *RGM* (Di Angelo and Di Stefano, 2010).

3.3 Nominal reference association

RGM aims at the evaluation of the *non-idealities* of the acquired object (form, orientation and location non-idealities). This operation is performed by means of a query to the *RGM*. Non-ideality evaluation always requires the knowledge of a nominal (or ideal) reference. In a previous work (Di Angelo et al. 2010) we classified the nominal references into two main categories: *explicit* and *intrinsic*. The *explicit* reference can be provided by a specification, also by using a CAD model through a registration process (Prieto et al. 2002, Gao et al. 2006). The *Intrinsic references* are nominal entities that are recognised in *non-ideal features* of the measured object. In relation with the type of *ideal feature*, we can recognise an *Intrinsic Shape Reference (ISR)*, one or more *Intrinsic Derived References (IDR)* and *Intrinsic Local References (ILR)*. An *ISR* is recognised in an *analytical feature*. Depending on the type of *ISR*, some dimensionable intrinsic geometric parameters, referred to as *Intrinsic Characteristics*, can be identified. Table 1 lists the *Intrinsic Characteristics* of the *ISRs*, which can be recognised by the methods implemented in the present work.

Table 1. The *Intrinsic Characteristics* of the *ISRs*

For a *non-analytical feature* the shape reference must be given explicitly. In some special types of non-ideal features, such as *GR* and *GA*, only *IDRs* and *ILRs* can be detected. An *IDR* is an analytically-known geometric entity deriving from the evaluation of some geometric properties of a feature. Generally speaking, these entities are not physical; for this reason, they are not directly measurable on the object, but can nevertheless be derived from the measured surface. Examples of *IDRs* are the axis and the circular sections of axially-symmetric surfaces. Whereas the circular sections are physical entities, the axis is not. In many cases these references are the situation features of the *non-ideal features*, since they serve as references for feature location and/or orientation. The situation features for each type of *non-ideal feature* here considered are reported in table 2. They are *ideal features* of the following types: point (ϕ), straight line (r) or plane (Π).

Table 2. The situation features for each type of non-ideal feature here considered

Other *IDRs* could be defined based on functional or manufacturing properties of the acquired object. For the gears shown in figure 2, the base cylinder or cone could represent the *Intrinsic Derived References* of the object and their axes could represent the location characteristic of the gear. A further example could be the symmetry plane of a *free-form* mirrored surface (figure 2 c). The implementation of these *IDRs* in the *RGM* requires the introduction of specific recognition rules.

Both shape and derived references are estimated by approximating the point cloud associated with a non-ideal feature through one or more association operations. Depending on the association rule which is used, different intrinsic references can be estimated.

The *Intrinsic Local Reference (ILR)* is an original type of intrinsic nominal reference since it is not considered in the current tolerancing standards. It deals with the uniformity of some intrinsic differential geometric properties, such as: regularity, curvature recurrence, and so on. These references do not pertain to the global analytical properties of a surface, but rather to properties which characterise it locally. In the *RGM* two *ILRs* are introduced. The first one is the *regularity* of the points pertaining to a *non-ideal feature*. Surface regularity locally measures the membership of each point to a regular surface which approximates its neighbourhood. Another *ILR* concerns the ruled surfaces (Di Angelo et al. 2010). This reference has to do with the recognition of the ruled property of a *non-ideal feature*. This kind of intrinsic reference can be recognised for cylinder, cone and *GR*.

Figure 2: Examples of *IDRs*: a) the base cylinder for a cylindrical gear, b) the base cone for a conical gear and c) the symmetry plane for a free-form specular surface.

Table 3 shows the *ISRs*, the *IDRs* and the *ILRs*, related to several ideal feature types.

The types of *ideal feature* which can be included within the *RGM* strictly depend on the capability of said software to recognise them. In fact, table 3 only reports those types of ideal features that the prototypal software here implemented can recognise. However, further categories of analytical and non-analytical features can be added to the *RGM*.

