An alternative to the classical filtering of the ECG signal: a polynomial approach based on shape feature

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"An alternative to the classical ECG filtering: a polynomial approach based on shape features"

O. Meste*, H. Rix* , R. Maniewski†

A new filtering technique for high-resolution ECG signals processing is proposed. The filter is based on approximation of the electrocardiogram segments by polynoms of low degree with possibility of including smoothing constraints at the frontier points. The segments localization, the polynomial degree and the continuity constraints are discussed in details. Finally it is shown that the proposed filter, without continuity constraints is FIR, linear and shift invariant. The filter may find application in analysis of the heart micropotentials, especially late potentials (LPs). In this case the idea is to match the filter to the high amplitude segments of the ECG in order to limit their influence in the part of the signal where micropotentials are expected. The preliminary results obtained for post-infarction patients show that the filtration allows for clear detection of abnormal electrical activity in the terminal part of the QRS complex.

Keywords: polynomial filtering, high-resolution ECG, late potentials

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1 Introduction

The aim of this research is to introduce a filtering technique with characteristics not issued from frequency domain but fitted to the global shape of the ECG signal. The idea is to match the filter to the parts of the signal with high amplitudes and abrupt slopes in order to limit the effect of their influence in the ST segment where ventricular late potentials (LPs) could be present. Another motivation is to separate the LPs from other microsignals looking at their specific behavior. This approach will take into account shape differences of the Q, R, S waves from subject to subject on the contrary to classical case where a unique filter (Simson, spline, ...) [1-3] is chosen independently of the leads X, Y, Z or the subject. The solution we propose is to approximate the segments of the ECG signal by polynoms of low degree, with possibility to obtain smoothing constraints at the frontier points. Several problems have to be solved: the segment localization, the continuity constraints and the polynomial degree. We will answer these questions in the following sections and we will show that filters we built are linear, shift invariant and finite impulse response (FIR) filters. Some examples will allow to illustrate their usefulness for the analysis of the micropotentials in the ST segment.

2 Polynomial approximation

The filtering (or smoothing) we propose consists in the approximation, using a least mean square criterion, of a segment with length N from signal $s[n]$ ($1 \leq n \leq L$ and $N \leq L$). Polynom $p_l[n]$ of degree m will approximate the signal $s[n]$ on the interval $[(l-1)(N-1)+1, (l-1)(N-1)+N]$ and is defined as follows:

$$p_l[n] = \sum_{k=0}^{m} \alpha_{lk} n^k \quad with \quad (1 \leq n \leq N) \;\; and \;\; l \geq 1 \tag{1}$$

Vector $\mathbf{p}_l$ defined by $\mathbf{p}_l = [p_l[1], \ldots, p_l[N]]^T$ can be written as:

$$\mathbf{p}_l = \mathbf{H}\boldsymbol{\alpha}_l \tag{2}$$

where $\mathbf{H}$ is a Van der Monde matrix defined by:

$$\mathbf{H} = (\mathbb{1} \mid \mathbf{n} \mid \ldots \mid \mathbf{n^m}) \quad with \quad \mathbf{n} = [1, \ldots, N]^T and \quad \mathbf{n}^m = [1^m, \ldots, N^m]^T \tag{3}$$

The minimization of the least mean squared error $J_l = \sum_{n=1}^{N}(s[(l-1)(N-1)+n] - p_l[n])^2$ for a fixed value of the polynomial degree m leads to a function of the α_{lk} (the components of $\boldsymbol{\alpha}_l$):

$$\hat{\boldsymbol{\alpha}}_l = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{s}_l \tag{4}$$

with

$$\mathbf{s}_l = [s[(l-1)(N-1)+1], s[(l-1)(N-1)+2], \ldots, s[(l-1)(N-1)+N]]^T \quad (5)$$

The polynomial approximation of vector $\mathbf{s}_l$ is then:

$$\mathbf{p}_l = \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{s}_l = \mathbf{M}\mathbf{s}_l \quad (6)$$

Going from one segment to the other may be done by imposing or not continuity constraints. We give the case of a zero order continuity, that is the continuity of the function.

