Divergence control of a one-level supply chain replenishment rule

Fernanda Strozzi, Carlo Noé, Jose M Zaldivar Comenges

Divergence control of a one-level supply chain replenishment rule

Journal:	*International Journal of Production Research*
Manuscript ID:	TPRS-2011-IJPR-0037
Manuscript Type:	Original Manuscript
Date Submitted by the Author:	09-Jan-2011
Complete List of Authors:	Strozzi, Fernanda; Cattaneo University-LIUC Noé, Carlo; Cattaneo University-LIUC Zaldivar Comenges, Jose; European Commission, Joint Research Centre
Keywords:	SUPPLY CHAIN DYNAMICS, SUPPLY CHAIN MANAGEMENT, CONTROL, BULLWIP EFFECT
Keywords (user):	

Divergence control of a one-level supply chain replenishment rule

Fernanda Strozzi, Carlo Noè and José-Manuel Zaldívar[†]

Carlo Cattaneo University (LIUC), Engineering Faculty, 21053-Castellanza, Italy. Fax: +39 0331 480476; Tel:+39 0331 572364; E-mail: fstrozzi@liuc.it ; cnoe@liuc.it

[†]*European Commission, Joint Research Centre, TP202, 21020- Ispra, Italy. Fax: +39 0332 789328; Tel: +39 0332 789202; E-mail: jose.zaldivar-comenges@jrc.ec.europa.eu*

Abstract. We present a general control and improvement strategy for one-level supply chain based on maintaining the divergence of the system close to zero at each time step. The on-line implementations as well as the results obtained are shown for a logistic chain model with an Order-Up-To policy using several demand patterns. The divergence can be obtained using the state space volume calculated with all the state variables of the model. However, it is also possible to calculate the divergence by applying state space reconstruction techniques using only one state variable. The results obtained with both approaches show that this strategy allows the reduction of the total cost.

Keywords. On-line control, supply chain management, order policy.

1. INTRODUCTION

An optimal ordering policy for a supply chain should be able to keep production levels close to demand and, at the same time minimize capacity requirements (Geary et al., 2006). However, a typical effect that arises in supply chains is the bullwhip effect. This effect refers to the phenomenon that occurs when orders to the supplier have larger variance than the ones from the customers, i.e., variance amplification. In addition, this effect is propagated along the supply chain producing different effects at each levels (Coppini et al., 2010).

For this reason, different techniques for reducing the bullwhip effects and the associated total cost in a supply chain have been developed based, amongst others, on: improving demand forecast (Gaalman and Disney, 2006, Disney and Lambrecht, 2008); improving the communication in the supply chain (Geary et al., 2006) and application of a proportional controller to the ordering policy (Magee, 1958; Deziel and Eilon, 1967; Towill 1982; Matsuyama, 1997; Chen and Disney 2003, amongst others). Furthermore, methodological approaches for studying the bullwhip problem have included the use of genetic algorithm (GA) to optimize the order policy (Lee et al., 1997, Disney et al., 2000; Sudhir and Chandrasekharan, 2005; Strozzi et al., 2007) and the application of control theory methodologies to the supply chain, recently summarized by Sarimveis et al. (2008). These control methodologies vary from the application of a simple proportional controller to the ordering policy (Chen and Disney, 2007) to highly sophisticated techniques such as model predictive control (Tzafestas et al., 1997).

In a recent paper, Strozzi et al. (2008) applied a new approach for total cost reduction in a single-product one-level supply chain with an Order-Up-To (OUT) replenishment policy, which is a standard algorithm used to balance demand and inventory. This is accomplished by reviewing and ordering, at a defined period, the goods necessary to keep the inventory up-to a defined level (Gilbert, 2005). The control strategy proposed was based on maintain the divergence of the system, which is related to the stability of the supply chain. The foundation of this approach is on maintaining the divergence of the system at a value lower than one, i.e. $|div| < 1$. The divergence of the system, which is a scalar quantity, is defined as the trace of the Jacobian of the ordinary differential equations that model the supply chain. The divergence is a measure of the stability of a dynamic system, the supply chain in our case, in the sense that it gives the rate of expansion or contraction of infinitesimal volume in the state space (Arnold, 1973). Applying this method, Strozzi et al. (2008) showed that it is possible to reduce the total cost as well as, in some cases, the bullwhip effect. This analysis was carried out off-line, using the analytical values of the divergence calculated from the model equations.

However, the analytical values of the divergence are not suitable for developing an on-line approach which would allow to control the order policy in real-time. For this reason, in this work, we have extended the methodology by using approximated values of the divergence at time *t* obtained from the time series of state space variables (Orders, Net Stock and Real Demand) and, then, used this value in the control algorithm to obtain a new order at time *t+1*. We have compared this approach, in terms of reduction of the total cost, with the case of a classical OUT order policy as in Chen and Disney (2003) and (2007).

Additionally, given that not all state variables are measurable in a real supply chain, we have also analysed the extension of this approach when only one state variable is known. In this case, we have applied state space reconstruction techniques based on time-delayed vectors (Takens, 1981; Packard et al., 1980). The results using several demand patterns (Gaussian, constant with a jump and periodic) are presented and discussed.

2. METHODS AND APPROACH

2. 1. The supply chain and order policy model

Let us consider the dynamics of a single level supply chain represented by orders (O_t), Net stock (NS_t) and the estimated demand (D_t). The replenishment lead-time is zero, with only a single, order-of-events review period. According to Cannella and Ciancimino (2010), this replenishment rule, also known as Inventory and Order Based Production Control System (IOBPCS) order policies (Coyle, 1977), has a long history in production and inventory control (Magee, 1958; Deziel and Eilon, 1967; Towill 1982; Matsuyama, 1997) and it has been recently applied and popularised by Chen and Disney (2003) and (2007). The equations of this model can be written as:

$$O_t = D_t + \frac{1}{T_i}(k \cdot \sigma_{D_t^R} - NS_t) \quad (1)$$

$$NS_t = NS_{t-1} + O_{t-1} - D_{t-1}^R \quad (2)$$

$$D_t = \rho(D_{t-1} - \mu) + \theta(D_{t-1}^R - D_{t-1}) + \varepsilon_t + \mu \quad (3)$$

where $1/T_i$ is the proportional controller proposed by Chen and Disney (2003) and (2007) in the inventory feedback loop, σ is the standard deviation of real demand (D_t^R) and k is the safety factor. The demand is estimated using Eq. (3), which is similar to an ARMA (Autoregressive moving average) model with a mean μ, an autoregressive constant ρ, a moving average constant θ and a forecast error $\varepsilon_t = D_t^R - D_t$ (assumed to be a white noise process). However, our approach has some differences to that of Chen and Disney (2003). We substitute ε_{t-1} at each time step with its real value, and we use an ARMA(1,1) to forecast the demand; whereas Chen and Disney (2003) and (2007) supposed that the real demand was an ARMA(1,1) and they used a moving average to forecast it.

