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Evaluation of the areal material distribution of paper from its optical transmission image

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Abstract

The goal of this study was to evaluate the areal mass distribution (defined as the x-ray transmission image) of paper from its optical transmission image. A Bayesian inversion framework was used in the related deconvolution process so as to combine indirect optical information with *a priori* knowledge about the type of paper imaged. The *a priori* knowledge was expressed in the form of an empirical Besov space prior distribution constructed in a computationally effective way using the wavelet transform. The estimation process took the form of a large-scale optimization problem, which was in turn solved using the gradient descent method of Barzilai and Borwein. It was demonstrated that optical transmission images can indeed be transformed so as to fairly closely resemble the ones that reflect the true areal distribution of mass. Furthermore, the Besov space prior was found to give better results than the classical Gaussian smoothness prior (here equivalent to Tikhonov regularization).

1 Introduction

The concept of formation has been introduced in paper research [1, 2, 3, 4] so as to describe the small-scale uniformity of the basis weight (areal distribution of solid material) in paper. In the papermaking process wood fibres, mineral fillers, and other additives, together form the basic structure of paper. Fibres themselves form a more or less random network with predominantly planar orientation of fibres. The resulting structure displays variation in its areal mass density, which depends on the inhomogeneity at small length scales of the distributions of the individual solid components, and on changes in process conditions at large length scales. The small-scale density variation, *i.e.*, formation, has traditionally been one of the quality parameters of paper, originally measured by visual inspection and later by analyzing in different ways its optical transmission image [1, 3]. Paper formation has also been measured using transmission images of x-rays and beta radiation (electrons) [1, 4]. In these latter two cases radiation passes through the sample with very little scattering, whilst visible electromagnetic radiation undergoes scattering in addition to absorption from interfaces inside the paper structure. In the visible region absorption is weak in comparison with scattering, *i.e.*, the scattering coefficient is much larger than the absorption coefficient [5].

In optical transmission of light through paper, (multiple) scattering events result in its effectively diffusive motion around the direction of propagation [5], the resulting image is 'blurred', and structural features, which would appear sharp if only absorption would take place, are not well detectable in the resulting transmission image. It has thus become evident that 'optical formation' is different from those determined by other means [1]. Apart from diffusive motion, fibres may also act as wave guides for light rays, thus adding another component to blurring. We do not consider here paper structures with anisotropic orientation of fibres, and both these contributions are thus isotropic [5]. It has not been possible so far to estimate the true areal mass distribution from optical transmission images. This would, however, be desirable as scanning of large areas on-line with x-rays or beta radiation is technically much more difficult than obtaining similar optical transmission data (such a technology already exists).

Fibres used in paper are chemically and structurally heterogeneous, and more than one type of fibre are also commonly used, so that their dielectric properties vary. Because the effective diffusive motion of light around its direction of propagation results from multiple scattering, we however assume

that the system of fibres can be described with its average dielectric properties (scattering and absorption coefficient) [1, 5]. The dielectric properties of fibres, fillers and coating pigments are as well different, which adds another complication to the problem. At this stage we do not address this complication, and only consider uncoated paper in which the concentration of fillers is relatively low.

Previously the effect of scattering on x-ray transmission has been described by a convolution process [6], and we adopt the same approach here. For the case of a single scattering during transmission (thin sample), it can be shown that the convolution kernel is Gaussian [7]. For thicker samples with multiple scattering the analytical form of the kernel is not known, but the Gaussian kernel has successfully been used in this case as well [6, 8]. In optical tomography based on recording optical transmission images a similar distribution has also been applied [9, 10]. Therefore, we assume in the following that a Gaussian kernel can be used when reducing scattering effects in the transmission of visible light by deconvolution.

For deconvolution we use the Bayesian inversion framework [11], and, as discussed above, the aim is to estimate the areal mass distribution of paper from its optical transmission image. As the scattering coefficient in paper of x-rays is very small, the true areal mass distribution is assumed to be obtained by an x-ray transmission image. The basic idea in Bayesian inversion is to augment the 'indirect' optical data with *a priori* knowledge about the mass distribution. Such knowledge is gained here by microtomographic reconstructions of paper samples. The prior distributions are constructed from the tomographic reconstructions using the recently introduced wavelet-based Besov space approach [12]. These distributions are generic enough so that they need to be determined only for the type of paper of interest.