2.1 Zero order continuity

The constraints for polynom $p_l[n]$ $(l > 1)$ is $p_l[1] = p_{l-1}[N]$. So equation (1) becomes:

$$\sum_{k=0}^{m} \alpha_{lk} = p_{l-1}[N] \quad (7)$$

giving:

$$\alpha_{lm} = p_{l-1}[N] - \sum_{k=0}^{m-1} \alpha_{lk} \quad (8)$$

Finally after replacement of α_{lm} in (1) by the expression in (8), we obtain:

$$\mathbf{p}_l = \left(\mathbb{1} - \mathbf{n}^\mathbf{m} \mid \mathbf{n} - \mathbf{n}^\mathbf{m} \mid \ldots \mid \mathbf{n}^{\mathbf{m-1}} - \mathbf{n}^\mathbf{m}\right) \begin{pmatrix} \alpha_{l1} \\ \alpha_{l2} \\ \vdots \\ \alpha_{l(m-1)} \end{pmatrix} + p_{l-1}[N]\mathbf{n}^m \quad (9)$$

$$= \mathbf{H}_0 \begin{pmatrix} \alpha_{l1} \\ \alpha_{l2} \\ \vdots \\ \alpha_{l(m-1)} \end{pmatrix} + p_{l-1}[N]\mathbf{n}^m \quad (10)$$

The polynomial approximation (in the least mean squared sense) of vector $\mathbf{s}_l$ is then:

$$\mathbf{p}_l = \mathbf{H}_0(\mathbf{H}_0^T\mathbf{H}_0)^{-1}\mathbf{H}_0^T(\mathbf{s}_l - p_{l-1}[N]\mathbf{n}^m) + p_{l-1}[N]\mathbf{n}^m \quad (11)$$

The continuities of higher orders (i. e. continuity of successive derivatives) can be obtained through a similar way. The filtering (or smoothing) obtained as presented before is, obviously, not shift invariant since the final result depends on the definition of the initial point $s[1]$. We will show that there exists a solution to get this invariance. It makes use of the averaging of all the polynomial approximations for different initial point $s[1]$.

3 Shift-invariant polynomial approximation

Although the smoothing proposed in the previous section is not shift invariant, the obtained results are used for approximation of each particular segment. What makes the difference is the computing of several polynomial approximations for various reference indexes (i.e. modifying the temporal position of $s[1]$). Fig. (1) illustrates this principle assuming that no continuity constraints is imposed.

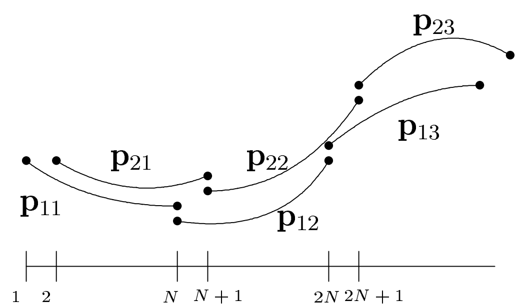

Figure 1: shifted polynomial approximations

In Fig. (1) $\mathbf{p}_{rl}$ represents the approximation of segment number l taking for the definition of $s[1]$ (the first sample used for the approximation) the rth point $(1 \leq r \leq N)$. In other words we introduce an average of approximations on shifted intervals in order to provide the shift-invariant property demonstrated below.

We will precise how to design the filter either without any constraints or when a zero order continuity is imposed.

3.1 No continuity imposed

When we do not impose any continuity constraints at the frontier points between the different segments, matrix $\mathbf{M}$ in (6) is independent of indexes r and l. So polynoms $\mathbf{p}_{rl}$ have all the same form:

$$\mathbf{p}_{rl} = \mathbf{M} \begin{pmatrix} s[(l-1)(N-1)+(r-1)+1] \\ s[(l-1)(N-1)+(r-1)+2] \\ \vdots \\ s[(l-1)(N-1)+(r-1)+N] \end{pmatrix} \tag{12}$$

We can remark that $\mathbf{p}_{Nr} = \mathbf{p}_{1(r+1)}$

Averaging the $\mathbf{p}_{rl}[n]$ values associated to the same signal sample, e.g. $n = N$, leads to:

$$\begin{aligned} r[N] &= p_{11}[N] + p_{21}[N-1] + \ldots + p_{N1}[1] & (13)\\ &= \sum_{l=1}^{N} p_{l1}[N-(l-1)] & (14) \end{aligned}$$