Other intrinsic properties can be recognised in the mutual geometric relationships between *non-ideal features*. These relationships can be subdivided into: *orientation properties* and *location properties*. Each of them has a corresponding nominal category with which an intrinsic reference (a specific orientation or location) can be associated. Both *Intrinsic Orientation* and *Intrinsic Location References* can be identified between *non-ideal features* which are located based on their own situation features. The orientation relationship is a property which can be evaluated between directions. The *Intrinsic Orientation Reference (IOR)* refers to the mutual geometric properties of parallelism and perpendicularity between *non-ideal features*. Furthermore, some angular values can be recognised to be very close to angular values which frequently occur in mechanical workpieces, such as 30°, 45° and 60°. The *Intrinsic Location Reference (ILoR)* includes special types of location relationships. They are: concentricity, coaxiality and coincidence. The recognition of these intrinsic mutual location relationships requires that a *localised* situation feature should be identified for each *non-ideal feature*. A situation feature is said to be localised if it is possible to unequivocally identify its location within an arbitrarily given reference frame. For example, the axis of a *GA* localises the surface to a plane which is orthogonal to the axis. On the contrary, the extrusion direction, that is to say, the situation feature recognised for a generic extruded surface, is not a localised situation feature. Therefore, for this kind of surface it is only possible to recognise orientation properties and not location relationships.

Table 3. Ideal feature types and the related *ISRs*, *IDRs* and *ILRs*

For the recognition of the orientation and location properties it seems more suitable to classify the features into planar features (henceforth referred to as *P – feature*) and features whose situation feature is an axis or an extrusion direction (henceforth referred to as *R – feature*).

3.4 Dimensionable intrinsic parameters’ evaluation

Within the *RGM*, dimensionable geometric parameters are both the *intrinsic characteristics* and the *intrinsic situation characteristics*. According to the definition given by the GPS standards, *situation parameters* describe “the relative situation (location or orientation) between two situation features” (ISO/TS 17450-1: 2005). They can be further divided into *location parameters* and *orientation parameters*. The former are expressed by length (distance) values; the latter are expressed by angular values. In *RGM* an *intrinsic location characteristic* is automatically associated whenever an intrinsic reference, either analytical or derived, is recognised to be parallel to another reference. The dimensionability of the parallelism is therefore an intrinsic reference. This dimensionable characteristic does not identify an intrinsic reference value for the dimension, but a reference dimension can nevertheless be specified for it. This operation is here referred to as *dimensional registration*.

4. The *RGM* data structure

RGM is an idealised representation of an acquired real object. This geometric representation can be described by means of a hypergraph structure denoted by $RGM(V, \epsilon)$, where V is the finite set of nodes v_i of the hypergraph. Each node is associated with a *non-ideal feature* of the measured object. Some labels are assigned to each node of the hypergraph, which describe the *non-ideal feature* attributes (type of ideal feature, intrinsic characteristics, situation features, *R – feature* or *P – feature*). In $RGM(V, \epsilon)$, ϵ is a family of two sub-sets of V ($\epsilon = \{V_A, V_{GR}\}$) which are known as *hyper-edges* and which respectively represent the sets of *non-ideal features* for which adjacency relationships and mutual geometric properties are respectively recognised. Whereas the adjacency relationships are recognised between *non-ideal features*, the mutual geometric relationships are recognised between the situation features of the *non-ideal features*.

In *RGM* both the *Adjacency Graph* (AG) and the *Mutual Geometric Relationship Hypergraph* (MGRH) can be identified. In the $AG = (V_A, E_A)$, an arc $e_i \in E_A$ represents an adjacency between neighbouring *non-ideal features*. For every pair of adjacent *non-ideal features* there exists unique sole arc e_i connecting the corresponding nodes.

In the hypergraph $MGRH = (V_{GR}, \epsilon_{GR})$, ϵ_{GR} is a family of two hyper-edges ($\epsilon_{GR} = \{V_{OR}, V_{LR}\}$). V_{OR} is the set of *non-ideal features* for which intrinsic orientation properties are recognised. V_{LR} in turn represents the set of *non-ideal features* for which intrinsic location properties are detected. Taking the MGRH as a starting point, several graphs can be automatically derived to represent each type of orientation and location property recognised between the *non-ideal features*. These graphs are defined in the following sections.