2 Materials and methods

2.1 Mathematical model for measured data

Let A' be a mapping from x-ray transmission data, $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, to the corresponding optical transmission data, $m : \mathbb{R}^2 \rightarrow \mathbb{R}$. Although the form of A' is in principle unknown and may even be nonlinear, we will assume here,

as discussed above, that it is a convolution operator A such that

$$(Af)(x) := \int_{\mathbb{R}^2} f(u) g(x - u) du . \quad (1)$$

Following the above discussion we assume furthermore that the convolution kernel is a Gaussian bell-shaped function $g : \mathbb{R}^2 \rightarrow \mathbb{R}$ with a width determined empirically from the measured edge-spread function (ESF), see figs. 1 and 2.

2.2 Optical transmission data

The optical measurements of paper were done by illuminating the sample with a parallel light beam on one side, and recording the light that passes through to the other side. The light source was a Durst CLS 450 darkroom enlarger. Images were recorded with a Canon 5D Mark II digital single lens reflex camera equipped with a Canon EF 100mm f2.8 USM macro objective with aperture stopped down to f16 for optimal resolution.

For measuring the ESF as illustrated in fig. 1 the sample was partly eclipsed with a strip of opaque metal tape. The resulting optical intensity profile was then used to determine the ESF of the sample as shown in fig. 2.

2.3 x-ray transmission and tomographic data

We used a SkyScan 1172 x-ray scanner to take a radiograph (x-ray transmission image) of the same partly eclipsed paper sample as in Subsection 2.2. The resolution was chosen to be $8.5 \mu\text{m}$ such that individual fibres would be visible in as large a scanned area as possible. The image together with the respective intensity profile across the edge of the eclipsing tape is shown in fig. 2.

Furthermore, we collected three-dimensional (3D) information about the paper structure by making x-ray tomographic reconstructions of similar samples. X-ray micro-computed tomography (μCT) is based on analyzing the absorption of x-rays in the sample while illuminating it with them from (up to about a thousand) different directions, and using the resulting radiographs to form a 3D reconstruction (image) of the sample. To this end we used an Xradia Micro XCT-400 device with a resolution of $1.2 \mu\text{m}$ and a standard filtered back-projection reconstruction algorithm.

Figure 1: A schematic setup of the experiment for measuring the edge spread function (ESF). The recorded ESF image has a blurred boundary between the dark and light areas due mainly to scattering of light inside the paper.

Figure 2: An optical transmission image (upper left), x-ray transmission image (upper right), both of a size of 500x400 pixels, and their normalized ESFs (lower panel) for a handsheet sample.

Figure 3: 3D-tomographic reconstruction of a paper sample (left) and the coefficients of its wavelet transform (right, the lighter the grayscale colour the bigger the magnitude of the transform coefficient).

We used the 3D reconstructions for the construction of Bayesian prior distributions: virtual x-ray transmission images of the samples were also created by taking projections of the tomographic reconstructions in the appropriate direction, yielding information about the Besov space properties of the samples as explained below.

One 3D reconstruction is shown on the left panel of fig. 3. Different grayscales represent different local x-ray absorption coefficients. As absorption depends on material density, different material components, such as solid components and the void space between them, can be identified in the image by their varying ranges of grayscale values using computational image analysis methods.

2.4 Bayesian inversion

The aim is to extract information about the x-ray transmission map f of a paper sample from the indirect (optical) measurement modelled by

$$m = Af + \epsilon, \quad (2)$$

in which m is the recorded optical transmission image and ϵ is random measurement noise. This is an ill-posed deconvolution problem, so *a priori* information is needed for its successful inversion in addition to the measurement data.

We first discretize the problem by letting m and f be $d \times d$ pixel images. We then follow the Bayesian inversion approach, where m and f are modelled as random variables that take values in $\mathbb{R}^{d \times d}$. Also, we model the measurement noise ϵ as a Gaussian random variable with independent, identically distributed components with zero mean and variance $\sigma^2 > 0$. The complete solution to the inverse problem is given by the *posterior distribution*

$$\pi(f | m) = \frac{\pi(f)\pi(m | f)}{\pi(m)}, \quad (3)$$

in which $\pi(m)$ can be understood as a normalization constant, $\pi(f)$ is called the prior distribution and $\pi(m | f)$ is called the likelihood distribution.

The likelihood distribution is essentially a measurement model that, for the assumptions made, takes a Gaussian form,

$$\pi(m | f) = C \exp\left(-\frac{1}{2\sigma^2} \|m - Af\|_2^2\right). \quad (4)$$

Here $\|\cdot\|_2$ denotes the standard Euclidean norm in $\mathbb{R}^{d \times d}$.

