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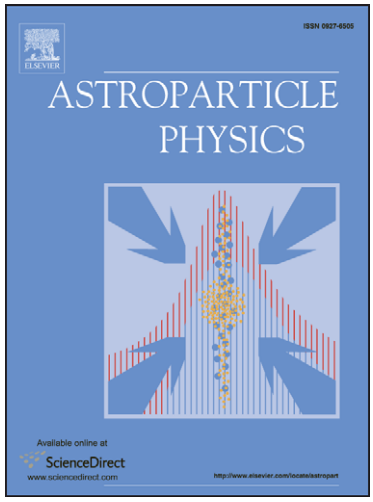

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Gravitational wave astronomy: the definitive test for the "Einstein frame versus Jordan frame" controversy

Christian Corda

18th October 2010

Associazione Scientifica Galileo Galilei, Via Bruno Buozzi 47, - 59100 PRATO, Italy

E-mail address: cordac.galilei@gmail.com

Abstract

The potential realization of a gravitational wave (GW) astronomy in next years is a great challenge for the scientific community. By giving a significant amount of new information, GWs will be a cornerstone for a better understanding of the universe and of the gravitational physics.

In this paper the author shows that the GW astronomy will permit to solve a captivating issue of gravitation as it will be the definitive test for the famous "Einstein frame versus Jordan frame" controversy.

In fact, we show that the motion of the test masses, i.e. the beam splitter and the mirror in the case of an interferometer, which is due to the scalar component of a GW, is different in the two frames. Thus, if a consistent GW astronomy will be realized, an eventual detection of signals of scalar GWs will permit to discriminate among the two frames. In this way, a direct evidence from observations will solve in an ultimate way the famous and long history of the "Einstein frame versus Jordan frame" controversy.

1 Introduction

The scientific community hopes in a first direct detection of GWs in next years [1]. The realization of a GW astronomy, by giving a significant amount of new information, will be a cornerstone for a better understanding of the universe and of the gravitational physics. In fact, the discovery of GW emission by the compact binary system PSR1913+16, composed by two neutron stars [2], has been, for physicists working in this field, the ultimate thrust allowing to reach

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the extremely sophisticated technology needed for investigating in this field of research.

In a recent research [3], the author showed that the GW astronomy will be the definitive test for general relativity, or, alternatively, a strong endorsement for extended theories of gravity. In this paper the analysis is improved by showing that, in addition, the GW astronomy will permit to solve a captivating issue of gravitation as it will be the ultimate test for the famous "Einstein frame versus Jordan frame" controversy.

In fact, the author shows that the motion of test masses, i.e. the beam splitter and the mirror in the case of an interferometer, in the field of a scalar GW is different in the two frames. Then, if a consistent GW astronomy will be realized, an eventual detection of signals of scalar GWs will permit to discriminate among the two frames.

In this way, a direct evidence from observations will solve in an ultimate way the famous and long history of the "Einstein frame versus Jordan frame" controversy.

The controversy on conformal frames started from early investigations [4], till recent analyses [5, 6], with lots of effort of famous physicists, see [6, 7, 8] for example. In the generalization of the Jordan-Fierz-Brans-Dicke theory of gravitation [9, 10, 11], which is known as scalar-tensor gravity [6, 12, 13, 14], the gravitational interaction is mediated by a scalar field together with the usual metric tensor. Scalar-tensor gravity is present in various frameworks of theoretical physics, like dilaton gravity in superstring and supergravity theories [15], like description of braneworld models [16], like conformal equivalents to modified f(R) gravity [17], or in attempts to realize inflation [18, 19, 20] and to obtain dark energy [21, 22]. Scalar-tensor gravity arises from the conviction of lots of scientists that every modern theoretical attempt to unify gravity with the remaining interactions requires the introduction of scalar fields [12]. An ultimate endorsement for the viability of scalar-tensor gravity could arrive from detection of GWs, see [3] for details.