Since matrix $\mathbf{M}$ in (6) can be written as:

$$\mathbf{M} = \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \vdots \\ \mathbf{m}_N^T \end{pmatrix} \qquad (15)$$

relation (13) becomes:

$$r[N] = \mathbf{m}_N^T \begin{pmatrix} s[1] \\ \vdots \\ s[N] \end{pmatrix} + \mathbf{m}_{N-1}^T \begin{pmatrix} s[2] \\ \vdots \\ s[N+1] \end{pmatrix} + \ldots + \mathbf{m}_1^T \begin{pmatrix} s[N] \\ \vdots \\ s[2N-1] \end{pmatrix} \qquad (16)$$

Finally, this leads to:

$$\begin{aligned} r[N] &= s[1]m_N[1] + s[2]\left(m_N[2] + m_{N-1}[1]\right) + \ldots + \\ & \quad s[N]\left(m_N[N] + m_{N-1}[N-1] + \ldots + m_1[1]\right) + \ldots + \\ & \quad s[2N-1]\left(m_1[N]\right) & (17) \end{aligned}$$

Setting $diag(\mathbf{M}, u)$ for the uth diagonal of $\mathbf{M}$ ($u > 0$ for the diagonals above the main one and $u < 0$ for the others), the previous relation becomes:

$$\begin{aligned} r[N] &= s[1]diag(\mathbf{M}, -(N-1)) + s[2]\sum diag(\mathbf{M}, -(N-2)) + \ldots + \\ & \quad s[N]\sum diag(\mathbf{M}, 0) + \ldots + s[2N-1]diag(\mathbf{M}, N-1) & (18) \end{aligned}$$

It is easy to show that matrix $\mathbf{M}$ is $N \times N$ and symmetric, so :

$$\sum diag(\mathbf{M}, \beta) = \sum diag(\mathbf{M}, -\beta) = f[\beta] \qquad (19)$$

Then relation (18) takes the form:

$$r[N] = s[1]f[N-1] + s[2]f[N-2] + \ldots + s[N]f[0] + \ldots + s[2N-1]f[N-1] \quad (20)$$

It is clear from this form that the involved operation is a convolution of vector $[s[1], \ldots, s[2N-1]]^T$ with the symmetric RIF $f[n]$. Hence, This filtering is linear and shift invariant. Obviously, the values of $r[n]$ for $N < n < L-N$ can be deduced from the later development. As examples, different impulse responses are given in Fig. 2, corresponding to different values of degree m.

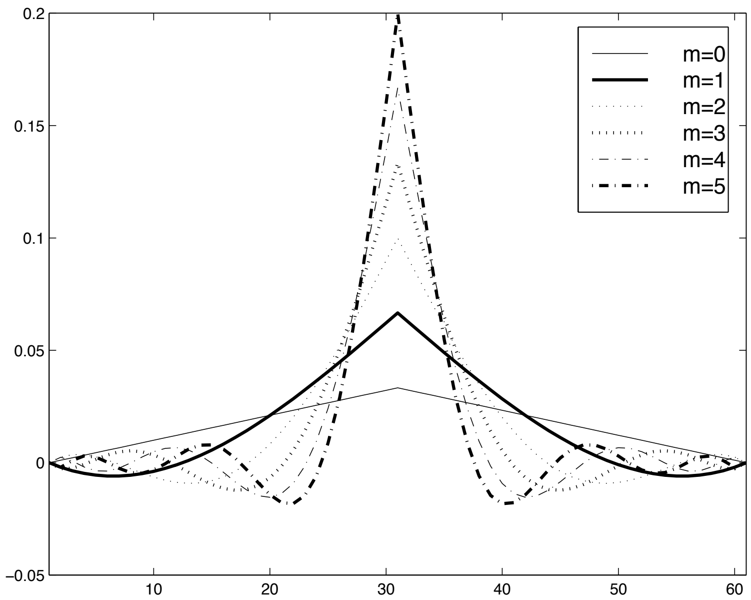

Figure 2: Impulse responses of the filter when no continuity is imposed, for $N = 30$ and different values of degree m