The prior distribution $\pi(f)$ is used to express our *a priori* knowledge in mathematical form. It should assign high probabilities to images $f \in \mathbb{R}^{d \times d}$ that are expected in light of the *a priori* information, and low probability to unusual images. In this work we use Besov space priors as explained in Subsection 2.5 below.

The posterior distribution eq. (3) contains full information about the ill-posed inverse problem at hand. However, for practical purposes we need to find a representative estimate for the unknown f from the posterior. There are many useful choices of estimates (and their confidence limits) to choose from; in this work we concentrate on the maximum *a posteriori* (MAP) estimate defined by

$$f_{\text{MAP}} = \arg \max_{f \in \mathbb{R}^{d \times d}} \pi(f | m), \quad (5)$$

i.e., f_{MAP} is the image in $\mathbb{R}^{d \times d}$, which gives the largest value for the posterior distribution $\pi(f | m)$ evaluated with a fixed (measured) realization of the random variable m .

2.5 Wavelet-based Besov space priors

Classical choices for prior distributions are Gaussian and are (formally) of the form $\pi(f) = C \exp(-\delta \|f\|_{L^2(\mathbb{R}^2)})$ or $\pi(f) = C \exp(-\delta \|\nabla f\|_{L^2(\mathbb{R}^2)})$. These

choices lead, however, to rather smooth estimates. In the present problem, prior distributions that promote variable degrees of smoothness are, however, desirable. We proceed to discuss the so-called Besov space priors (formally) of the form $\pi(f) = C \exp(-\delta \|f\|_{B_{pq}^s})$ with parameters $s \in \mathbb{R}$ and $1 \leq p, q \leq \infty$. These priors have been shown to be discretization invariant [12], ensuring numerical reliability when working at different resolutions.

Out of the three parameters of a Besov space, s and p are the most important. Roughly speaking, if $f \in B_{pq}^s$ then we could say that 'derivatives up to order s are in L^p ', see e.g. [13, 14]. The parameter q is for fine tuning, and as the solution is not very sensitive to this parameter we choose here $p = q$, which leads to a simpler form for the norm.

Evaluation of the Besov space norm is most easily done in the wavelet domain. We give here only a brief description of wavelets; more details can be found, *e.g.*, in [13, 14]. The wavelet expansion for a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is given by

$$f = \sum_{j=0}^{\infty} \sum_{G \in G_j} \sum_{m \in \mathbb{Z}^n} c_{j,m}^{(G)} \psi_{j,m}^{(G)}, \quad (6)$$

where the coefficients are defined by

$$c_{j,m}^{(G)} = \left\langle f, \psi_{j,m}^{(G)} \right\rangle, \quad (7)$$

and the index set G_j has 2^n elements when $j = 0$, and $2^n - 1$ elements when $j > 0$. The functions $\psi_{j,m}^{(G)}$ form an orthogonal basis for $L^2(\mathbb{R}^n)$ and can be chosen to be compactly supported in cubes of sizes 2^{-j} .

As an example we show in fig. 3 a tomographic reconstruction of about a $0.5 \times 0.5 \text{ mm}^2$ piece of paper (left panel) together with its wavelet coefficients as a 3D Mallat pyramid (right panel) [15]. The smallest cubes contain coefficients of $\psi_{1,m}^{(G)}$ and the size of the cubes grow together with j . Notice that most of the large coefficients are related to small values of j , which is natural for a wavelet transform.

The Besov space norm can now be defined using the wavelet coefficients such that

$$\|f\|_{B_{pp}^s}^p := \sum_{j=0}^{\infty} 2^{j(s - \frac{n}{p} + \frac{n}{2})p} \sum_{G \in G_j} \sum_{m \in \mathbb{Z}^n} \left| c_{j,m}^{(G)} \right|^p. \quad (8)$$

Since $\log_2 \left(\sum_{G \in G_j} \sum_{m \in \mathbb{Z}^n} \left| c_{j,m}^{(G)} \right|^p \right)$ tends to behave as $\alpha j + C$ for some con-

stants α and C , a necessary bound for s is given by

$$s \leq \frac{n}{p} - \frac{n}{2} - \frac{\alpha}{p}. \quad (9)$$

Equation (8) concerns functions defined on a continuum. A discrete (and approximate) version of the norm can be computed for pixel images or voxel volumes by truncating the wavelet transform. There are fast algorithms for evaluating the coefficients eq. (7) for discrete data sets. We will use the prior distribution

$$\pi(f) = C \exp(-\delta \|f\|_{B_{pp}^s}^p) \quad (10)$$

for $f \in \mathbb{R}^{d \times d}$ with a truncated wavelet transform and a suitable weighting parameter $0 < \delta < \infty$ together with the corresponding normalization constant C .