The "Einstein frame versus Jordan frame" controversy started because some authors claimed that scalar-tensor gravity is unreliable in the Jordan frame, leading to the problem of negative kinetic energies [23, 24, 25]. On the other hand, the Einstein frame version of scalar-tensor gravity, which is obtained by the conformal rescaling of the metric [26, 27, 28, 29]

$$\tilde{g}_{ab} = \varphi g_{ab} \tag{1}$$

and a nonlinear scalar field redefinition [26, 28]

$$d\tilde{\varphi} = \frac{1}{k}\frac{d\varphi}{\varphi} \quad \Longrightarrow \quad \tilde{\varphi} = \tilde{\varphi}_0 + \frac{1}{k}\ln\frac{\varphi}{\varphi_0}, \tag{2}$$

has a positive definite energy [27]. In this paper Latin indices are used for 4-dimensional quantities, Greek indices for 3-dimensional ones and the author works with $G = 1$, $c = 1$ and $\hbar = 1$ (natural units). k in Eqs. (2) is defined like $k \equiv \sqrt{\frac{16\pi}{|2\omega+3|}}$ and such a notation has not to be confused with other notations in

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the literature (in various books and papers k represents the spatial curvature of Universe, see [30] for example). φ is the fundamental scalar field of scalar-tensor gravity [6, 12, 13, 14], ω is the Brans-Dicke parameter [11], $\tilde{\varphi}$ is the "conformal scalar field" [26] and φ_0 and $\tilde{\varphi}_0$ are constants that represent the "zero values" of φ and $\tilde{\varphi}$.

In general, analyses in the Einstein frame are simpler concerning the field equations, but the connection with particle physics is more difficult than in the Jordan frame. Thus, there are authors who use the Einstein frame as a mathematical artifice to solve the field equations and then return in the Jordan frame to compare with astrophysics observations [17, 22]. Other authors claim that the two conformal frames are equivalent [28]. Others again are not interested in the problem [5]. Different positions of various authors have been discussed in [27] and, at the present time, the debate remains open [5, 6, 17, 22, 28, 29]. The controversy on conformal frames could appear a purely technical one. Actually, it is very important as the physical predictions of a classical theory of gravity, or of a dark energy cosmological scenario, are deeply affected by the choice of the conformal frame. Thus, the fundamental question is: which is *the physical frame* of observations? Using of conformal transformations to perform analyses in the Einstein frame abounds in the literature, with divergence of opinions between different authors [5, 6, 17, 22, 23, 24, 25, 28, 29]. The motion in the Einstein frame *is not geodesic* [26], a key point which strongly endorses deviations from equivalence principle and non-metric gravity theories in the Einstein frame [6, 26, 31, 32]. Thus, some authors claim that physics must be different in the two different frames, see [31, 32] for example. Another important point concerns doubts on the physical equivalence in respect to the Cauchy problem [33, 34].

2 A review of some important issues

2.1 Gravitational waves in scalar-tensor gravity: derivation in the Jordan frame

In order to better understand the results of this paper it is useful to sketch the derivation of GWs in scalar-tensor gravity and in the Jordan frame [39].

The most general action of scalar-tensor theories of gravity in four dimensions and in the Jordan frame is given by [33, 36]

$$S = \int d^4x\sqrt{-g}[f(\phi)R + \frac{1}{2}g^{mn}\phi_{;m}\phi_{;n} - V(\phi) + \mathcal{L}_{(matter)}]. \tag{3}$$

Choosing

$$\varphi = f(\phi) \quad \omega(\varphi) = \frac{f(\phi)}{2f'(\phi)} \quad W(\varphi) = V(\phi(\varphi)) \tag{4}$$

Eq. (3) reads

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$$S = \int d^4x\sqrt{-g}[\varphi R - \frac{\omega(\varphi)}{\varphi}g^{mn}\varphi_{;m}\varphi_{;n} - W(\varphi) + \mathcal{L}_{(matter)}], \tag{5}$$

which is a generalization of the Jordan-Fierz-Brans-Dicke theory [9, 10, 11].