The Eqs. (1)-(3) can be written in matrix form:

$$\begin{bmatrix} O_t \\ NS_t \\ \overline{D}_t \end{bmatrix} = \begin{bmatrix} -1/T_i & -1/T_i & \rho-\theta \\ 1 & 1 & 0 \\ 0 & 0 & \rho-\theta \end{bmatrix} \begin{bmatrix} O_{t-1} \\ NS_{t-1} \\ \overline{D}_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{T_i}(k\cdot\sigma + D_{t-1}) - \mu\cdot\rho + \theta\cdot D_{t-1} + \varepsilon_t + \mu \\ -D_{t-1} \\ -\mu\cdot\rho + \theta\cdot D_{t-1} + \varepsilon_t + \mu \end{bmatrix} \quad (4)$$

The (3x3) matrix on the right-hand side is the Jacobian matrix, *J*, of the system i.e., the matrix of all first-order partial derivatives. The stability of the dynamics is given by the eigenvalues $\lambda_1, \lambda_2, \lambda_3$ of *J* matrix which are given by:

$$\lambda_1 = 0, \lambda_2 = \rho - \theta, \lambda_3 = 1 - \frac{1}{T_i} \quad (5)$$

Local stability is ensured if all the eigenvalues are smaller than one in absolute value, then

$|\rho - \theta| < 1$, implying $-1 < \rho - \theta < 1$, $\left|1 - \frac{1}{\mathrm{T_i}}\right| < 1$ and therefore $T_i > \frac{1}{2}$.

The divergence of the dynamic system, given by Eqs. (1)-(3), is:

$$div = \lambda_1 + \lambda_2 + \lambda_3 = \rho - \theta - \frac{1}{T_i} + 1 \quad (6)$$

For the case T_i=1 (i.e., no proportional control), our baseline scenario, the divergence is identical to λ_2, and so the required $|div| < 1$ stability condition is satisfied.

In Strozzi et al. (2008) we applied $|div| < 1$, by choosing T_i such as the right-hand side of Eq. (6) becomes <1, as the control goal function and we showed that a reduction of costs was achieved. In the present paper our objective is to develop a control and improvement strategy that reduces the *div* of the system, by approximating the divergence using only measured variables. Therefore, in this case, the knowledge of the analytical expression, Eq. (6), is not necessary.

To assess the performances of the different control strategies, we have defined, following Chen and Disney (2007), the total cost as the sum of ordering, and the holding and shortage costs. These costs may be calculated as:

$$\text{Ordering costs}: OC_t = \begin{cases} c \cdot O_t & \text{if } O_t < K \\ c \cdot K + c_0 \cdot (O_t\text{-}K) & \text{if } O_t \geq K \end{cases} \tag{7}$$

$$\text{Holding costs}: HC_t = \begin{cases} c' \cdot NS_t & \text{if } NS_t > 0 \\ 0 & \text{if } NS_t \leq 0 \end{cases} \tag{8}$$

$$\text{Shortage costs}: SC_t = \begin{cases} c'' \cdot |NS_t| & \text{if } NS_t < 0 \\ 0 & \text{if } NS_t \geq 0 \end{cases} \tag{9}$$

$$\text{Total cost}: TC_t = OC_t + HC_t + SC_t \tag{10}$$

In addition, we have considered a specific case defined, similarly to Chen and Disney, (2003) and (2007) as: the average demand is μ=4 units per period, the normal production cost per unit in normal production is c=100 € per production period, whereas c_0 is 200 € per unit per period for overtime production . The inventory holding cost is c'=10 € per unit per period, the shortage cost is c''=50 € per unit per period, and the capacity limit K=12 units per period. The inventory safety factor is set to $k\sigma_D = 0.2\mu$. Finally, several demand patterns have been analyzed: Gaussian, constant with a jump and periodic. We have considered the case of Gaussian demand as Chen and Disney, (2003) and (2007) did, which represents a stationary demand with noise; the jump demand represents a sudden variation in the former stationary demand which is normally difficult to predict and the periodic demand represents a seasonality in the demand. In this way we have check the robustness of the control strategy proposed against typical demand patterns.

2.2. State space divergence reconstruction using experimental data

The divergence of a dynamic system is related to the evolution of an infinitesimal volume in the state space by the Liouville theorem as follows (Arnold, 1973)

$$\frac{dV_t}{dt} = \int_{\Gamma(t)} div[F(\mathbf{x})]dx_1 \dots dx_n \tag{11}$$

where x_1, x_2,...x_n are the state variables in an n-dimensional state space. By solving the Liouville equation, we obtain

$$V_{t+h} = V_t \cdot \exp\left[\int_t^{t+h} div[F(\mathbf{x})]d\tau\right] \tag{12}$$

where h is a infinitesimal time interval, then

$$div(t) = \frac{1}{h}\log\left(\frac{V_{t+h}}{V_t}\right) \tag{13}$$

and, taking the first term of the Taylor expansion of the logarithm, the divergence of a dynamic system can be calculated using the state space volume, V_t, (Strozzi et al., 1999):

$$div = \frac{\dot{V}_t}{V_t}\,. \tag{14}$$

Using the state variables values of the system, it is possible to calculate the state space volume, V_t, using different realizations of the system (Eckman and Ruelle, 1985) or different points along the same realization as in Bosch et al. (2004 a,b):

$$V_t = \left|\det\begin{bmatrix} O_t - O_{t-\Delta t} & 0 & 0 \\ 0 & NS_t - NS_{t-\Delta t} & 0 \\ 0 & 0 & D_t - D_{t-\Delta t} \end{bmatrix}\right| \tag{15}$$

where Δt is a short time delay for which it is assumed that the Jacobian of the system has not substantially changed. In this paper, we impose that Δt is equal to two weeks.

If not all the state variables are accessible for the calculation, then it is possible to use the theory of embedding, which allows us to express a temporal time series of measurements, $s(t)=h[\boldsymbol{x}(t)]$, of the state variables, $\boldsymbol{x}(t)$, on an equivalent state space that is -in a topological sense- similar to the original dynamic system. Techniques of state space reconstruction have been introduced by Packard et al. (1980) and Takens (1981), who showed that it is possible to address this problem using time delay embedding vectors of the original measurements, i.e. $\{s(t), s(t\text{-}\Delta t), s(t\text{-}2\Delta t),\ldots, s(t\text{-}(d_E\text{-}1)\Delta t)\}$. Δt and d_E are respectively the time delay and the dimension of the space required to preserve equivalent topology.