5. The recognition of the intrinsic orientation properties

Some intrinsic orientation properties can be recognised between non-ideal features. They are parallelism, perpendicularity and special angular orientations (such as, for example, 30°, 45° and 60°). These properties are recognisable between *non ideal features* whose situation features include a spatial direction, such as plane, cylinder, cone, GA and generic extruded (table 2).

In the RGM a set of N parallel features of the same type constitutes a *system of homogeneous parallel entities* ($S_{h//}$). Two types of systems can be identified: the system including R – features, which is denoted by $S_{//R}$, and that including P – features, which is denoted by $S_{//P}$. These systems can be automatically detected by analysing the *Mutual Parallelism Relationship Graph* for R – features $G_{//R} = (V_{//R}, E_{//R})$ and the *Mutual Parallelism Relationship Graph* for P – features $G_{//P} = (V_{//P}, E_{//P})$, respectively. $G_{//R}$ ($G_{//P}$) is a graph where each node represents an R – feature (P – feature) and each edge represents a mutual parallelism property recognised between R – features (P – features). In $G_{//R}$ ($G_{//P}$) there are as many *components* (i.e. maximal connected sub-graphs (Gross et al. 2004)) as spatial directions for which a parallelism property has been recognised. A component of $G_{//R}$ ($G_{//P}$), characterised by N nodes, is a *system of homogeneous parallel entities* $S_{//R}$ ($S_{//P}$) if the *degree* of each node is equal to $N-1$. For example, for the object shown in figure 3a, four $S_{h//}$ systems are recognised (figure 3b). Each $S_{h//}$ in $G_{//P}$ or $G_{//R}$ is associated with a *reference spatial direction*.

In order to recognise an intrinsic orientation property between non-ideal features, some rules must be firstly defined. In this work, the rules to recognise these properties are based on the evaluation of the dot product between the spatial directions of the *ideal features*. These rules vary depending on whether the features are of the same type (P – features or R – features) or of a different type (see table 4). Due to the non-ideality of real objects, the dot product never exactly matches the ideal value. Consequently, the recognition of the orientation property cannot be deduced from the mathematical verification of an equality. In this paper, however, the values are considered to ‘match’ if the dot product value falls within a properly given tolerated range around the ideal value (reported in table 4 for the various properties). A further control consists in verifying the coherence of the parallelism relationships when these are recognised in pairs between N features ($N \geq 3$) of the same type (R – features or P – features). In other words, the transitive property must be verified. For instance, for $N = 3$ the transitive property can be enunciated in the following way:

$$\text{if feature } A // \text{feature } B \text{ and feature } B // \text{feature } C \rightarrow \text{feature } A // \text{feature } C \quad (1)$$

Table 4. The rules to recognise the intrinsic orientation properties

Figure 3: An example of recognition of orientation properties and related graphs

The transitive property, in practical cases, cannot be verified and ambiguities arise due to errors of *type I* and errors of *type II*. An error of *type I* occurs when the orientation property is true but nonetheless fails to be recognised. An error of *type II*

originates when an orientation property is recognised between features, but is not true. This incoherence must be solved. For this purpose, a first approach consists in applying the missing parallelism conditions so that the transitive property is verified. A second approach consists in splitting the incoherent set of parallel entities into two or more *systems of homogeneous parallel entities*. The first approach may give rise to errors of *type II*, whereas the second one may generate errors of *type I*. In figure 4b, for example, the parallelism between P_D and P_C does not seem to be recognised; as a consequence, the set of homogeneous entities $\{P_A, P_B, P_C, P_D, P_E\}$ is not a *system of parallel entities*. The graph of figure 4b is amended either by adding the missing parallelism condition (as shown in figure 4c) or by removing one node, either P_C or P_D (as shown in figures 4d and 4e, respectively). These criteria which are to be adopted to solve the above-mentioned incoherence are part of the parallelism recognition rule. In either case the criterion adopted produces uncertainties.