By varying the action (5) with respect to g_{mn} and to the scalar field φ the field equations are obtained [33, 36]

$$\begin{gathered} G_{mn} = -\frac{4\pi\tilde{G}}{\varphi}T_{mn} + \frac{\omega(\varphi)}{\varphi^2}(\varphi_{;m}\varphi_{;n} - \frac{1}{2}g_{mn}g^{ab}\varphi_{;a}\varphi_{;b}) + \\ +\frac{1}{\varphi}(\varphi_{;mn} - g_{mn}\Box\varphi) + \frac{1}{2\varphi}g_{mn}W(\varphi) \end{gathered} \tag{6}$$

with associated a Klein - Gordon equation for the scalar field

$$\Box\varphi = \frac{1}{2\omega(\varphi)+3}(-4\pi\tilde{G}T + 2W(\varphi) + \varphi W'(\varphi) + \frac{d\omega(\varphi)}{d\varphi}g^{mn}\varphi_{;m}\varphi_{;n}. \tag{7}$$

In the above equations T_{mn} is the ordinary stress-energy tensor of the matter and $\tilde{G}$ is a dimensional, strictly positive, constant. The Newton constant is replaced by the effective coupling

$$G_{eff} = -\frac{1}{2\varphi}, \tag{8}$$

which is, in general, different from G. General relativity is obtained when the scalar field coupling is

$$\varphi = const. = -\frac{1}{2}. \tag{9}$$

To study GWs, the linearized theory in vacuum ($T_{mn} = 0$) with a little perturbation of the background has to be analysed [30, 36]. The background is assumed given by the Minkowskian background plus $\varphi = \varphi_0$ and φ_0 is also assumed to be a minimum for W [36]

$$W \simeq \frac{1}{2}\alpha\delta\varphi^2 \Rightarrow W' \simeq \alpha\delta\varphi. \tag{10}$$

Putting

$$\begin{gathered} g_{mn} = \eta_{\mu mn} + h_{mn} \\ \varphi = \varphi_0 + \delta\varphi. \end{gathered} \tag{11}$$

and, to first order in h_{mn} and $\delta\varphi$, if one calls $\widetilde{R}_{mnrs}$, $\widetilde{R}_{mn}$ and $\widetilde{R}$ the linearized quantity which correspond to R_{mnns} , R_{mn} and R, the linearized field equations are obtained [36]

$$\begin{gathered} \widetilde{R}_{mn} - \frac{\widetilde{R}}{2}\eta_{mn} = -\partial_m\partial_n\Phi + \eta_{mn}\Box\Phi \\ \Box\Phi = m^2\Phi, \end{gathered} \tag{12}$$

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where

$$\Phi \equiv -\frac{\delta\varphi}{\varphi_0}$$
$$m^2 \equiv \frac{\alpha\varphi_0}{2\omega+3}. \tag{13}$$

The case in which it is $\omega = const.$ and $W = 0$ in Eqs. (6) and (7) has been analysed in [36] with a treatment which generalized the "canonical" linearization of general relativity [30].

For a sake of completeness, let us complete the linearization process by following [36].

The linearized field equations become

$$\widetilde{R}_{mn} - \frac{\widetilde{R}}{2}\eta_{mn} = \partial_m\partial_n\Phi + \eta_{mn}\Box\Phi$$
$$\Box\Phi = 0 \tag{14}$$

Let us put

$$\bar{h}_{mn} \equiv h_{mn} - \frac{h}{2}\eta_{mn} + \eta_{mn}\Phi$$
$$\bar{h} \equiv \eta^{mn}\bar{h}_{mn} = -h + 4\Phi, \tag{15}$$

with $h \equiv \eta^{mn}h_{mn}$, where the inverse transform is the same

$$h_{mn} = \bar{h}_{mn} - \frac{\bar{h}}{2}\eta_{mn} + \eta_{mn}\Phi$$
$$h = \eta^{mn}h_{mn} = -\bar{h} + 4\Phi. \tag{16}$$