3.2 Zero order continuity

We will show that, when the polynomial approximations are imposed to be continuous at the junction of $\mathbf{p}_{r(l-1)}$ with $\mathbf{p}_{rl}$, the obtained filter does not have the characteristics of the precedent filter. So the imposed constraints are $\mathbf{p}_{rl}[1] = \mathbf{p}_{r(l-1)}[N]$, with no hypothesis on the constraint on $\mathbf{p}_{r1}[1]$ ($\forall r \in [1, N]$). For example, let us look at the average of polynoms at sample $2N-1$. We have:

$$r[2N-1] = p_{12}[N] + p_{22}[N-1] + p_{32}[N-2] + \ldots + p_{N2}[1] \qquad (21)$$

Using (11), relation (21) becomes:

$$\begin{aligned} r[2N-1] &= 2\mathbf{m}_{0N}^T \begin{pmatrix} s[N] \\ \vdots \\ s[2N-1] \end{pmatrix} + 2p_{11}[N]j[N] + \\ & \mathbf{m}_{0(N-1)}^T \begin{pmatrix} s[N+1] \\ \vdots \\ s[2N] \end{pmatrix} + p_{21}[N]j[N-1] + \ldots + \end{aligned}$$

$$\mathbf{m}_{02}^T \begin{pmatrix} s[2N] \\ \vdots \\ s[3N-1] \end{pmatrix} + p_{(N-1)1}[N]j[2] \tag{22}$$

with

$$\begin{pmatrix} \mathbf{m}_{01}^T \\ \mathbf{m}_{02}^T \\ \vdots \\ \mathbf{m}_{0N}^T \end{pmatrix} = \mathbf{H}_0(\mathbf{H}_0^T\mathbf{H}_0)^{-1}\mathbf{H}_0^T \;\; et \;\; \mathbf{j} = (\mathbf{I} - \mathbf{H}_0)\mathbf{n}^m \tag{23}$$

We may develop this relation in order to be close to the form obtained in (17), so we obtain:

$$\begin{aligned} r[2N-1] \;\; = \;\; & 2s[N]m_{0N}[1] + s[N+1](2m_{0N}[2] + m_{0(N-1)}[1]) + \\ & s[N+2](2m_{0N}[3] + m_{0(N-1)}[2] + m_{0(N-2)}[1]) + \ldots + \\ & s[3N-1]m_{02}[N] + \\ & 2p_{11}[N]j[N] + p_{21}[N]j[N-1] + \ldots + p_{(N-1)1}j[2] \end{aligned} \tag{24}$$

One can observe that this formula contains one part which is shift invariant (the coefficients do not change for $r[2N]$, etc...) and another part which is a function of the values taken by the polynoms approximating the previous segments. This later part due to the continuity constraints makes the filtering dependant on the signal values in the first segment. In the case of ECG signals this leads for example to the approximation of ST segment taking Q-wave into account. This behavior makes this filtering inappropriate for our application. Thus, we make use of the first proposed filter.

4 Application

We applied the filtering based on polynomial approach to high resolution ECG signals with the aim to enhance the presence of late potentials [1-5]. In the ECG signal filtering, the length N is visually fitted for each patient and each lead (X,Y,Z). Degree m of the polynoms has been taken equal to 2. Twenty post myocardial infarction patients have been chosen for filter performance: ten with clinically diagnosed Ventricular Tachycardia (VT) and ten without clinically diagnosed arrhythmia. Patients (P_1,P_2) belong to the first group and (P_3,P_4) to the second group. In Fig. 3, 5, 7, 9 one can see the ECG signals recorded from X,Y and Z leads on these four patients. The beginning and end of the segment used to determine the filter length N are marked by circles. The choice of the interval used to build our filter is made empirically, looking at the time interval where LP may be present and where the variation of QRS signal implies a contribution to high frequencies of the QRS spectrum.

The results of our filtering are given in Fig. 4, 6, 8, 10. They correspond to the vector magnitude $\sqrt{X_f^2 + Y_f^2 + Z_f^2}$ where X_f,Y_f,Z_f are the ECG signals from orthogonal leads filtered using the polynomial technique. For Fig. 3 to Fig. 10, we have determined the maximum of the QRS (represented by the first vertical line) by taking the maximum (MA) of the vector magnitude of the three unfiltered leads and the end of the high-frequency electrical activity (represented by the second vertical line) such that the value of $\sqrt{X_f^2 + Y_f^2 + Z_f^2}$ is below 1.5/1000*MA. This value has been choosen regards to the noise level and the expected micropotentials magnitude. We can remark that for the group (P_1,P_2) the spreading of the electrical activity (second vertical line) out of the QRS is clear on the contrary to the second group (P_3,P_4).