A detailed discussion of the influence of reconstruction parameters on the results and the application of other reconstruction techniques is out of the scope of this paper. The interested reader is referred to the books of Abarbanel (1996) and Kantz and Shreiber (1997) and references therein, that provide a comprehensive discussion of the assumptions and validity of the approach. In the context of non-stationarity, the notion of a "correct" embedding or delay is inappropriate, as has been demonstrated by Grassberger et al. (1991). Instead it is important to remember that a sufficiently large embedding should be chosen in order to describe the relevant dynamics as well as to take into account the effects of noise that tends to artificially inflate the dimension. In the present work we have used $d_E = 3$ and $\Delta t = 2$ (two weeks). We tested different values of the reconstruction parameters, time delay and embedding dimension, both values were selected being the minimal values that allowed a proper reconstruction when compared with analytical values. Smaller values did not allow a proper reconstruction of the state space while higher ones did not improve the results and need longer time series. With these parameters, we have obtained:

$$V_t = \left| \det \begin{bmatrix} NS_t - NS_{t-\Delta t} & 0 & 0 \\ 0 & NS_{t-\Delta t} - NS_{t-2\Delta t} & 0 \\ 0 & 0 & NS_{t-2\Delta t} - NS_{t-3\Delta t} \end{bmatrix} \right| \quad (16)$$

then

$$\dot{V}_t = \frac{\Delta V_t}{h} = \frac{V_t - V_{t-h}}{h} \quad (17)$$

where h is a convenient time step. In this work h is chosen equal to one time step, i.e. one week; however other numerical approximations of the derivatives can be selected (Burden and Faires, 1996).

2.3. Control and on-line improvement strategy

The numerical calculation of the divergence for dissipative systems and sign checking ($div < 0$) may give rise to numerical problems due to the fact that dissipation shrinks state space volumes exponentially (see Eq. (12) and Zaldívar et al., 2005). For this reason, instead of calculating $div = \dot{V}_t / V_t$, we have proposed a simple control strategy that consists in trying to keep both values: V_t and ΔV_t close to zero, by acting simultaneously in a sort of PD (Proportional-Derivative) controller as follows:

$$T_i^{t+1} = T_i^t + K_p \cdot (0 - V_{t+1}) + K_d (0 - \Delta V_{t+1}) \quad (18)$$

In this sense, we have acted on the parameter T_i in order to adapt the order policy depending on the dynamics of the supply chain. The objective is, therefore, to change, at each time step, the proportional control constant introduced by Chen and Disney (2003, 2007) as a function of the state space volume and its variation. It should be noted that this control strategy is different from the one defined in Strozzi et al. (2008) whereby $|div| < 1$ was applied. The objective of this strategy is to try to reduce the state-space volume, and its variation, at each time step. The basic idea behind this is that the divergence measures the relative rate of increase of the state space volume of the dynamic system under study. When one is trying to control a dynamic system, the final objective is, usually, to avoid sudden or abrupt changes.

In mechanical systems it can be shown that the total energy of the system is directly proportional to the state space volume and its variation is related to changes of total energy (Zaldívar and Strozzi, 2010). In this sense, we assume that, by maintaining the analogous system energy low, one should be able to have a smooth operation reducing the total cost and the bullwhip effect. On the other hand, if one allows the system energy to increase its reaction to a sudden change will be more abrupt and therefore it will tend to increase the operation cost and the bullwhip effect.

Mechanic/Hydraulic analogies (e.g., “water hammer” proposed by Holweg et al., 2005; and “tap in a shower” proposed by Disney et al. , 2006) for tuning the weight of the proportional controller in the ordering policy have been already reviewed in Cannella and Ciancimino (2010). However, in our case the divergence has a clear physical meaning since it is related to the energy of a system.

It should be noted that Eq. (18) is only one of many control strategies that could be implemented. However, in this work we are interested in assessing if the general approach is feasible, therefore, we have not explored other control algorithms that could provide better results. Probably, the control strategy should to be selected and adapted depending on the specific real system considered and on the data available.

3. RESULTS AND DISCUSSION

In order to show the characteristics of the proposed methodology, we have considered a typical case in which a Gaussian demand with a step occurs. Figure 1 represents one simulation of the real and forecast demand, the orders and the net stocks when no control is applied, T_i=1, whereas in Fig. 2 the values of the state space volume and its variation are shown. It is possible to observe that the values obtained using the three state variables: O_t, NS_t and D_t, are qualitatively similar compared to the ones obtained using only one state space variable, NS_t, and its delayed vectors. Figure 3 compares the divergence calculated using Eq. (14) with the analytical divergence provided in Eq. (6).

[Figure 1]

[Figure 2]

[Figure 3]

As demonstrated in Strozzi et al. (1999), the divergence is preserved under state space reconstruction. However, this holds only for the case in which there is no noise and the dynamic system has reached the final attractor (Takens, 1981). Since in the example considered these conditions are not satisfied, one should expect that the divergence will not be perfectly reconstructed. This can be observed during the step period, where there is a considerable discrepancy between the numerical and analytical values. As mentioned earlier, the calculation of the divergence can pose numerical problems (Zaldívar et al., 2005) that increase with the amount of noise present in the system. For this reason, it is preferable to use state space volume and its variation, ΔV_t.

A general discussion on techniques to reduce the problems of noise as well as other state space reconstruction approaches can be found in Bosch et al. (2004a) and references therein. In the present work, we are only interested in assessing the validity of the approach applied to supply chains and for these reasons we have not tried to optimize systematically the reconstruction, the embedding parameters or the control strategy. In any case this problem is highly dependent of the system under study and general rules are difficult to develop.

The idea of the control strategy given by Eq (18) is to reduce V_t and ΔV_t. In this sense, maintaining both values close to zero at each time step is similar to maintaining the system in the stability region given by the condition $|div|<1$ that, as has been shown by Strozzi et al. (2008), this reduces the total cost.

The reduction of costs by the application of the control approach defined by Eq. (18) using the complete state space, Eq. (15), and the reconstructed state space based on one variable, Eq. (16), is shown in Figs. 4-6. In this case, three demand patterns have been considered: Gaussian noise (Fig. 4a), constant with a jump (Fig. 5a) and periodical (Fig. 6a). The logarithmic value of the mean total cost by using the proportional control $(1/T_i)$ is also represented as well as the values obtained using $|div|<1$, as in Strozzi et al. (2008). To calculate the mean total cost, we have performed 20 runs (of 30 weeks length), for each pair of ρ and θ values between -1 and 1. The intervals [-1,1] are divided in sub-intervals of widths 0.1. It can be observed that the total cost is considerable reduced in all the cases, by using the proposed control strategies, in particular, for T_i=1, which corresponds to the case without proportional control, see Table 1. The only exception is the case of T_i=0.5 and Gaussian noise demand but, in this case, all values obtained are quite similar. In conclusion the two control strategies, based on reducing the values of V_t and ΔV_t, give similar results, even without knowledge of the complete set of the state variables, i.e. when the state space is reconstructed using only one measurement.