By putting the first of Eqs. (16) in the first of the field equations (14) we get

$$\Box\bar{h}_{mn} - \partial_m(\partial^a\bar{h}_{an}) - \partial_n(\partial^a\bar{h}_{an}) + \eta_{mn}\partial^b(\partial^a\bar{h}_{ab}). \tag{17}$$

Now, let us consider the gauge transform (Lorenz condition)

$$\bar{h}_{mn} \to \bar{h}'_{mn} = \bar{h}_{mn} - \partial_{(m}\epsilon_{n)} + \eta_{mn}\partial^a\epsilon_a$$
$$\bar{h} \to \bar{h}' = \bar{h} + 2\partial^a\epsilon_a \tag{18}$$
$$\Phi \to \Phi' = \Phi$$

with the condition $\Box\epsilon_n = \partial^m\bar{h}_{mn}$ for the parameter ϵ^μ. We obtain

$$\partial^\mu\bar{h}'_{mn} = 0, \tag{19}$$

and, omitting the $'$, the field equations can be rewritten like

$$\Box\bar{h}_{mn} = 0 \tag{20}$$

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$$\Box\Phi = 0; \tag{21}$$

solutions of Eqs. (20) and (21) are plan waves:

$$\bar{h}_{mn} = A_{mn}(\overrightarrow{k})\exp(ik^a x_a) + c.c. \tag{22}$$

$$\Phi = a(\overrightarrow{k})\exp(ik^a x_a) + c.c. \tag{23}$$

Thus, Eqs. (20) and (22) are the equation and the solution for the tensor waves exactly like in general relativity [30], while Eqs. (21) and (23) are respectively the equation and the solution for the scalar massless mode [36].

The solutions (22) and (23) take the conditions

$$\begin{aligned} k^a k_a &= 0 \\ k^m A_{mn} &= 0, \end{aligned} \tag{24}$$

which arises respectively from the field equations and from Eq. (19).

The first of Eqs. (24) shows that perturbations have the speed of light, the second the transverse effect of the field.

Fixed the Lorenz gauge, another transformation with $\Box\epsilon^m = 0$ can be made; let us take

$$\begin{aligned} \Box\epsilon^m &= 0 \\ \partial_m\epsilon^m &= -\tfrac{\bar{h}}{2} + \Phi, \end{aligned} \tag{25}$$

which is permitted because $\Box\Phi = 0 = \Box\bar{h}$. We obtain

$$\bar{h} = 2\Phi \quad \Rightarrow \quad \bar{h}_{mn} = h_{mn} \ , \tag{26}$$

i.e. h_{mn} is a transverse plane wave too. The gauge transformations [36]

$$\begin{aligned} \Box\epsilon^m &= 0 \\ \partial_m\epsilon^m &= 0, \end{aligned} \tag{27}$$

enable the conditions

$$\begin{aligned} \partial^m\bar{h}_{mn} &= 0 \\ \bar{h} &= 2\Phi. \end{aligned} \tag{28}$$

Considering a wave propagating in the positive z direction

$$k^m = (k, 0, 0k), \tag{29}$$

the second of Eqs. (24) implies

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$$A_{0\nu} = -A_{3\nu}$$

$$A_{\nu 0} = -A_{\nu 3} \tag{30}$$

$$A_{00} = -A_{30} + A_{33}.$$

Now, let us see the freedom degrees of A_{mn}. We were started with 10 components (A_{mn} is a symmetric tensor); 3 components have been lost for the transverse condition, more, the condition (26) reduces the components to 6. One can take A_{00}, A_{11}, A_{22}, A_{21}, A_{31}, A_{32} like independent components; another gauge freedom can be used to put to zero three more components (i.e. only three of ϵ^m can be chosen, the fourth component depends from the others by $\partial_m \epsilon^m = 0$).