The global result is given in Fig. 11 where the proposed filtering technique is compared to the classical Simson method [3] using the mean and the standard deviation of the duration of the high-frequency electrical activity from the twenty patients. In both cases we have determined the length (in ms) separating the maximum of the QRS and the end of the high-frequency electrical activity (see the description above) computed on the filtered signals from the Simson filter and the polynomial filter. One can note a small improvement in the separation of the two groups for the polynomial filter compared to the Simson one.

5 Conclusions

In this paper we have introduced an alternative to classical filtering. This alternative is based on a representation of signal segments (QRS complex of ECG in our application) approximated by polynoms. The provided filter at this first step being not shift-invariant we proposed in a second step to average filtered signals corresponding to different choices of the first sample. We showed that combining these two steps leads to a shift-invariant filter. This property is true only if no continuity constraints is imposed to frontier points. The goal of this study was not to compare a lot of methods but to introduce a new approach where the parameters directly depends on the morphology of the QRS complex. In fact, on the presented results we show we are able to detect abnormal activity with better sensitivity and specificity. The criterion we take i.e. the duration of abnormal micropotentials gives a better separation between the two groups of patients with our filtering than with Simson filtering.

6 References

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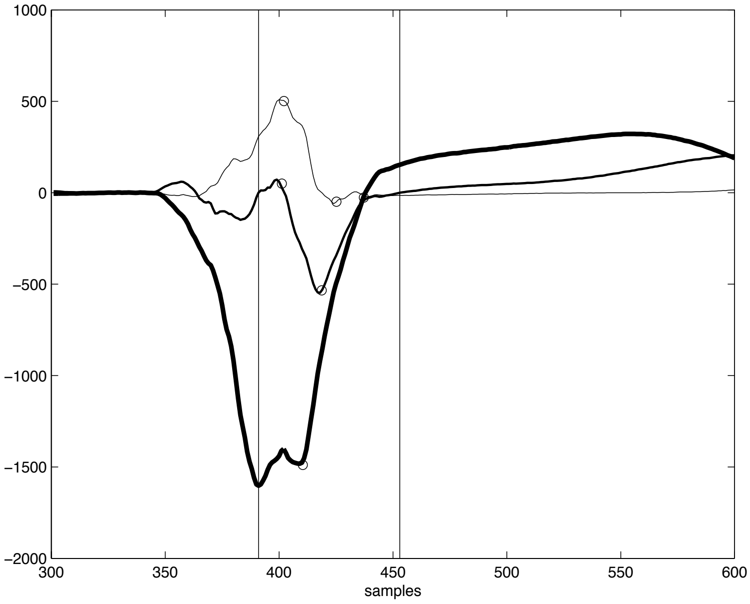

Figure 3: Units: ordinates in μV and abscissa in ms. ECG signals from leads X,Y,Z for patient P_1. Circles indicate the segments involved in the filters definition. Vertical lines indicate the maximum of the QRS and the end of high-frequency electrical activity