Figure 4: Derivation of several *systems of homogenous parallel entities* from one system that is not $S_{h//}$

The transitive property for parallelism (1) cannot be verified between different types of non-ideal features. For instance, if A is an *R – feature*, B is a *P – feature*, and C is an *R – feature*, the above property is not necessarily satisfied for any directions of *R – features*. For a given *P – feature* there could be *R – features* which are parallel to it, but not parallel to each other. For the same reason, the transitive property is not verified between *P – features* which are all parallel to the same *R – feature*.

5.1 The Mutual Orientation Relationship Graph (MORG)

The concept of *system of homogeneous parallel entities* is functional to build the *Mutual Orientation Relationship Graph* $MORG = (V_{OR}, E_{OR})$. The nodes of $MORG$ (V_{OR}) are *systems of homogeneous parallel entities* $S_{h//}$ or single features which do not belong to any $S_{h//}$. The arcs (E_{OR}) identify mutual orientation properties between connected nodes. A label is assigned to each edge which specifies the type of recognised orientation (parallelism, perpendicularity, 30° , 45° and 60°). Figure 3c illustrates an example of $MORG$. It refers to the object shown in figure 3a.

Thanks to the concept of *system of homogeneous parallel entities*, if an orientation property is recognised between one feature of a system $S_{h//1}$ and one feature of a system $S_{h//2}$, then it is possible to deduce the satisfaction of this property between each feature of $S_{h//1}$ and each feature of $S_{h//2}$.

6. The recognition of the Intrinsic Location Characteristic

For each parallelism property which is recognised between two localised situation features, there can be a dimensional parameter referred to as *intrinsic location characteristic*. This parameter represents the distance between two parallel ideal entities approximating the two situation features recognised to be parallel to each other. For example, for the system $S_{//P}$ of three planar features shown in figure 5, three intrinsic location characteristics can be identified. In the same figure, d_{A-B} is the distance between the parallel ideal planes $\Pi_{A//A-B}$ and $\Pi_{B//A-B}$ respectively approximating the non-ideal features P_A and P_B , which are recognised to be parallel to

each other. This pair of ideally parallel features will be referred to as “virtual gauge”, working analogously to a physical gauge, which matches the features to be measured with the two parallel surfaces.

In an ideal model, the coherence between the distances of three or more situation features must be verified in order to satisfy the non-contradiction principle so that: $d_{A-B} + d_{B-C} = d_{A-C}$. On the contrary, in an acquired object in which the features are recognised to be parallel despite the fact that they are not really parallel to each other, every dimension can be evaluated independently from the others. For this reason, $d_{A-B} + d_{B-C} \neq d_{A-C}$ and the representation of the three distances are not really a redundancy.

In the *RGM* query, several values for the *Intrinsic Location* parameter can be obtained depending on the association rule used for approximating the two *non-ideal features* (for example, the least squares fitting L_2 , the upper or inner envelope fitting, etc.).

In order to represent the *intrinsic location characteristics* in the *RGM*, a specific graph is built which is referred to as *Intrinsic Location Parameters Graph (ILPG)*. It is defined further down in this paper, in section 7.1.

Figure 5: The three intrinsic location characteristics identified for a system $S_{//P}$ of three planar features

7. The recognition of the intrinsic location properties

Other intrinsic location properties are: coaxiality, concentricity and coincidence. Table 5 shows the intrinsic location properties which are here recognised. In order to detect them some *recognition rules* must be firstly defined.

Table 5. The intrinsic location properties here recognised for the several feature types

Coaxiality is the property of two ideal axially-symmetric features having the same axis. Some *recognition rules* need to be defined for the recognition of this property between non-ideal features. The rule which is here being followed involves first the parallelism recognition and then the evaluation of the related *intrinsic location characteristic* (representing the distance between the axes of the features). Coaxiality is recognised if the distance value (estimated by approximating the axial-symmetric features by the L_2 -rule) falls within a properly given tolerated range around 0.

In the GPS standards, the concept of concentricity is defined between features projected onto a plane (ISO 1101: 2004). In this work this concept is extended to include 3D features. Concentricity is then the property of two spherical ideal features having the same centre. As far as the rule for the recognition of this property is concerned, it should be based on the comparison of the distance between the centres of the two spheres (analytically identified by the L_2 -rule) with a properly given tolerated value.

