1d WCIP and FEM hybridization

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***Abstract* — An hybridization between two numerical methods, the 1d Wave Concept Iterative Procedure (WCIP) and the 2d Finite Element Method (FEM), is developed. Using two examples, comparisons are provided between the new hybrid method and an analytic solution, when available, or the WCIP alone.**

I. INTRODUCTION

The Wave Concept Iterative Procedure (WCIP) is a numerical method dedicated to the study of planar homogeneous multi-layer circuits [1]. For instance, analyses can be performed on frequency selective surfaces [2], but not high impedance surfaces that contains "mushrooms", *i.e.* patches with via-holes, because the WCIP alone cannot deal with via-holes. Currently, the method is also inappropriate for inhomogeneous dielectric substrates despite some attempts [3]. To overcome this difficulty, an hybridization with the Transmission Line Matrix method has been achieved in [4] but it is restricted to regular rectangular meshes. This limitation prevents the method from being efficient when applied to the study of innovative substrates. Hybridization with the Finite Element Method (FEM) is a way to cope with this issue especially thanks to the use of unstructured triangular meshes. In this paper, a first attempt to devise a hybrid WCIP/FEM method is introduced by means of a quadrangular mesh to facilitate the implementation. Numerical examples are provided to assess the accuracy of this approach.

II. METHODS

A. 1d WCIP principle

The purpose of the WCIP is to solve Maxwell's equations in guided and stratified structures. The method relies on the behavior of incoming B and outgoing A waves upon an interface Σ which separates two domains 1 and 2; see Fig. 1. As the WCIP requires homogeneous dielectric layers, we must have in domain 2 $\varepsilon_{r,1} = \varepsilon_{r,2}$, where ε_r is the relative dielectric permittivity. Waves A and B are generated by a source wave B_0. After discretization, the equations to consider are

$$\begin{cases} B_1 = \Gamma_1^W A_1 + B_0 \\ B_2 = \Gamma_2^W A_2 \end{cases}, \quad \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = S \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}, \tag{1}$$

where the operator S contains the transmission/reflection conditions on Σ, and $(\Gamma_i^W)_{i=1,2}$ are the scattering operators for homogeneous media in domain i. The discrete operators $(\Gamma_i^W)_{i=1,2}$ are based on the Fast Modal Transform (FMT) [5] and modal scattering operator $(\widehat{\Gamma}_i)$ according to

$$\Gamma_i^W = \mathrm{FMT}^{-1}\widehat{\Gamma}_i\mathrm{FMT}. \tag{2}$$

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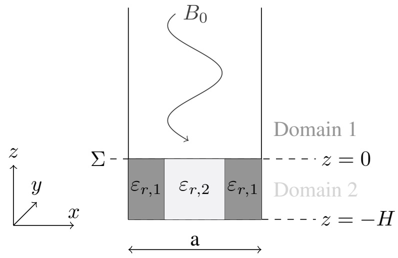

Fig. 1. A multi-layer circuit.

Combining equations in (1), the linear system to solve is

$$\left(\mathrm{Id} - \begin{pmatrix} \Gamma_1^W & 0 \\ 0 & \Gamma_2^W \end{pmatrix} S\right)\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} B_0 \\ 0 \end{pmatrix}, \tag{3}$$

where Id is the identity matrix. Note that when the Transverse Electric (TE) case is considered, B_0 is along the y-axis (see Fig. 1 for the axis orientation) and in Transverse Magnetic (TM) case, B_0 is along the x-axis. In 1d, TE and TM modes are uncoupled which entails that the generated waves are also along the y-axis in the TE case.