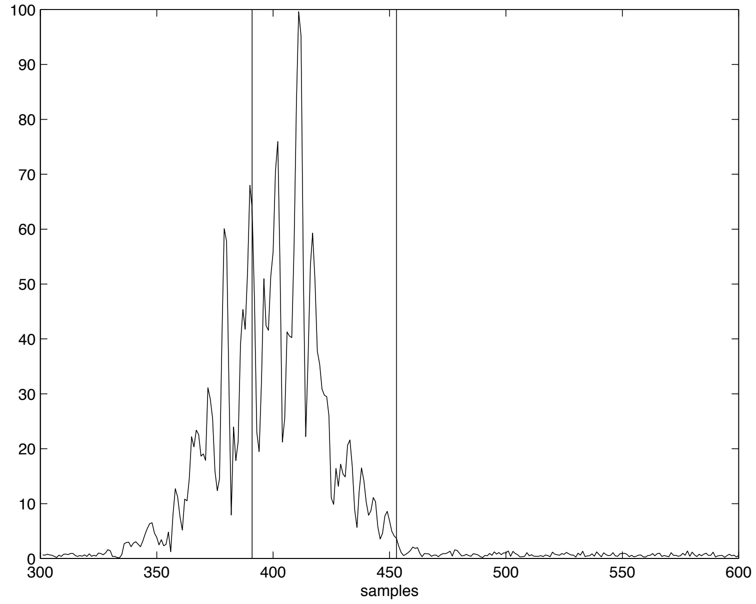

Figure 4: Units: ordinates in μV and abscissa in ms. Vector magnitude of the filtered signals from leads X, Y, Z for patient P_1. Vertical lines indicate the maximum of the QRS and the end of high-frequency electrical activity

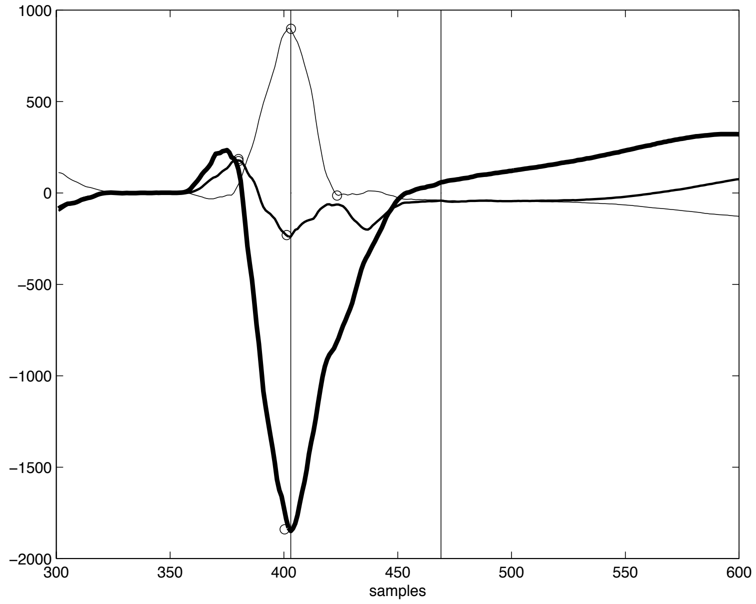

Figure 5: Units: ordinates in μV and abscissa in ms. ECG signals from leads X,Y,Z for patient P_2. Circles indicate the segments involved in the filters definition. Vertical lines indicate the maximum of the QRS and the end of high-frequency electrical activity

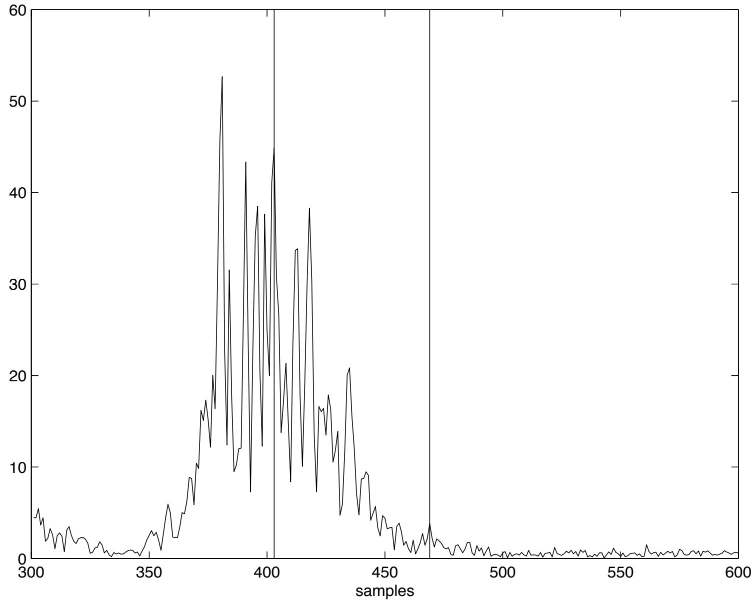

Figure 6: Units: ordinates in μV and abscissa in ms. Vector magnitude of the filtered signals from leads X, Y, Z for patient P_2. Vertical lines indicate the maximum of the QRS and the end of high-frequency electrical activity