Coincidence is the property of two features which are one on the prosecution of the other. Some recognition rules are also defined in order to detect this property

between some analytical features (plane, cone, cylinder, sphere). These rules vary depending on the non-ideal feature type. For analytical axially – symmetric features (cone and cylinder) the rule involves first the coaxiality-property recognition and then the evaluation of the difference between the corresponding intrinsic characteristics (estimated by approximating the features by the L_2 -rule). If said difference value falls within a properly given tolerated range around 0, the features can be recognised to be coincident with each other. An analogous recognition rule can be defined for spherical features. In this case, the pre-emptive recognition of the concentricity property, rather than the coaxiality property, is required. For planar features the recognition rule involves first the parallelism detection and then the evaluation of the corresponding *intrinsic location characteristic* (representing the distance between the planar features). If this value (estimated by approximating the features by the L_2 -rule) falls within a properly given tolerated range around 0, the coincidence property is recognised (table 6).

Table 6. The coincidence property recognition

When intrinsic location properties of a specific type (concentricity, coaxiality or coincidence) are recognised in pairs between N features ($N \geq 3$), the transitive property must be verified. Any set of N features for which $\sum_{i=1}^N (N-i)$ location properties are recognised in pairs is said to automatically satisfy the coherence imposed by the transitive property. This set of features is referred to as *system of coherent localised entities* (S_{Lo}).

In order to represent intrinsic location properties and identify the different types of *system of coherent localised entities*, a specific graph is defined. The *mutual location relationship graph* $MLRG = (V_{LR}, E_{LR})$ is a graph where each node identifies mutual location properties between connected nodes. Each edge is assigned a label which reports the type of location recognised (coaxiality, concentricity and coincidence).

The *systems of coherent localised entities* can be automatically detected by analysing the above-mentioned graph. In the graph there are as many *components* as non-ideal features for which an intrinsic location property has been recognised. A component, characterised by N nodes, represents a *system of coherent localised entities* S_{Lo} (for a specific type of location property) if the *degree* of each node is equal to $N-1$. If the degree of any nodes of the component is less than $N-1$, the coherence verification, imposed by the need to satisfy the transitive property, cannot be possibly carried out. The subsequent incoherence could be solved either by imposing the missing location conditions or by splitting the component into two or more *systems of coherent localised entities* S_{Lo} . The first approach could give rise to errors of *type II*, whereas the second one could generate errors of *type I*.

A *reference situation feature* is associated with each S_{Lo} . This turns out to be useful during the phase of RGM query whenever the user asks for the evaluation of the intrinsic situation characteristics between two non-ideal features. If one or both non-ideal features belong to a system of *coherent localised entities* S_{Lo} , it is possible to consider the reference situation feature (associated with S_{Lo}) rather than the situation feature of the single feature. This allows us to obtain a value for the situation characteristic which is coherent with satisfying the transitive property.

7.1 Intrinsic Location Parameters Graph (ILPG)

Thanks to the concept of *system of coherent localised entities*, the *Intrinsic Location Parameters Graph* ($ILPG = (V_{LP}, E_{LP})$), which has been introduced in section 6, can be efficiently defined. This graph is the data structure used for describing, within the RGM, the *intrinsic location characteristics* identified in the acquired object.

The nodes of *ILPG* are *systems of coherent localised entities* S_{Lo} or single features which do not belong to any S_{Lo} . In either case the features can be planar or axial-symmetric since the intrinsic location characteristic can only be defined (see section 6) for these types. The arcs of *ILPG* represent the intrinsic location parameters identified between the connected nodes. According to the concept of *system of coherent localised entities*, if an intrinsic location parameter is recognised to exist between one feature of a system S_{Lo1} and one feature of a system S_{Lo2} , an unequivocally identified dimensional parameter can be associated between each feature of S_{Lo1} and each feature of S_{Lo} . The identification of this single intrinsic location parameter is coherent with satisfying the transitive property.

8. Application example

The methodology described in the previous sections has been implemented in an original software, coded in C++, by using a library dedicated to the processing of tessellated geometric models, which has been developed at the University of L'Aquila.