B. Hybridization principle

Instead of using the WCIP in both domains, the FEM is implemented in the domain that contains the inhomogeneity (domain 2 in Fig. 1). In the TE case, the wave A_{y2} (the index 2 refers to domain 2) is the outgoing wave seen from the interface. This wave is introduced as a source term in the weak formulation. The electric field E_{y2} is computed using the FEM. The corresponding weak formulation is given by

$$\begin{aligned} &\int_{D_2} \nabla E_{y2} \cdot \nabla w \, ds - k_0^2 \int_{D_2} E_{y2} w \, ds \\ &\quad + jk_0 \int_\Sigma E_{y2} w \, dl = 2jk_0\sqrt{Z_0}\int_\Sigma A_{y2} w \, dl, \end{aligned} \tag{4}$$

where w stands for a test function, k_0 is the free space wavenumber and Z_0 the free space impedance. This equation enables to calculate the incoming wave B_{y2} using

$$B_{y2} = \frac{1}{2\sqrt{Z_0}}\Big(E_{y2} - \frac{Z_0}{j\omega\mu_0}\frac{\partial E_{y2}}{\partial z}\Big). \tag{5}$$

From (4) and (5) with the FEM, a discrete operator Γ_2^F is built according to

$$B_{y2} = \Gamma_2^F A_{y2}. \tag{6}$$

The new linear system of the hybrid method is obtained by replacing Γ_2^W by Γ_2^F in (3). The linear system (3) and the corresponding one for the hybrid method are not explicitly assembled and consequently they are solved by a Krylov subspace solver as GMRes which only requires a matrix-vector product [6].

III. NUMERICAL RESULTS

The propagation of a mode of order n in vacuum between two metallic slabs separated by a distance a ended by a short-circuit is first considered. The solution is defined by

$$E_y(x,z) = E_0\Big[2\sinh\left(p_n(H+z)\right)e^{-p_nH}f_n\Big], \text{with:}$$
$$p_n = \sqrt{\left(\frac{n\pi}{a}\right)^2 - k_0^2} \text{ and } f_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right), \tag{7}$$

where E_0 is the incident wave amplitude, f_n the function of order n of the modal basis, p_n the propagation constant along the z-axis. Here $a = 1.27\text{cm}$ and the working frequency is 16GHz, which implies only one propagating mode. Discretization error, *i.e.* error between the analytic and the computed solutions, is calculated when using the WCIP alone. Error is 10^{-16} whatever mesh size, *i.e.* the error is of the order of the machine accuracy as expected.

The same problem is simulated with the hybrid method. A quadrangular mesh is used in the FEM domain. Contrary to the previous case where a 1d structure was considered, the FEM is implemented on a 2d structure. Consequently, the discretization error in L^2-norm is calculated while refining the mesh along both axes (x and z-axis), and is evaluated on the upper surface S_1. Results concerning electric and tangential magnetic fields are presented in Table I. The indices H and Ex stand for Hybrid and Exact and Mesh size represents the edge length of the rectangles compared to the initial mesh. For instance, $1/2$ means that the step size is twice smaller than the initial step size in both axes. Initial mesh is characterized by a step size of $794\mu\text{m}$ in both directions which is achieved taking $H = 1.27\text{cm}$. N_{dof} is the number of degrees of freedom. Convergence order is estimated taking into account the errors from two consecutive mesh sizes, which explains the empty boxes of the tables.

TABLE I. ERROR E AND H-FIELDS ON S_1, HYBRID METHOD.

Mesh size	N_{dof}	$\|\|\mathbf{E}_{\text{H}} - \mathbf{E}_{\text{Ex}}\|\|_{L^2}$		$\|\|\mathbf{H}^{\mathbf{t}}_{\text{H}} - \mathbf{H}^{\mathbf{t}}_{\text{Ex}}\|\|_{L^2}$	
		Error	Order	Error	Order
$1/2$	1089	$1.27.10^{-2}$	-	$4.55.10^{-5}$	-
$1/4$	4225	$5.20.10^{-3}$	1.29	$1.86.10^{-5}$	1.29
$1/8$	16641	$2.34.10^{-3}$	1.15	$8.37.10^{-6}$	1.15
$1/16$	66049	$1.11.10^{-3}$	1.07	$3.97.10^{-6}$	1.07

A second test problem is considered consisting of a centered microstrip line of width $w = 6.3\text{mm}$ which is inserted on the surface S_2 into the FEM domain (see Fig. 2). A comparison between the electric and tangential magnetic fields obtained by the WCIP alone (denoted by the index W) and with the hybrid method is presented in Table II as an analytic solution is not known in this particular case.