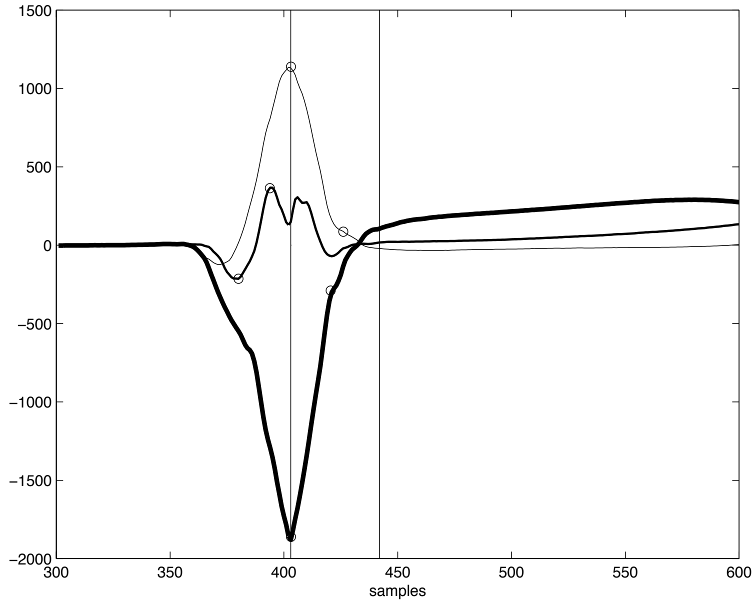

Figure 7: Units: ordinates in μV and abscissa in ms. ECG signals from leads X,Y,Z for patient P_3. Circles indicate the segments involved in the filters definition. Vertical lines indicate the maximum of the QRS and the end of high-frequency electrical activity

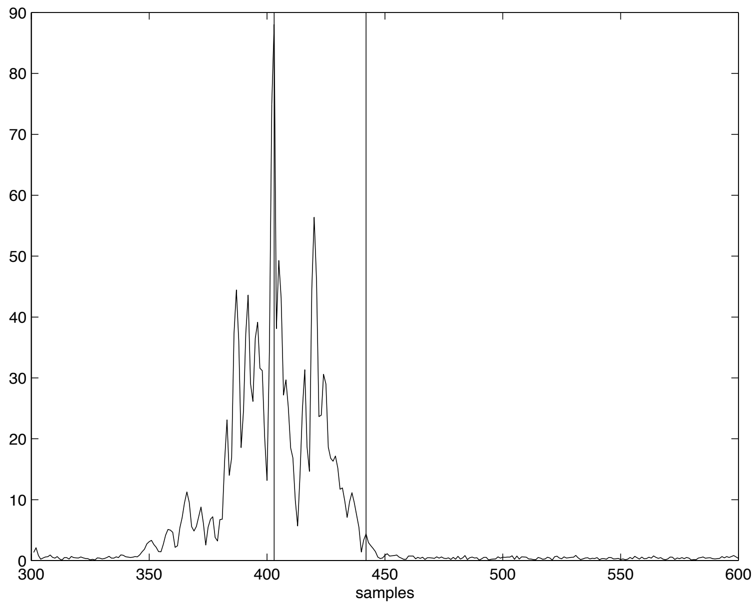

Figure 8: Units: ordinates in μV and abscissa in ms. Vector magnitude of the filtered signals from leads X, Y, Z for patient P_3. Vertical lines indicate the maximum of the QRS and the end of high-frequency electrical activity