Then, by taking

$$\begin{gathered} \epsilon_m = \tilde{\epsilon}_m(\overrightarrow{k}) \exp(ik^a x_a) + c.c. \\ k^m \tilde{\epsilon}_m = 0, \end{gathered} \tag{31}$$

the transform law for A_{mn} is (see Eqs. (18) and (22))

$$A_{mn} \to A'_{mn} = A_{mn} - 2ik(_m \tilde{\epsilon}_{n)}. \tag{32}$$

Thus, the six components of interest are

$$\begin{array}{rcl} A_{00} & \to & A_{00} + 2ik\tilde{\epsilon}_0 \\ A_{11} & \to & A_{11} \\ A_{22} & \to & A_{22} \\ A_{21} & \to & A_{21} \\ A_{31} & \to & A_{31} - ik\tilde{\epsilon}_1 \\ A_{32} & \to & A_{32} - ik\tilde{\epsilon}_2. \end{array} \tag{33}$$

The physical components of A_{mn} are the gauge-invariants A_{11}, A_{22} and A_{21}. One can choose $\tilde{\epsilon}_n$ to put equal to zero the others.

The scalar field is obtained by Eq. (26):

$$\bar{h} = h = h_{11} + h_{22} = +2\Phi. \tag{34}$$

In this way, the total perturbation of a GW propagating in the $z-$ direction in this gauge is

$$h_{\mu\nu}(t+z) = h^+(t+z)e^{(+)}_{\mu\nu} + h^\times(t+z)e^{(\times)}_{\mu\nu} + \Phi(t+z)e^{(s)}_{\mu\nu}. \tag{35}$$

The term $h^+(t+z)e^{(+)}_{\mu\nu} + h^\times(t+z)e^{(\times)}_{\mu\nu}$ describes the two standard (i.e. tensor) polarizations of GWs which arises from general relativity in the TT gauge [30], while the term $\Phi(t+z)e^{(s)}_{\mu\nu}$ is the extension of the TT gauge to the scalar-tensor case [36]. The correspondent line element results [36]

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$$ds^2 = -dt^2 + dz^2 + (1 + h^+ + \Phi)dx^2 + (1 - h^+ + \Phi)dy^2 + 2h^\times dxdy. \quad (36)$$

This is the case of massless GWs in scalar-tensor gravity.

By removing the assumptions $\omega = const.$ and $W = 0$ in Eqs. (6) and (7) the analysis can be realized for the case of massive GWs.

In that case, again $\widetilde{R}_{mnrs}$ and Eqs. (12) are invariants for gauge transformations [35]

$$\begin{gathered} h_{mn} \to h'_{mn} = h_{mn} - \partial_{(m}\epsilon_{n)} \\ \Phi \to \Phi' = \Phi; \end{gathered} \quad (37)$$

then

$$\bar{h}_{mn} \equiv h_{mn} - \frac{h}{2}\eta_{mn} + \eta_{mn}\Phi \quad (38)$$

can be defined, and, by considering the transform for the parameter ϵ^μ

$$\Box\epsilon_n = \partial^m \bar{h}_{mn}, \quad (39)$$

a gauge similar to the Lorenz one of electromagnetic waves can be chosen in this case too

$$\partial^m \bar{h}_{mn} = 0. \quad (40)$$

Thus, the field equations read like

$$\Box\bar{h}_{mn} = 0 \quad (41)$$

$$\Box\Phi = m^2\Phi. \quad (42)$$

Solutions of Eqs. (41) and (42) are plan waves again

$$\bar{h}_{mn} = A_{mn}(\overrightarrow{p})\exp(ip^a x_a) + c.c. \quad (43)$$

$$\Phi = a(\overrightarrow{p})\exp(iq^a x_a) + c.c. \quad (44)$$

where now

$$\begin{gathered} k^a \equiv (\omega, \overrightarrow{p}) \qquad \omega = p \equiv |\overrightarrow{p}| \\ q^a \equiv (\omega_{mass}, \overrightarrow{p}) \qquad \omega_{mass} = \sqrt{m^2 + p^2}. \end{gathered} \quad (45)$$

Again, in Eqs. (41) and (43) the equation and the solution for the tensor waves exactly like in general relativity [30] have been obtained, while Eqs. (42) and (44) are respectively the equation and the solution for the scalar mode which now is massive [35].