TABLE II. DISTANCE BETWEEN HYBRID METHOD AND WCIP.

Mesh size	N_{dof}	$\|\|\mathbf{E}_{\text{H}} - \mathbf{E}_{\text{W}}\|\|_{L^2}$		$\|\|\mathbf{H}^{\mathbf{t}}_{\text{H}} - \mathbf{H}^{\mathbf{t}}_{\text{W}}\|\|_{L^2}$	
		Error	Order	Error	Order
$1/2$	1089	$1.23.10^{-1}$	-	$4.42.10^{-4}$	-
$1/4$	4225	$6.25.10^{-2}$	0.982	$2.24.10^{-4}$	0.982
$1/8$	16641	$3.14.10^{-2}$	0.992	$1.12.10^{-4}$	0.992
$1/16$	66049	$1.57.10^{-2}$	0.996	$5.64.10^{-5}$	0.996

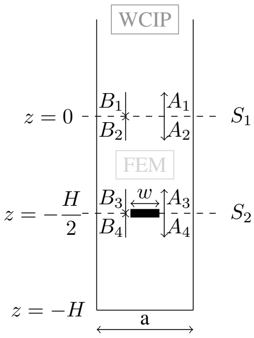

Fig. 2. Insertion of a microstrip in the FEM domain.

IV. CONCLUSION

Electric and magnetic fields obtained with the hybrid method in the case of a mode propagation over an interface separating two air volumes converge to the exact solution. When the FEM domain contains a microstrip line, a comparison between the hybrid method and the WCIP alone illustrates that both methods converge to the same electromagnetic field. The corresponding convergence rate is consistent with the convergence rate estimated during the comparison with the exact solution in the first experiment.

Other structures have to be tested namely circuits with dielectric substrate change with vertical slope (see Fig. 1) or not, and via metallization connecting two metallic surfaces, where the use of the FEM is mandatory. Algorithm acceleration is also being studied.

REFERENCES

[1] S. Wane, D. Bajon, H. Baudrand and P. Gammand, “A new full-wave hybrid differential-integral approach for the investigation of multilayer structures including nonuniformly doped diffusions”, *IEEE MTT.*, vol. 53, no. 1, pp. 200–213, 2005.

[2] E.-A. Hajlaoui, H. Trabelsi, A. Gharsallah and H. Baudrand, “Analysis of Novel Dual-Resonant and Dual-Polarized Frequency Selective Surface using Periodic contribution of Wave Concept Iterative Process: PPMS-WCIP”, in *Information and Communication Technologies: From Theory to Applications, 2008. ICTTA 2008. 3rd International Conference on* , pp. 1–6, 2008.

[3] M. Titaouine, N. Raveu and H. Baudrand, “Wave Concept Iterative Procedure for inhomogeneous multi-layered circuits”, in *Circuits and Systems and TAISA Conference, 2009. NEWCAS-TAISA '09. Joint IEEE North-East Workshop on* , pp. 1–3, 2009.

[4] A. Zugari, M. Khalladi, M. Iben Yaich, N. Raveu and H. Baudrand, “New approach: WCIP and FDTLM hybridization”, in *Microwave Symposium (MMS), 2009 Mediterrannean* , pp. 1–4, 2009.

[5] R.S. N'gongo and H. Baudrand, “A new approach for microstrip active antennas using modal F.F.T-algorithm”, in *Antennas and Propagation Society International Symposium*, IEEE, vol. 3, pp. 1700–1703, 1999.

[6] N. Raveu, L. Giraud and H. Baudrand, “WCIP acceleration”, in *Microwave Conference Proceedings (APMC), 2010 Asia-Pacific* , pp. 971–974, 2010.