ELECTRO-MAGNETO-FLUID-STRUCTURAL COUPLING PROBLEM: THE VIBRATING VISCOMETER

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ABSTRACT

This study presents a viscosity sensor that converts the vibration amplitude at the resonance frequency of a needle immersed in viscous fluid, into electric current. The goal of this work is to provide a dedicated numerical tool for the sensor design, that couples fluid, structure and magnetism.

$\vec{n}$	Normal pointing out.
$\vec{V}$	Velocity.
ς	Charge density.
μ_r	Relative permeability.
μ_0	Vacuum permeability.

NOMENCLATURE

Q	Quality factor.
ω	Angular frequency.
R	Radius of the needle.
X	Vibration amplitude.
k_m	Added mass.
k_d	Damping coefficient.
K_0, K_1	Modified Bessel functions of second kind.
Re	Reynolds number.
ρ	Fluid density.
R	Radius of the beam.
μ	Kinematic viscosity.
$\wedge$	Hydrodynamic function.
Γ	Hydrodynamic coefficient.
F	Hydrodynamic force.
$\vec{A}$	Magnetic potentiel vector.
$\vec{H}$	Magnetic field.
$\vec{B}$	Magnetic flux density.
$\vec{E}$	Electric field.
$\vec{D}$	Electric displacement field.
σ	Electric conductivity.
T_2	Stress tensor of air.
$\vec{J}_e$	External current density.

INTRODUCTION

Viscosity sensors using piezoelectric material and magnetostrictive material have been developed to be used in oil, food or cosmetic industries to control on-line processes [1]. However they require high-frequency system. Cantilever structures commonly used in atomic force microscopy [2] as mass-sensitive devices [3], and biosensors [4], in both liquid and gaseous environments are used increasingly in applications involving the viscosity sensing. In this work, a numerical model of a viscometer based on cantilever structures (Fig. 1), including all the physics (magnetism, vibration, fluid mechanics) is proposed. The vibration of the oscillator is due to an electric current through a coil which is under a static magnetic field. When the mechanical oscillator is immersed in a viscous fluid, the vibration amplitude decreases because of the energy dissipation. In these conditions, the quality factor Q that is the ratio of the energy stored in the oscillator to the energy supplied can be significantly reduced. Thereby the goal of this work is the modeling and the semi numerical computation of a fluid-structure-magnetic interaction problem, where commercial finite element programs show huge difficulties in an adequate resolution of fluid, beam and magnetic dynamics due to computational limitations.

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FIGURE 1. VIBRATING VISCOMETER.

FIGURE 2. Schematic view of the sensor.

METHODS AND MATERIALS

Fig. 2 shows schematic view of the sensor. It is composed of a mechanical oscillator, permanent magnets and two coils. A driven alternative current at fixed frequency is applied through the first coil as a vibration source for the oscillator. This vibration generates induced current in the second coil, that is proportional to the vibration amplitude and therefore to fluid viscosity. In this study, Bernoulli-Euler equations are used to model the needle. The action of the surrounding fluid is replaced by the hydrodynamic resistance coefficients reflecting the added mass and

damping coefficients proposed by Tuck [5]. The excitation system is modeled as magneto-structure interaction using Maxwell equations.

Fluid-structure interaction

In case of small vibration, the hydrodynamic force is a function of the vibration amplitude meaning that

$$F = \Lambda(\omega)X(x, \omega) \quad (1)$$

in time harmonic terms [5, 6] where X is the vibration amplitude and ω the angular frequency. Tuck has shown that [5]:

$$\Lambda(\omega) = (k_m - jk_d)\pi\rho R^2\omega^2 \quad (2)$$

where k_m is the added mass coefficient, k_d the damping coefficient, R the radius of the beam. In these conditions the Reynolds number is:

$$Re = \frac{\rho\omega D^2}{4\mu} \quad (3)$$

where ρ is the density of the fluid, D the diameter of the beam, ω the angular frequency and μ the kinematic viscosity. Stokes [7] provided a solution to Tuck's formulation:

$$k_m - jk_d = 1 + \frac{4iK_1(-i\sqrt{iRe})}{\sqrt{iRe}K_0(-i\sqrt{iRe})} \quad (4)$$

where K_0 , K_1 are modified Bessel functions of second kind,

$$\Gamma(\omega) = k_m - jk_d \quad (5)$$

is a hydrodynamic coefficient.