In order to verify the reliability of the proposed methodology, a specific test case has been analysed which refers to a real object whose acquisition has been carried out by means of an optical scanner (www.scansystems.it). The proposed method has been verified in the identification of some qualitative aspects, such as the features identification, the SHPE recognition, the MORG, MLRG and ILPG resulting. The figure 6 shows the results for the features identification and their respective labels. The areas coloured in black are recognised as non – regular and are therefore automatically excluded from the error evaluation. The situation features are all evaluated by using the L_2 -rule (Di Angelo et al., 2010).

Figure 6. The results of the features identification and their respective labels

Figure 7 shows the results obtained for the $S_{h//}$ recognition of the object in figure 6 when using the software here implemented:

- the SHPE Report dialogue box (SHPE being an acronym for *system of homogeneous parallel entities*) reports the total number of $S_{//R}$ and $S_{//P}$ recognised;
- in figure 7b, a SHPE label is assigned to each $S_{h//}$. The features belonging to the same $S_{h//}$ are depicted in the same colour;
- in figures 7c and 7d, the features belonging to each $S_{//p}$ and $S_{//R}$, respectively, are listed with the corresponding *reference spatial direction*.

Figure 7. The results of the SHPE recognition for the test case under examination

Figure 8 displays the report screen and the graph resulting from the recognition of orientation properties for the test case here analysed (the nodes are depicted in the same colour as the corresponding *SHPE* reported in figure 7). Figure 9, in turn, shows the report screen and the *MLRG* obtained when analysing the test case being considered (the graph nodes are the same colour as the corresponding features in figure 6). Finally, the resulting *ILPG* is represented in figure 10. Each arc of the graph is associated with the distance between the corresponding non-ideal features.

The method has been verified also in the evaluation of some quantitative aspects, such as the analytical identification of the situation features and the evaluation of intrinsic characteristics for each recognised non ideal feature. In table 7 the situation feature and the intrinsic parameter related to each feature recognized for the object under examination (fig. 6) are reported. The differences of orientation from ideal condition are due to the geometric errors of the object. The four holes (cy_0, cy_1, cy_2 and cy_3) which are nominally equal, have different values of their intrinsic parameters (the radius of the cylinder). This variability is due to the error in the holes manufacturing. Furthermore, from some intrinsic situation characteristics the related dimension has been evaluated by using two different *Virtual Gauges* based on different rules (table 8). It is evident that both the value of each distance and the direction of the corresponding *Virtual Gauge* depend on the rule used to evaluate the *Virtual Gauge*.

Table 7. The situation features and the intrinsic characteristics of the non – ideal features recognised

Figure 8. The *MORG* resulting for the test case analysed.

Figure 9. The *MLRG* resulting for the test case analysed.

Figure 10. The resulting *ILPG* for the test case analysed.

Table 8. The effect of the rule in the distance evaluation

9. Conclusion

For some years now, our research group has been developing a new methodology for automatic tolerance inspection starting from an acquired high-density 3D model. In a previous paper (Di Angelo et al. 2010), we proposed a general approach for automatic form error evaluation and specification. That approach was founded on the concepts of *non-ideal feature* and *intrinsic nominal reference*. The

object to be inspected was segmented into a set of *non-ideal features*, for each of which one or more *intrinsic nominal references* were identified. Based on those concepts, new and old categories of form errors were defined and some procedures were proposed for a more robust process of verification of traditional tolerance categories.

In order to complete the category of *intrinsic nominal references*, this paper goes further and introduces new mutual intrinsic orientation, location and dimensional properties pertaining to 3D features. Moreover, with a view to grouping together all the information recognisable in a scanned object, a new data structure called *Recognised Geometric Model (RGM)* is presented. The recognition of *nominal categories* in a scanned object is carried out based on some *rules* which therefore play a leading role in the definition of the domain of the representable entities within *RGM*. In particular, this paper puts forth the rules concerning the recognition of intrinsic properties in the orientation and location relationships existing between *non ideal features*. The concept of *Virtual Gauge* is also introduced and the effect of the rule which has been used to evaluate it is highlighted in the analysis of a real case.