Let us calculate the relevant region in the sp plane. Figure 4 shows the scaling of the magnitude of the wavelet coefficients as a function of wavelet scale for the x-ray transmission data of the insert in fig. 5 with $p = 2$. Now when α , which is the slope of the regression line, is estimated from data, the inequality eq. (9) gives the boundary for the acceptable region of (s, p) pairs, as illustrated in fig. 5.

Notice that the smoothness parameter can be determined for both a tomographic reconstruction of the sample ($n = 3$) and its 2D projection or a radiograph ($n = 2$). However, these parameters might not be the same.

2.6 Computing the maximum *a posteriori* estimate

Determination of the MAP estimate eq. (5) that corresponds to a given Besov space prior is equivalent to solving the minimization problem

$$f_{\text{MAP}} = \arg \min_{f \in \mathbb{R}^{d \times d}} \left\{ \|Af - m\|^2 + \delta \|f\|_{B_{pp}^s}^p \right\}. \quad (11)$$

The solution to eq. (11) was also found in the wavelet domain since the evaluation of the Besov norm and its gradient is done there. From the computational point of view the values $p > 1$ should only be considered since in the case $p = 1$ the objective functional is not differentiable.

The two-point gradient method of Barzilai and Borwein [16] was used for the optimization. This method always chooses the step in the direction of negative gradient of the objective function. However, owing to an effective

Figure 4: Magnitude of wavelet coefficients as a function of wavelet scale as calculated with $p = 2$ for the x-ray transmission data inserted in fig. 5. The slope of the regression line (*i.e.* α) is about 1.26.

Figure 5: The upper bound for s as a function of p for the x-ray transmission image shown in the inset, and the region of possible smoothness parameters (shaded area). The point (s, p) chosen for the Besov norm is marked by a cross.

step length choice, it converges faster and is less affected by ill-conditioning than the classical steepest-descents method. Also, the method only uses vectors in the search space and gradients of the objective function, both of the size d^2 . No matrices of size $d^2 \times d^2$ are explicitly needed, which allows large-scale implementation of the method.

The Barzilai-Borwein method is defined iteratively by the updating rule

$$x_{k+1} = x_k - \alpha_k \nabla g(x_k) \quad , \quad (12)$$

where $g : \mathbb{R}^n \rightarrow \mathbb{R}$ and the increment α_k is given by

$$\alpha_k = \frac{s_k^T s_k}{s_k^T h_k} \quad (13)$$

with $s_k = x_k - x_{k-1}$, $h_k = \nabla g(x_k) - \nabla g(x_{k-1})$, and s_k^T the transpose of s_k .

3 Results

In fig. 6 we show an optical transmission image (m , left) and its deconvoluted map (f , right) as determined from eq. (2) such that the inversion was performed by solving eq. (11) in which the Barzilai-Borwein optimization method with Besov regularization was used. The parameter p was chosen as 2, and the corresponding s value 0.2 (see the point crossed in fig. 5) was found to give the minimum SEME value of eq. (14) for several test samples. The x-ray transmission image is shown in the middle panel of the figure. The effect of the method applied is, as desired, to remove most of the 'diffusion' the transmitted light has undergone inside the sample as a result of reflections from fibre surfaces. The degree of similarity of the three transmission images, optical, x-ray, and deconvoluted optical, must be analyzed quantitatively in order to see how much closer the deconvoluted optical image is to the x-ray image than the original optical image.

To this end, the entropies of the error signals (*i.e.* pointwise grayscale differences between two compared images) were determined by SEME as defined in eq. (14) [17]. The SEME function was chosen as the measure of similarity as it is specifically designed for determining the degree of image distortion unlike the more common measures such as, *e.g.*, mean squared error (MSE) and peak signal-to-noise ratio (PSNR)[17]. The absolute values

Figure 6: Optical transmission image (left) of a paper sample, its x-ray transmission image (middle), and a deconvoluted (corrected) optical transmission image (right).

of the minimum and maximum intensities were used in this expression so as to prevent certain types of differences from canceling out each other.