[Figure 4]

[Figure 5]

[Figure 6]

[Table 1]

To analyse in detail the costs reduction obtained, Figures 7-9 represent the mean logarithmic total cost surfaces as a function of the parameters ρ and θ for the three types of control considered, in the case of a periodic demand. The white regions in the figures are those in which the total cost is reduced applying one of the control strategies compared with the case of no control strategy, i.e., $T_i=1$. Similar results could be obtained for the other demand patterns analyzed: Gaussian and constant with a jump. In the vast majority of cases, the application of our control strategies reduces the total cost especially in the corners $0<\theta<1$ and $-1<\rho<0$ where the total cost is higher. As it can be observed, the reduction of the total cost increases using control strategy given by Eq (18), when compared with that of control using $|div|<1$ (Strozzi et al., 2008), see Figs 7-9. This is probably due to the fact that by trying to maintain V_t and ΔV_t as close to zero as possible, one is imposing a more restrictive criterion. It seems that better results are obtained using state space reconstruction, but this is probably due to the different values of the constants, K_p and K_d, of the controller.

[Figure 7]

[Figure 8]

[Figure 9]

4. CONCLUSIONS

A general control and improvement strategy is presented for a single-product one-echelon supply chain, in which an Order-Up-To (OUT) order policy is applied. In the case analysed the replenishment lead-time is zero, with only a single order-of-events review period. The strategy is based on reducing state space volume and its first derivative close to zero, at each time step. The implicit assumption is that changes in the state space volume would produce an increase in the costs and amplify the bullwhip effect in the supply chain. This is due to the fact that the state space volume of a system is related to its energy, at least in mechanical and chemical systems (Zaldívar and Strozzi, 2010), and therefore we try to avoid changes in these values.

The on-line implementation and the results obtained have been discussed using an OUT policy with several demand patterns. The analysis of the results has demonstrated that this type of control allows the reduction of costs. In a first step, state space volume has been obtained using the state variables obtained from the model. In addition state space reconstruction techniques from nonlinear dynamic systems theory has also been implemented to show that it is possible to use only the knowledge of one state variable for controlling the supply chain. This allows the extension of this methodology to real supply chains by using on-line monitoring data. Future work will be devoted to exploring this possibility.

Noise is always a problem when applying state space reconstruction techniques (Kantz and Schreiber, 1997) and when trying to develop a control strategy. Therefore, with high levels of noise our approach will probably need the application of non-linear noise reduction schemes (Schreiber and Ritcher, 1999) that have been proven to outperform traditional filtering techniques for dynamic systems.

Acknowledgements. The authors gratefully acknowledge Drs. C.N. Murray and E. Gutiérrez for the revision of English language, and the financial support of the European Commission DG RTD funded MANMADE NEST Project (Contract n° 043363) and Fondazione CARIPLO.

REFERENCES

Abarbanel, H.D.I., 1996. *Analysis of Observed Chaotic Data*. Springer-Verlag, New York.

Arnold, V. I., 1973, Ordinary differential equations, MIT Press, Cambridge.

Bosch, J., Strozzi, F., Zbilut, J. P., Zaldívar, J. M., 2004a. On line application of the divergence criterion for runaway detection in isoperibolic batch reactors: Simulated and experimental results. Computers and Chemical Engineering 28, 527-544.

Bosch J., Kerr, D. Snee, T. J., Strozzi, F. and Zaldívar, J. M., 2004b. Runaway detection in a pilot plant facility (Pilot-scale evaluation of runaway detection methodology). Industrial & Engineering Chemistry Research 43, 7019-7024.

Burden, R. L., Faires, J. D., 1996. *Numerical analysis* (3rd ed.). Boston: PWS.

Cannella, S. and Ciancimino, E., 2010. On the bullwhip Avoidance Phase: supply chain collaboration and order smoothing. International Journal of Production Research 48, 6739 – 6776.

Chen, Y. F., Disney, M. S., 2003. The order-up-to policy "sweet spot" - Using proportional controllers to eliminate the bullwhip problem, in EUROMA POMS Conference, Como Lake, Italy (2003) Vol. 2, 551-560.

Chen Y.F., Disney S.M. 2007. The myopic order-up-to policy with a feedback controller, International Journal of Production Research 45, 351-368.

Coppini M., Rossignoli C., Rossi T., Strozzi F., 2010, Bullwhip effect and inventory oscillations analysis using Beer Game model. International Journal and Production Research, 48, 3943 – 3956.

Coyle, R.G., 1977. Management System Dynamics. New York: John Wiley & Sons.

Deziel, D.P., and Eilon, S., 1967. A linear production-inventory control rule. The Production Engineer, 43, 93-104.

Disney S.M., Naim M.M. and Towill D.R., 2000, Genetic algorithm optimization of a class of inventory control systems, International Journal of Production Economics 68, 259–278.

Disney, S.M, Farasyn, I., Lambrecht, M., Towill, D.R., de Velde, W. Van, 2006. Taming the bullwhip effect whilst watching customer service in a single supply chain echelon, European Journal of Operational Research, 173, 151-172.

Disney, S.M. and Lambrecht, M.R., 2008. On replenishment rules, forecasting, and the bullwhip effect in supply chains. Foundations and Trends in Technology, Information and Operations Management 2, 1–80.

Eckman, J.-P., Ruelle, D., 1985. Ergodic theory of chaos. Review of Modern Physics 57, 617–656.

Gaalman, G., Disney, S. M. 2006. State space investigation of the bullwhip problem with ARMA(1,1) demand process. International Journal of Production Economics 104, 327-339.

Geary, S., Disney, S.M., Towill, D.R., 2006. On bullwhip in supply chains-historical review, present practice and expected future impact. International Journal of Production Economics 101, 2-18.

Gilbert, K., 2005. An ARIMA supply chain model. Management Science 51, 305-310.

Grassberger, P., Schreiber, T., Schaffrath, C. 1991. Nonlinear time sequence analysis. International Journal of Bifurcations and Chaos 1, 521–547.

Holweg, M., et al., 2005. Supply chain collaboration: making sense of the strategy continuum. European Management Journal, 23 (2), 170-181.

Kantz, H., Schreiber, T., 1997. Nonlinear Time Series Analysis. Cambridge University Press.

Lee, H.L., Padmanabhan, V., and Whang, S., 1997. Information distortion in a supply chain: the bullwhip effect. Management Science, 43 (4), 546–558.

Magee, J.F. 1958. Production Planning and Inventory Control. New York: McGraw-Hill.

Matsuyama, K., 1997. Maintaining optimal inventory level by feedbacks. International Journal of Production Economics, 53 57-69.

Packard, N., Crutchfield, J., Farmer D, Shaw, R., 1980. Geometry from a time series. Physics Review Letters 45, 712-716.

Sarimveis, H., Patrinos, P., Tarantilis, C. D., Kiranoudis, C. T., 2008. Dynamic modeling and control of supply chain systems: A review. Computers & Operations Research 35, 3530-3561.

Schreiber, T., Ritcher, M. 1999. Fast nonlinear projective filtering in a data stream. International Journal of Bifurcations and Chaos 9, 2039-2045.