Based on the geometric product specification, the *RGM* data structure can be *queried* to gather some quantitative information concerning special intrinsic geometric parameters and/or non-idealities. Future work should address how to specify the tolerated errors in accordance with the *RGM* data structure and how to measure the non-idealities of the object.

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Figure 1: Flowchart for the process of derivation and query of the RGM

Figure 2: Examples of IDRs: a) the base cylinder for a cylindrical gear, b) the base cone for a conical gear and c) the symmetry plane for a free-form specular surface.

Figure 3: An example of recognition of orientation properties and related graphs

Figure 4: Derivation of several systems of homogenous parallel entities from one system that is not $S_{h//}$

Figure 5: The three intrinsic location characteristics identified for a system $S//P$ of three planar features

Figure 6. The results of the features identification and their respective labels

Figure 7. The results of the SHPE recognition for the test case under examination

Figure 8. The *MORG* resulting for the test case analysed.

Figure 9. The *MLRG* resulting for the test case analysed.

Figure 10. The resulting *ILPG* for the test case analysed.

Table 1. The *Intrinsic Characteristics* of the *ISRs*

Type of ISRs	Intrinsic Characteristics
<i>Plane</i>	none
<i>Sphere</i>	diameter
<i>Cylinder</i>	diameter
<i>Cone</i>	apex angle

Table 2. The situation features for each type of non-ideal feature here considered

Type of non ideal feature	Situation features
<i>Plane</i>	○ the plane II ;
<i>Sphere</i>	○ the centre ϕ ;
<i>Cylinder</i>	○ the axis r ;
<i>Cone</i>	○ the axis r ; ○ the apex ϕ ;
<i>Generic axial-symmetric</i>	○ the axis r ;
<i>Generic extruded</i>	○ the extrusion direction r ;
<i>Generic cone</i>	○ the apex ϕ ;
<i>Free form</i>	○ -----

Table 3. Ideal feature types and the related *ISRs*, *IDRs* and *ILRs*

Ideal feature type		Intrinsic Reference Types		
		ISR	IDR	ILR
Analytical	<i>Plane</i>	Analytical shape	Normal direction-	Surface regularity
	<i>Sphere</i>	Analytical shape	Centre	Surface regularity
	<i>Cylinder</i>	Analytical shape	Axis Ideal circumferential line	Surface regularity Profile regularity Ruledness
	<i>Cone</i>	Analytical shape	Axis Ideal circumferential line	Surface regularity Profile regularity Ruledness
Non Analytical	<i>Generic Ruled (GR)</i>	<i>generic extruded</i>	Extrusion Direction	Surface regularity Profile regularity Ruledness
		<i>generic cone</i>	Apex vertex	Surface regularity Ruledness
	<i>Generic Axially – symmetric (GA)</i>		- Axis Ideal circumferential line	Surface regularity Profile regularity
	<i>Free form</i>		-	Surface regularity

Table 4. The rules to recognise the intrinsic orientation properties

dot product value		Intrinsic Orientation Property
Features of the same type	Features of a different type	
0	1	perpendicularity
± 0.5	$\pm \frac{\sqrt{3}}{2}$	angular orientation of 60°, -60°, 120° and 240°
$\pm \frac{\sqrt{2}}{2}$	$\pm \frac{\sqrt{2}}{2}$	angular orientation of 45°, -45°, 135° and 225°
$\pm \frac{\sqrt{3}}{2}$	± 0.5	angular orientation of 30°, -30°, 150° and 210°
± 1	0	parallelism

Table 5. The intrinsic location properties here recognised for the several feature types

Feature type	Plane	Sphere	Cylinder	Cone	Generic Axially – symmetric (GA)
Plane	<i>Coincidence</i>	-	-	-	-
Sphere	-	<i>Concentricity Coincidence</i>	-	-	
Cylinder	-	-	<i>Coaxiality Coincidence</i>	<i>Coaxiality</i>	<i>Coaxiality</i>
Cone	-	-	<i>Coaxiality</i>	<i>Coaxiality Coincidence</i>	<i>Coaxiality</i>
Generic Axially – symmetric (GA)			<i>Coaxiality</i>	<i>Coaxiality</i>	<i>Coaxiality</i>