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The fact that the dispersion law for the modes of the scalar massive field Φ is not linear has to be emphasized. The velocity of every tensor mode $\bar{h}_{mn}$ is the light speed c, but the dispersion law (the second of Eq. (45)) for the modes of Φ is that of a massive field which can be discussed like a wave-packet [35]. Also, the group-velocity of a wave-packet of Φ centred in $\overrightarrow{p}$ is [35]

$$\overrightarrow{v_G} = \frac{\overrightarrow{p}}{\omega_{mass}}, \tag{46}$$

which is exactly the velocity of a massive particle with mass m and momentum $\overrightarrow{p}$.

From the second of Eqs. (45) and Eq. (46) it is simple to obtain:

$$v_G = \frac{\sqrt{\omega_{mass}^2 - m^2}}{\omega_{mass}}. \tag{47}$$

If one wants a constant speed of the wave-packet, it has to be [35]

$$m = \sqrt{(1 - v_G^2)}\omega_{mass}. \tag{48}$$

Again, the analysis can remain in the Lorenz gauge with transformations of the type $\Box\epsilon_\nu = 0$; this gauge gives a condition of transverse effect for the tensor part of the field: $k^m A_{mn} = 0$, but it does not give the transverse effect for the total field h_{mn}. From Eq. (38) we get

$$h_{mn} = \bar{h}_{mn} - \frac{\bar{h}}{2}\eta_{mn} + \eta_{mn}\Phi. \tag{49}$$

At this point, in the massless case we could put

$$\begin{aligned} \Box\epsilon^m &= 0 \\ \partial_m\epsilon^m &= -\tfrac{\bar{h}}{2} + \Phi, \end{aligned} \tag{50}$$

which gives the total transverse effect of the field. But in the massive case this is impossible. In fact, by applying the D' Alembertian operator to the second of Eqs. (50) and by using the field equations (41) and (42) one obtains

$$\Box\epsilon^m = +m^2\Phi, \tag{51}$$

which is in contrast with the first of Eqs. (50). In the same way, it is possible to show that it does not exist any linear relation between the tensor field $\bar{h}_{mn}$ and the scalar field Φ [35]. Thus, a gauge in which h_{mn} is purely spatial cannot be chosen (i.e. we cannot choose $h_{m0} = 0$, see eq. (49)). But the traceless condition to the field $\bar{h}_{mn}$ can be enabled [35]

$$\begin{aligned} \Box\epsilon^m &= 0 \\ \partial_m\epsilon^m &= -\tfrac{\bar{h}}{2}. \end{aligned} \tag{52}$$

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These equations imply

$$\partial^m \bar{h}_{mn} = 0. \tag{53}$$

To enable the conditions $\partial_m \bar{h}^{mn}$ and $\bar{h} = 0$ transformations like

$$\begin{gathered} \Box \epsilon^m = 0 \\ \partial_m \epsilon^m = 0 \end{gathered} \tag{54}$$

can be used and, taking $\overrightarrow{p}$ in the z direction, a gauge in which only A_{11}, A_{22}, and $A_{12} = A_{21}$ are different to zero can be chosen. The condition $\bar{h} = 0$ gives $A_{11} = -A_{22}$. Now, by putting these equations in Eq. (49) we obtain

$$h_{mn}(t,z) = h^+(t-z)e^{(+)}_{mn} + h^\times(t-z)e^{(\times)}_{mn} + \Phi(t - v_G z)\eta_{mn}. \tag{55}$$

Again, the term $h^+(t-z)e^{(+)}_{mn} + h^\times(t-z)e^{(\times)}_{mn}$ describes the two standard (i.e. tensor) polarizations of GWs which arise from general relativity [30], while the term $\Phi(t - v_G z)\eta_{mn}$ is the scalar massive field arising from scalar-tensor gravity. In this case the associated line element results

$$ds^2 = -(1+\Phi)dt^2 + (1+\Phi)dz^2 + (1+h^+ + \Phi)dx^2 + (1-h^+ + \Phi)dy^2 + 2h^\times dxdy. \tag{56}$$

2.2 Quadrupole, dipole and monopole modes

We emphasize that in this Subsection we closely follow the papers [40, 41].