Strozzi, F., Bosch, J., Zaldívar, J.M., 2007. Beer Game Order Policy Optimization under Changing Customer Demand. Decision Support Systems 42, 2153-2163.

Strozzi, F., Noè, C., Zaldívar, J.M., 2008. Stability control in a supply chain: Total costs and bullwhip effect reduction. The Open Journal of Operational Research 2, 51-59.

Strozzi, F., Zaldívar, J. M., Westerterp, K. R., 1999. Towards on-line runaway prevention in chemical reactors using chaos theory techniques. AIChE Journal 45, 2429-2443.

Sudhir, J. R. D., Chandrasekharan, R., 2005. A simulation-based genetic algorithm for inventory optimization in a serial supply chain. International Transactions in Operational Research 2, 101-127.

Takens F. Detecting strange attractors in fluid turbulence. In Dynamic systems and Turbulence, Berlin: Springer, 1981, pp. 366-381.

Towill, D.R., 1982. Dynamic analysis of an inventory and order based production control system. International Journal of Production Research, 20(6), 671-687.

Tzafestas, S., Kapsiotis, G., Kyriannakis, E., 1997. Model-based predictive control for generalized production planning problems. Computers in Industry 34, 201-210.

Zaldívar, J.M., Bosh, J. Strozzi, F., 2005, Early warning detection of runaway initiation using non-linear approaches. Communication in Nonlinear Science and Numerical Simulation 10, 299-311.

Zaldívar, J.M., Strozzi, F., 2010, Phase-space volume based control of semibatch reactors. Chemical Engineering Research and Design 88, 320–330.

FIGURES AND TABLES CAPTIONS

Table 1. Mean total cost (*TC*) and percentage reduction when compared with the application of the classical OUT policy varying ρ and θ parameters using different control strategies applied to the three demands analysed: random (D_{rand}), random with a jump (D_{rand_j}), and periodic ($D_{periodic}$) represented in Figures 4, 5, 6, respectively.

Figure 1. Demand, D_t, (Real -continuous- and forecasted -connected points-), Orders (O_t) and Net Stock (NS_t). Parameters: $\rho = 1$, $\theta = 1$, $T_i = 1$ (no control); Real demand: Gaussian, $\mu = 4$ and $\sigma = 0.1$ with a step at week 20 from 4 to 8.

Figure 2. State space volume, V_t, and its difference, ΔV_t, for the simulation represented in Fig. 1, obtained using the three state space variables (a,b), Eq. (15) and one state space variable (c,d), Eq. (16) with $d_E = 3$ and $\Delta t = 2$ weeks.

Figure 3. Reconstructed divergences, Eq. (11), obtained using V_t calculated applying Eq. (12) and Eq. (13), for the simulations presented in Fig 1. In the top figure the divergence is approximated using D_t, O_t and NS_t, while, in the bottom one, only NS_t is used. In this case, the analytical divergence, Eq. (6), is constant and equal to zero.

Figure 4. a) Real demand: D_t^R is a Gaussian noise with mean $\mu = 4$ and variance $\sigma = 0.5$; b) Logarithm of mean Total Cost, with different control strategies: proportional control with constant T_i (continuous line), T_i modified to stay with $|div|<1$ (Strozzi et al., 2008) using the analytical divergence (…); T_i modified with control strategy given by Eq. 18 and state space volume calculated using three state space variables, Eq. (15) (-.-) , or applying state space reconstruction techniques, Eq. (16) (-*-).

Figure 5. a) Real demand: the same as fig. 4 but adding a jump at $t = 20$ weeks. ; b) Logarithm of mean Total Cost (*TC*), with different control strategies: proportional control with constant T_i (continuous line), T_i modified to stay with $|div|<1$ (Strozzi et

al., 2008) using the analytical divergence (…); T_i modified with control strategy given by Eq. 18 and state space volume calculated using three state space variables, Eq. (15), (-.-) , or applying state space reconstruction techniques, Eq. (16) (-*-).

Figure 6. a) Demand: $D_t^R = 3\cos(1/2t) + \mu + gs$ where gs is a Gaussian noise with mean $\mu = 4$ and variance $\sigma = 0.5$; b) Logarithm of mean Total Cost, with different control strategies: proportional control with constant T_i (continuous line), T_i modified to stay with $|div|<1$ (Strozzi et al., 2008) using the analytical divergence (… line); T_i modified with control strategy given by Eq. 18 and state space volume calculated using three state space variables, Eq. (15), (-.-) , or applying state space reconstruction techniques, Eq. (16) (-*-).

Figure 7. Logarithm of the mean Total Cost surfaces without control, T_i=1 (white surface) and with the control strategy $|div|<1$ (Strozzi et al., 2008) black surface, σ=0.5 as a function of ρ and θ.

Figure 8. Logarithmic of the mean Total Cost surfaces without control, T_i=1 (white surface) and with the control strategy given by Eq. (14) and state space volume calculated using all state space variables, Eq. (12) (black surface), as a function of ρ and θ.

Figure 9. Logarithmic of the mean Total Cost surfaces without control , T_i=1 (white surface) and with the control strategy given by Eq. (14) and state space volume calculated using one state space variables and applying state space reconstruction techniques, Eq. (13) (black surface), as a function of ρ and θ.

Table 1.

	OUT (T_i=1)	a/ |*div*|<1	TC reduction (%)	b/ V and ΔV Eqs. (15),(18)	TC reduction (%)	c/ V and ΔV Eqs.(16),(18)	TC reduction (%)
D_{rand}	$0.429 \cdot 10^8$	$0.261 \cdot 10^8$	39.23	$0.312 \cdot 10^8$	27.33	$0.272 \cdot 10^8$	36.64
D_{rand_j}	$0.582 \cdot 10^8$	$0.477 \cdot 10^8$	17.98	$0.413 \cdot 10^8$	29.05	$0.303 \cdot 10^8$	47.97
$D_{periodic}$	$2.169 \cdot 10^8$	$1.562 \cdot 10^8$	27.97	$1.562 \cdot 10^8$	27.95	$1.359 \cdot 10^8$	37.31