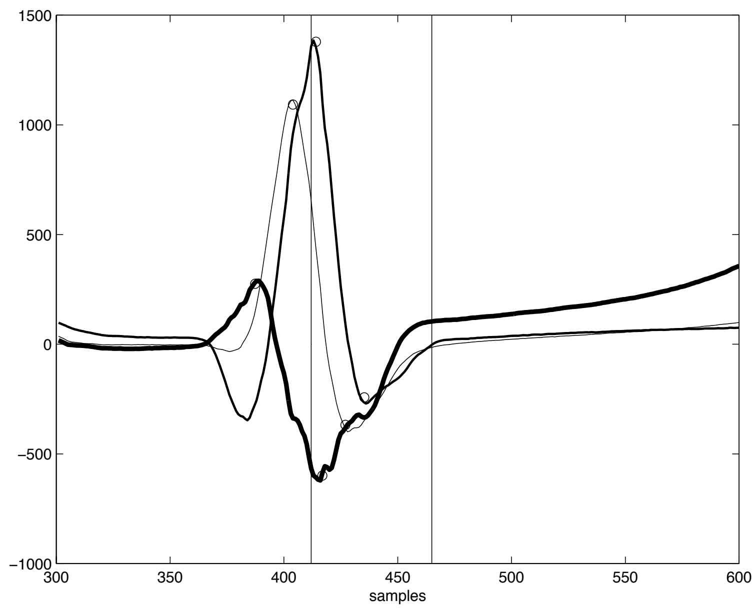

Figure 9: Units: ordinates in μV and abscissa in ms. ECG signals from leads X,Y,Z for patient P_4. Circles indicate the segments involved in the filters definition. Vertical lines indicate the maximum of the QRS and the end of high-frequency electrical activity

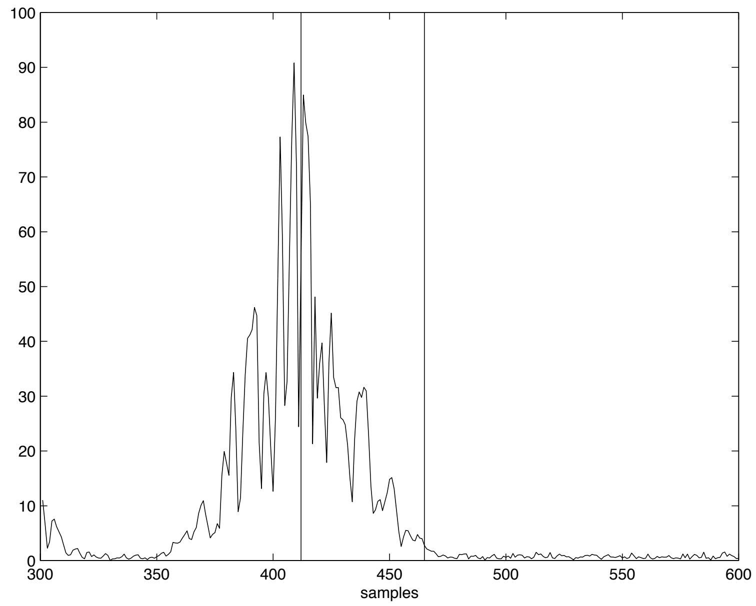

Figure 10: Units: ordinates in μV and abscissa in ms. Vector magnitude of the filtered signals from leads X, Y, Z for patient P_4. Vertical lines indicate the maximum of the QRS and the end of high-frequency electrical activity

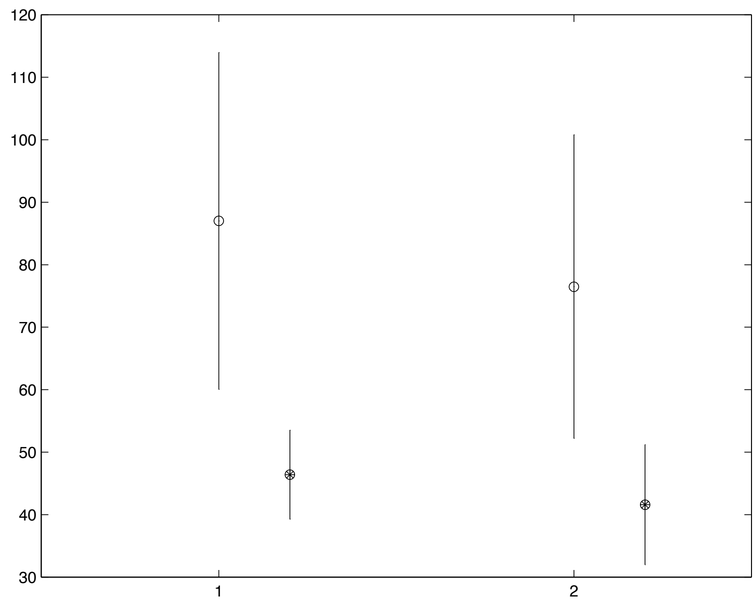

Figure 11: Mean and standard deviation (ms) of the electrical activity duration of the proposed filtering technique (abscissa=1) compared with the Simson filter (abscissa=2). Results from VT patients and non VT patients are indicated by the circle and the star respectively.