Magneto-structural interaction

In this paper, all the numerical computations are performed using the multiphysic coupling code COMSOL [8]. Many environments modeled with finite elements are unbounded or open, meaning that the electromagnetic fields extend toward infinity (see Fig. 3). Several approaches are used to model unbounded domains [9]. Here implementation of infinite element approach is used and the elements are referred to mapped infinite elements [10]. This implementation maps the model coordinates from the local, finite-sized domain to a stretched domain.

Under the quasi-static approximation, the electromagnetic problem on macroscopic level consists of solving Maxwell's equations. These equations for general time-varying fields in terms of

magnetic potentiel vector can be written as

$$\begin{cases} \sigma \frac{\partial \vec{A}}{\partial t} + \nabla \times (\mu_0^{-1} \mu_r^{-1} \vec{B}) + \sigma (\vec{V} \times \vec{B}) = \vec{J}_e \\ \vec{B} = \nabla \times \vec{A} \\ \vec{B} = \mu_0 \mu_r \vec{H} \end{cases} \quad (6)$$

for Ampere's law while Faraday's Law is

$$\begin{cases} \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{D} = \varsigma \end{cases} \quad (7)$$

where σ is the electric conductivity, ς is a space charge density, μ_0 and μ_r are respectively vacuum and relative permeability, $\vec{A}$ is the magnetic potentiel vector, $\vec{B}$ is the magnetic field, $\vec{V}$ represents the velocity of the moving geometries, $\vec{J}_e$ is generated externally current density, $\vec{E}$ is the electric field and $\vec{D}$ the electric displacement field.

When a structure is surrounded by air in Fig. 4, the stress tensor must be continuous. This gives rise to a surface force acting on the boundary between the two materials. In this case, the air stress tensor is

$$T_2 = -pI - \frac{1}{2}(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B})I + \vec{E}\vec{D}^t + \vec{H}\vec{B}^t \quad (8)$$

where p is the air pressure, I is the identity 3×3 tensor, and $\vec{B}$ and $\vec{H}$ are 3×1 vectors.

When air is approximated by vacuum, $p = 0$ and the equation (8) with $p = 0$ is known as the Maxwell stress tensor. In these conditions, the vibration source force acting on the moving geometries linked to the needle is computed by projection of the stress tensor on the outside of these solid surfaces.

$$\vec{n}T_2 = -p\vec{n} - \frac{1}{2}(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B})\vec{n} + (\vec{n} \cdot \vec{E})\vec{D}^t + (\vec{n} \cdot \vec{H})\vec{B}^t \quad (9)$$

where $\vec{n}$ is the boundary normal pointing out from the needle, T_2 is the stress tensor of air, $\vec{H}$ is the magnetic field and $\vec{B}$ the magnetic flux density.

RESULTS

In the present work, different viscous fluids were studied. As shown in Fig. 5 the vibration amplitude is decreased with the increase of the fluid viscosity. The driven current is imposed to investigate the effect of vibration amplitude on the induced current. The sensing current around the second coil and the vibration amplitude at the free-end of the needle are increased/decreased depending on the fluid viscosity surrounding the beam (Fig. 6).

FIGURE 3. SENSOR SURROUNDED BY AIR DOMAINE.

FIGURE 4. SOLID MATERIAL SURROUNDED BY AIR.

CONCLUSION AND PROSPECTS

The numerical vibrating viscosity sensor is developed for investigation of newtonian fluid viscosity. The output signals obtained from the sensing coil are proportional to the vibration amplitude. Obtained results are validated by experiments on the existing viscometer. A parametric study of the viscometer is now being performed to better understand the sensor behavior for its miniaturization.

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FIGURE 5. FREQUENCY RESPONSE OF THE VISCOMETER.

FIGURE 6. VIBRATION AMPLITUDE AND SENSING CURRENT.

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