In the framework of GWs, the more important difference between general relativity and scalar-tensor gravity is the existence, in the latter, of dipole and monopole radiation [40]. In general relativity, for slowly moving systems, the leading multipole contribution to gravitational radiation is the quadrupole one, with the result that the dominant radiation-reaction effects are at order $(\frac{v}{c})^5$, where v is the orbital velocity. The rate, due to quadrupole radiation in general relativity, at which a binary system loses energy is given by [40]

$$(\frac{dE}{dt})_{quadrupole} = -\frac{8}{15}\eta^2 \frac{m^4}{r^4}(12v^2 - 11\dot{r}^2). \tag{57}$$

η and m are the reduced mass parameter and total mass, respectively, given by $\eta = \frac{m_1 m_2}{(m_1+m_2)^2}$, and $m = m_1 + m_2$.

r, v, and $\dot{r}$ represent the orbital separation, relative orbital velocity, and radial velocity, respectively.

In scalar-tensor gravity, Eq. (57) is modified by corrections to the coefficients of $O(\frac{1}{\omega})$, where ω is the Brans-Dicke parameter (scalar-tensor gravity also predicts monopole radiation, but in binary systems it contributes only to these $O(\frac{1}{\omega})$ corrections) [40]. The important modification in scalar-tensor gravity is the additional energy loss caused by dipole modes. By analogy with electrodynamics, dipole radiation is a $(v/c)^3$ effect, potentially much stronger than

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quadrupole radiation. However, in scalar-tensor gravity, the gravitational "*dipole moment*" is governed by the difference $s_1 - s_2$ between the bodies, where s_i is a measure of the self-gravitational binding energy per unit rest mass of each body [40]. s_i represents the "*sensitivity*" of the total mass of the body to variations in the background value of the Newton constant, which, in this theory, is a function of the scalar field [40]:

$$s_i = \left(\frac{\partial(\ln m_i)}{\partial(\ln G)} \right)_N . \tag{58}$$

G is the effective Newtonian constant at the star and the subscript N denotes holding baryon number fixed.

Defining $S \equiv s_1 - s_2$, to first order in $\frac{1}{\omega}$ the energy loss caused by dipole radiation is given by [40]

$$(\frac{dE}{dt})_{dipole} = -\frac{2}{3}\eta^2 \frac{m^4}{r^4}(12v^2 - 11\dot{r}^2). \tag{59}$$

In scalar-tensor gravity, the sensitivity of a black hole is always $s_{BH} = 0.5$ [40], while the sensitivity of a neutron star varies with the equation of state and mass. For example, $s_{NS} \approx 0.12$ for a neutron star of mass order $1.4M_\odot$, being $M_\odot$ the solar mass [40].

Binary black-hole systems are not at all promising for studying dipole modes because $s_{BH1} - s_{BH2} = 0$, a consequence of the no-hair theorems for black holes [40]. In fact, black holes radiate away any scalar field, so that a binary black hole system in scalar-tensor gravity behaves as if general relativity. Similarly, binary neutron star systems are also not effective testing grounds for dipole radiation [40]. This is because neutron star masses tend to cluster around the Chandrasekhar limit of $1.4M_\odot$, and the sensitivity of neutron stars is not a strong function of mass for a given equation of state. Thus, in systems like the binary pulsar, dipole radiation is naturally suppressed by symmetry, and the bound achievable cannot compete with those from the solar system [40]. Hence the most promising systems are mixed: BH-NS, BH-WD, or NS-WD.