Table 6. The coincidence property recognition

Type of analytical features	Required mutual geometric relationship	Further condition to be verified
Plane	<i>Parallelism</i>	<i>Distance between planar features</i>
Sphere	<i>Concentricity</i>	<i>Difference between diameters</i>
Cylinder	<i>Coaxiality</i>	<i>Difference between diameters</i>
Cone	<i>Coaxiality</i>	<i>Difference between apex angles</i>

Table 7. The situation features and the intrinsic characteristics of the non – ideal features recognised

Feature name	Situation features						Intrinsic parameters
	Base point [mm]			Direction			
	x	y	z	x	y	z	Radius [mm]
cy_0	25.44	82.27	5.36	0.009	0.041	0.999	11.05
cy_1	-25.76	82.35	8.16	0.009	0.003	0.999	10.83
cy_2	-25.41	-81.18	1.15	0.004	-0.001	-0.999	11.13
cy_3	25.81	-81.21	6.81	0.013	0.015	-0.999	11.03
cy_4	-0.067	0.23	-0.42	-0.001	0.002	0.999	31.66
pl_5	50.38	1.07	17.07	0.999	-0.001	-0.001	--
pl_6	0.41	2.01	0.09	0.001	0.000	-1	--
pl_7	-0.19	-0.51	40.20	0.002	-0.001	0.999	--
pl_8	-0.27	-80.59	19.67	-0.001	-0.000	1	--
pl_9	0.06	80.93	19.70	-0.000	0.000	1	--
pl_10	-50.36	-1.00	16.97	-1.0	-0.002	-0.002	--
pl_11	-0.88	50.52	29.84	0.002	0.999	-0.013	--
pl_12	0.42	-0.42	14.56	0.002	-0.001	0.999	--
pl_13	-0.14	-50.26	29.83	-0.002	-0.999	-0.027	--
pl_14	0.05	111.91	9.69	0.002	0.999	0.007	--
pl_15	0.07	-111.09	10.23	0.002	-1	0.001	--
pl_16	45.14	106.66	9.85	0.707	0.707	0.013	--
pl_17	-45.12	106.64	10.15	-0.707	0.707	0.003	--
pl_18	45.02	-105.88	10.04	0.707	-0.707	0.004	--
pl_19	-45.13	-105.77	10.13	-0.706	-0.708	-0.001	--
cy_20	0.28	-0.35	-143.34	-0.003	0.004	0.999	21.24

Table 8. The effect of the rule in the distance evaluation

Parallel features		L ₁ -rule				L ₂ -rule			
		Virtual Gauge direction			distance [mm]	Virtual Gauge direction			distance [mm]
		x	y	z		x	y	z	
pl_10	cy_2	$1.36 \cdot 10^{-4}$	$-1.23 \cdot 10^{-4}$	1.000	24.38	$2.71 \cdot 10^{-4}$	$-1.59 \cdot 10^{-4}$	1.000	24.67
pl_10	cy_3	$5.34 \cdot 10^{-3}$	0.03811	0.999	76.38	0.010	0.042	0.999	76.29
pl_10	pl_5	0.999	$7.4 \cdot 10^{-4}$	$-1.32 \cdot 10^{-4}$	100.92	1.000	$2.9 \cdot 10^{-5}$	$-8.11 \cdot 10^{-4}$	100.98
cy_2	cy_3	$-8.2 \cdot 10^{-5}$	$-2.53 \cdot 10^{-3}$	-0.999	51.31	$-3.6 \cdot 10^{-3}$	-0.016	-0.999	51.28
cy_2	pl_5	$-1.26 \cdot 10^{-3}$	$-1.60 \cdot 10^{-3}$	-0.999	75.87	$-2.13 \cdot 10^{-4}$	$-3.42 \cdot 10^{-3}$	-0.999	75.98
cy_3	pl_5	$-1.26 \cdot 10^{-3}$	$-1.60 \cdot 10^{-3}$	-0.999	24.06	$2.78 \cdot 10^{-3}$	0.010	-0.999	24.38