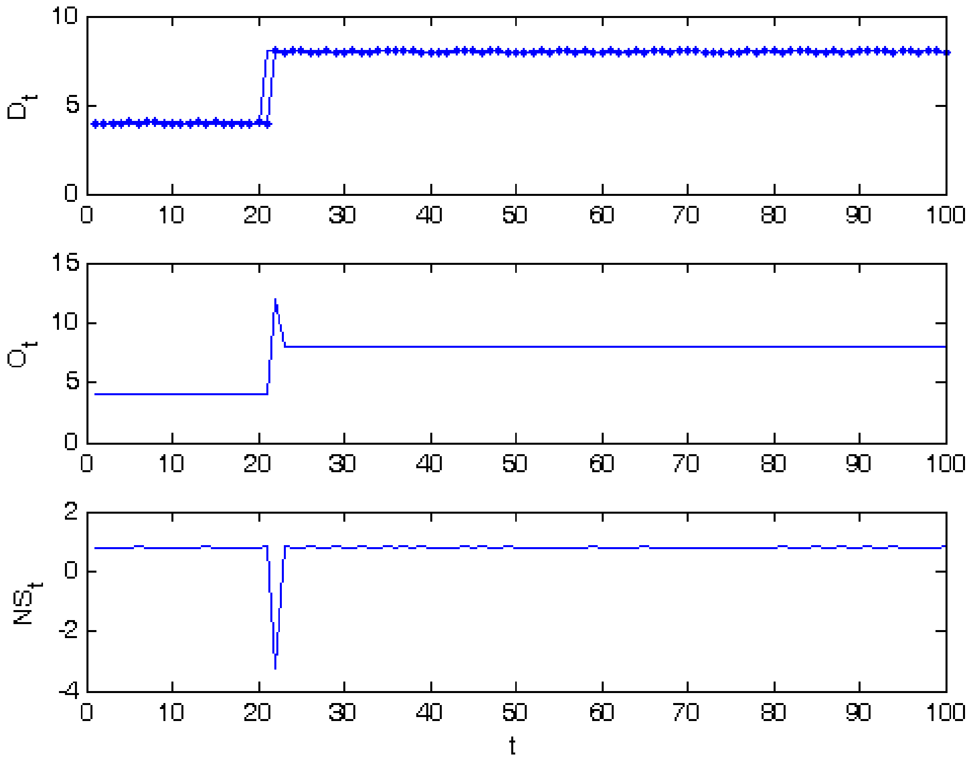
10
5
0
D_t
0 10 20 30 40 50 60 70 80 90 100
15
10
5
0
O_t
0 10 20 30 40 50 60 70 80 90 100
2
0
-2
-4
NS_t
0 10 20 30 40 50 60 70 80 90 100
t

Figure 1.

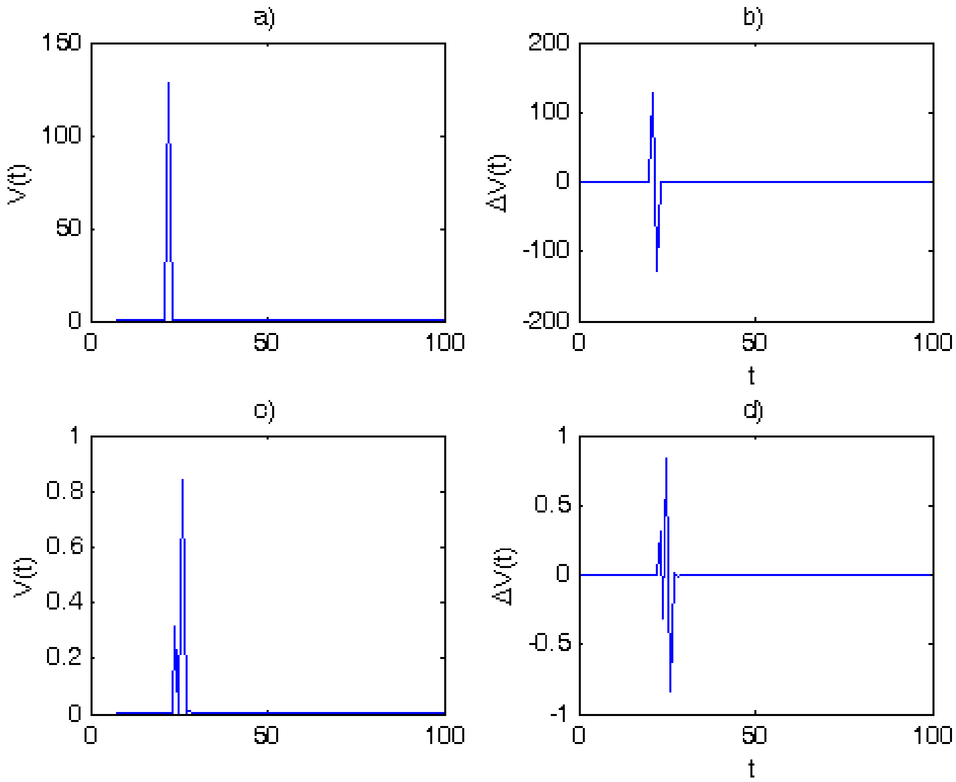

Figure 2.

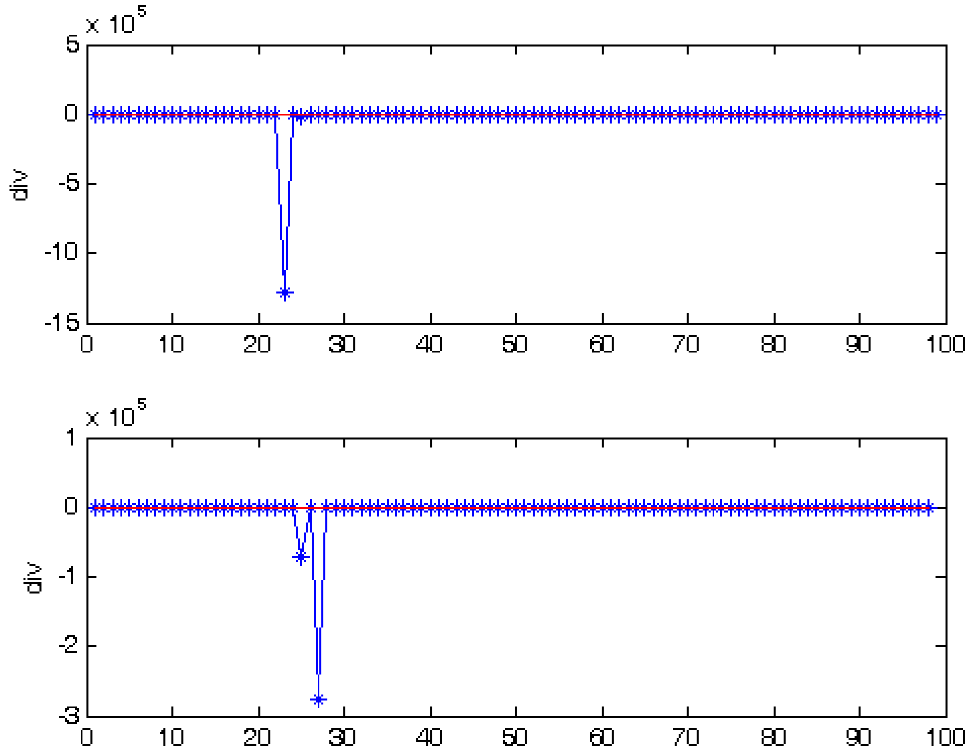
x 10^5
5
0
-5
-10
-15
div
0
10
20
30
40
50
60
70
80
90
100
x 10^5
1
0
-1
-2
-3
div
0
10
20
30
40
50
60
70
80
90
100

Figure 3.