$$\text{SEME} = \frac{1}{k_1 k_2} \sum_{j=1}^{k_2} \sum_{i=1}^{k_1} \left[\frac{|I_{max;i,j}^w| - |I_{min;i,j}^w|}{|I_{max;i,j}^w| + |I_{min;i,j}^w|} \text{MSE}(I_{i,j}^w) \right], \quad (14)$$

in which I is the error signal, max and min values were calculated for 4×4 pixel blocks, *i.e.*,

$$I_{max;i,j}^w = \max \{ I_{i,j}^{1,1}, I_{i,j}^{1,2}, \dots, I_{i,j}^{4,3}, I_{i,j}^{4,4} \}$$

$$I_{min;i,j}^w = \min \{ I_{i,j}^{1,1}, I_{i,j}^{1,2}, \dots, I_{i,j}^{4,3}, I_{i,j}^{4,4} \},$$

and

$$\text{MSE}(I_{i,j}^w) = \frac{1}{16} \left[\sum_{m,n=1}^4 (I_{i,j}^{m,n})^2 \right].$$

The smaller the SEME value, the more similar are the two images.

For SEME we found a value of 38.98 (2.00) when comparing the optical (deconvoluted optical) image with the x-ray transmission image. It is evident that when the present deconvolution method is applied to optical transmission images, the deconvoluted images resemble the corresponding x-ray transmission images significantly better (for the measure of similarity used) than the original optical images.

To compare the images further, we also applied a gradient-based orientation analysis method, the so-called structure tensor (ST) analysis [18, 19].

ST attempts to find the directions θ_m for which the L^2 norms of the local directional derivatives are maximized. The orientations of the fibres are then perpendicular to the directions of the gradients. The orientation distribution curves such determined for the original optical, x-ray and deconvoluted optical images are depicted in fig. 7. It is evident that there is no particular orientation visible in the distribution determined for the original optical image, although there is a clear preferred orientation of fibres in a direction close to zero degrees (horizontal direction in the image) in the distribution determined for the x-ray transmission image. In the distribution determined for the deconvoluted optical image this preferred orientation is however recovered to a fairly high accuracy. Only the orientational anisotropy is somewhat smaller than in the distribution for the x-ray image.

We also made a comparison of the Besov space based regularization with the more traditional Tikhonov regularization method [20], or equivalently, used $\pi(f) = C \exp(-\delta \|\nabla f\|_{L^2(\mathbb{R}^2)})$ as the prior. The results of this comparison are shown in Table 1. It is evident from this table that the Besov space based method gives better results for the deconvoluted images. We found in particular that the Besov space based method can reproduce the finest details of the figure better than the Tikhonov method.

Table 1: SEME values for the similarity of x-ray transmission and corrected optical transmission images as determined by the Besov space based and Tikhonov methods. The values for the two methods with the same δ value are not directly comparable; the main feature is that the Besov space approach gives the smallest SEME value.

δ	Besov	Tikhonov
0.1	9.03	13.80
0.01	5.72	9.62
0.001	2.00	16.64
0.0001	16.66	17.93
0.00001	17.13	27.20
0.000001	17.05	24.32

Figure 7: Orientation distribution for the original optical, deconvoluted optical and x-ray transmission images as determined by a structure tensor analysis.

4 Conclusions

Disparity of optical transmission images of sheets of inhomogeneous materials such as paper with the corresponding transmission images obtained by x-rays or beta radiation, is an old unsolved problem. In order to at least partly solve this problem a new inversion method was introduced by which the optical transmission images can be corrected so as to better resemble, *e.g.*, the x-ray transmission images. In this method a prior based on a Besov space classification of the actual structure, was used for defining the regularization scheme. This regularization is known to be less affected by sharp edges (singularities) in the structure. In the optimization the Barzilai-Borwein method was applied, which has proved to converge faster and to need less resources than more traditional optimization methods.

It was demonstrated for paper samples that, with the methods introduced, optical transmission images can indeed be transformed so that they fairly closely resemble the ones that reflect the true areal distribution of mass, determined here by x-ray transmission. Similar results were obtained for other samples, but those results are not shown here. It was also demonstrated that the Besov space based regularization introduced gives better results than the traditional Tikhonov regularization. As a further measure of similarity an orientational analysis of the three different transmission images was performed. The clear preferred orientation of fibres evident in the x-ray transmission image was lost in the optical transmission image, but it was fairly accurately recovered in the deconvoluted optical image. We can conclude that the method introduced here can provide rather reliable results for properties related to the areal mass distribution even when they are determined from optical transmission images. It is evident, however, that this method is only the first step towards an actual retrieval of the true (2D) material distribution from optical transmission data, especially for systems with multiple material components with varying dielectric properties.

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