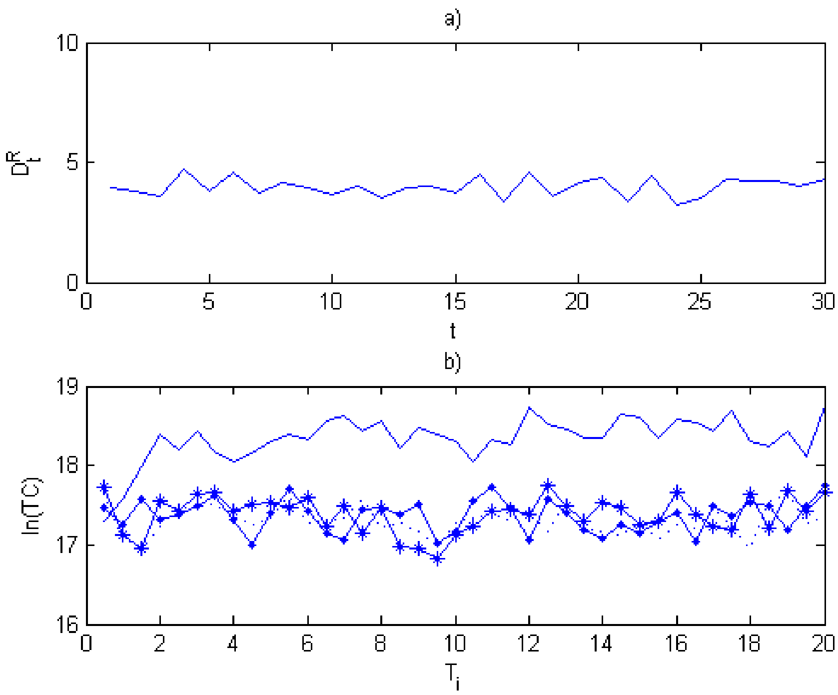
a)
10
5
0
0
5
10
15
20
25
30
t
D_t^R
b)
19
18
17
16
0
2
4
6
8
10
12
14
16
18
20
T_i
ln(TC)

Figure 4.

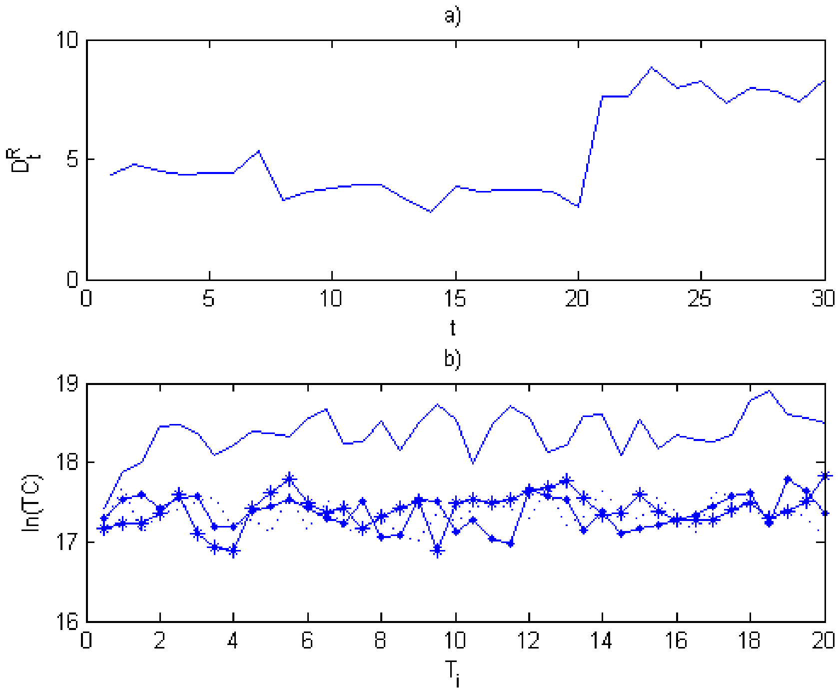
a)
10
5
0
D_t^R
0
5
10
15
20
25
30
t
b)
19
18
17
16
ln(TC)
0
2
4
6
8
10
12
14
16
18
20
T_i

Figure 5.

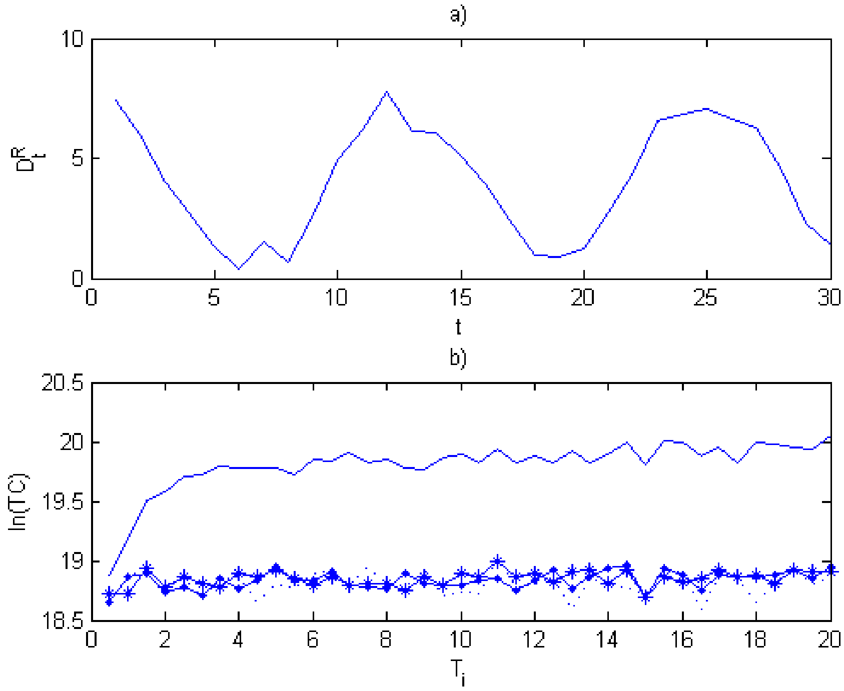
a)
10
5
0
D_t^R
0
5
10
15
20
25
30
t
b)
20.5
20
19.5
19
18.5
ln(TC)
0
2
4
6
8
10
12
14
16
18
20
T_i

Figure 6.

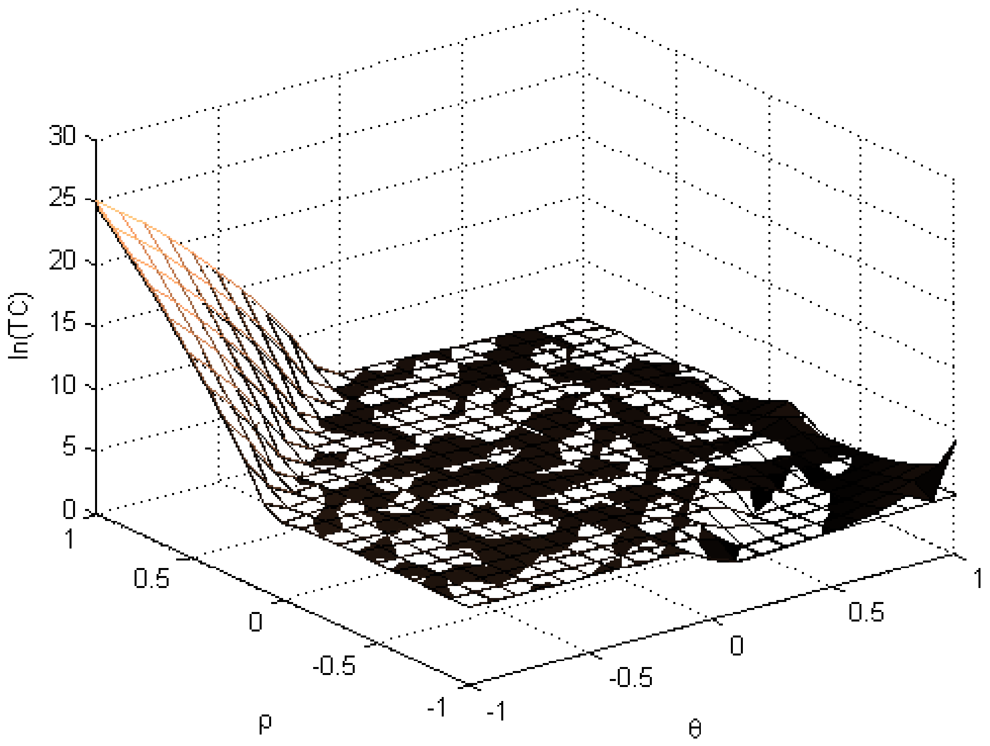

Figure 7.

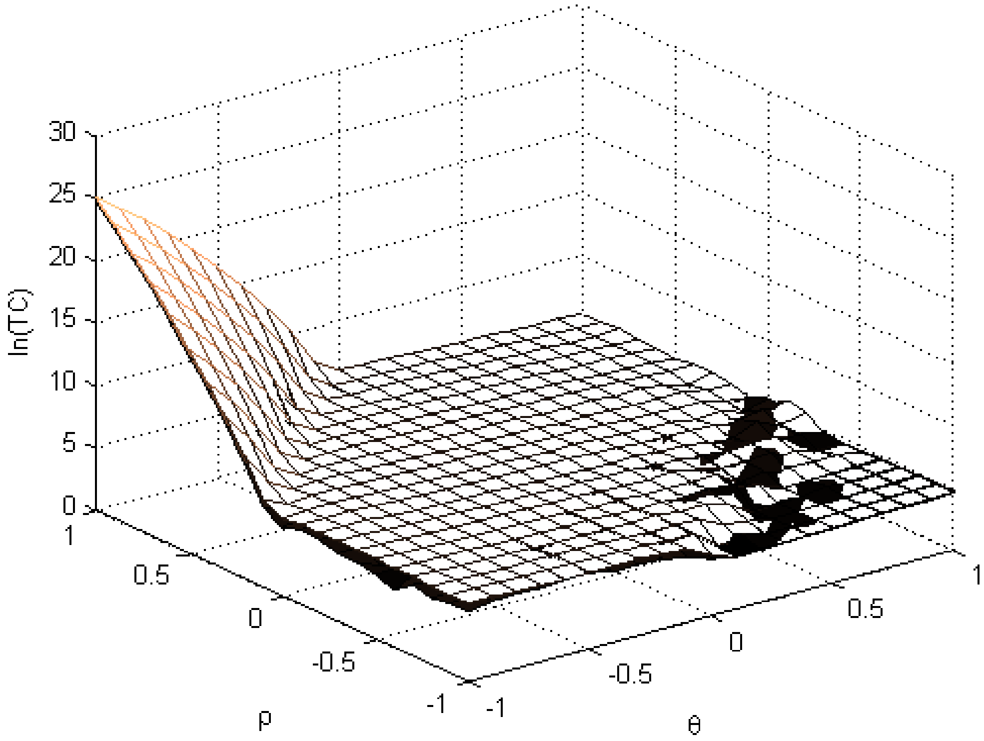

Figure 8.

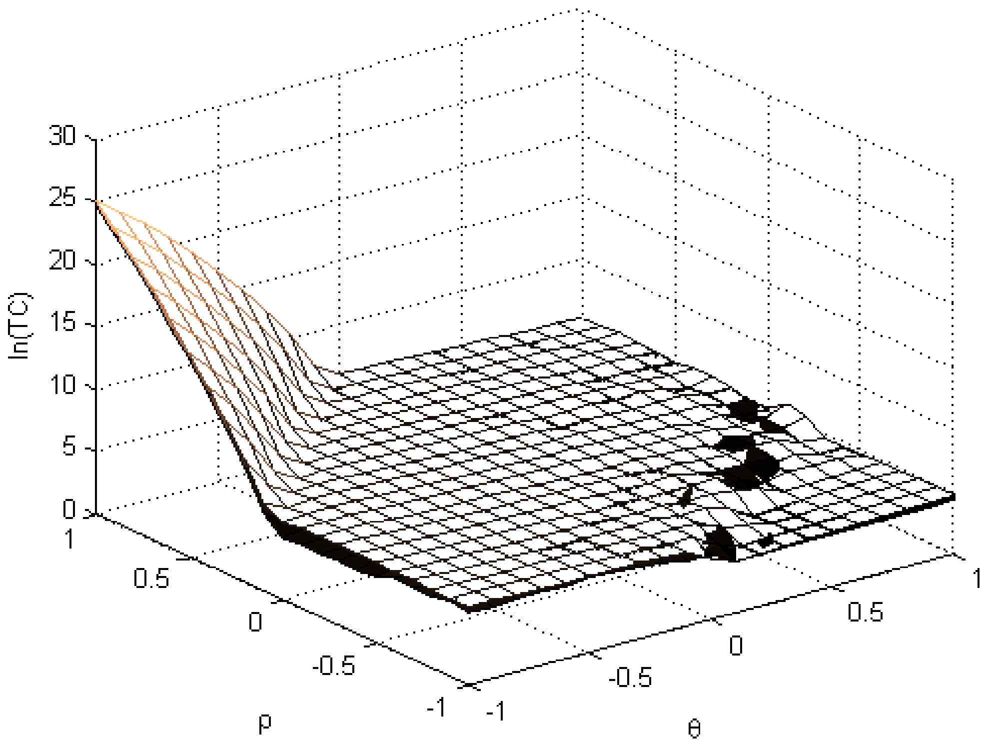

Figure 9.