The emission of monopole radiation from scalar-tensor gravity is very important in the collapse of quasi-spherical astrophysical objects because in this case the energy emitted by quadrupole modes can be neglected [30, 41]. The authors of [41] have shown that, in the formation of a neutron star, monopole waves interact with the detectors as well as quadrupole ones. In that case, the field-dependent coupling strength between matter and the scalar field has been assumed to be a linear function. In the notation of this paper such a coupling strength is given by $k^{-1} = \sqrt{\frac{|2\omega+3|}{16\pi}}$ in Eq. (2). Then [41]

$$k^{-1} = \alpha_0 + \beta_0(\varphi - \varphi_0) \tag{60}$$

and the amplitude of the scalar polarization results [41]

$$\Phi \propto \frac{\alpha_0}{d} \tag{61}$$

where d is the distance of the collapsing neutron star expressed in meters.

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2.3 Conformal invariance of the + and × polarizations

It is also important to reviewing that the quadrupole modes, i.e. + and ×, are conformal invariants [39].

In standard general relativity the GW-equations in the TT gauge are [30]

$$\Box h^{\alpha}_{\beta} = 0, \tag{62}$$

where $\Box \equiv (-g)^{-1/2}\partial_a(-g)^{1/2}g^{ab}\partial_b$ is the usual D'Alembert operator. Clearly, matter perturbations do not appear in (62) since scalar and tensor perturbations do not couple with tensor perturbations in Einstein equations. The task is now to derive the analogous of Eqs. (62) considering the action of scalar-tensor gravity (5). Matter contributions will be discarded as GWs are analysed in the linearized theory in vacuum. By following [38], a conformal analysis helps in this goal. In fact, by considering the conformal transformation (1), we obtain the conformal equivalent Hilbert-Einstein action

$$A = \frac{1}{2k}\int d^4x\sqrt{-\widetilde{g}}[\widetilde{R} + L(\ln\varphi, (\ln\varphi)_{;a})], \tag{63}$$

in the Einstein frame, where $L(\ln\varphi, (\ln\varphi)_{;a})$ is the conformal scalar field contribution derived from [38]

$$\tilde{R}_{ab} = R_{ab} + 2((\ln\varphi)_{;a}(\ln\varphi)_{;b} - g_{ab}(\ln\varphi)_{;d}(\ln\varphi)^{;d} - \frac{1}{2}g_{ab}(\ln\varphi)^{;d}{}_{;d}) \tag{64}$$

and

$$\tilde{R} = \varphi^{-2} + (R - 6\Box(\ln\varphi) - 6(\ln\varphi)_{;d}(\ln\varphi)^{;d}). \tag{65}$$

In any case, the $L(\ln\varphi, (\ln\varphi)_{;d})$-term does not affect the GWs-tensor equations, thus it will not be considered any longer [38].

By starting from the action (63) and deriving the Einstein-like conformal equations, the GWs equations are

$$\widetilde{\Box}\widetilde{h}^{\alpha}_{\beta} = 0, \tag{66}$$

expressed in the conformal metric $\tilde{g}_{ab}$. As scalar perturbation does not couple to the tensor part of gravitational waves, it is [38]

$$\widetilde{h}^{\alpha}_{\beta} = \widetilde{g}^{\delta\alpha}\delta\widetilde{g}_{\beta\delta} = \varphi^{-2}g^{\delta\alpha}\varphi^2\delta g_{\beta\delta} = h^{\alpha}_{\beta}, \tag{67}$$

which means that h^{α}_{β} is a conformal invariant.

As a consequence, the plane wave amplitude $h^{\alpha}_{\beta} = h(t)e^{\alpha}_{\beta}\exp(ik_\beta x^\alpha)$, where e^{α}_{β} is the polarization tensor, are the same in both the Jordan and Einstein frame. The D'Alembert operator transforms as [38]

$$\widetilde{\Box} = \varphi^{-2}(\Box + 2(\ln\varphi)^{;a}\partial_{;a}) \tag{68}$$

and this means that the background is changing while the tensor wave amplitude is fixed.

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3 Geodesic deviation

The following analysis concerns potential observable effects due by GWs in order to discriminate *the physical frame*. For this goal, let us use the geodesic deviation equation, which governs GWs signals in the gauge of the local observer. This gauge is the locally inertial coordinate system of a laboratory environment on Earth, where GWs experiments are performed [30, 35, 36]. The geodesic deviation equation in the Jordan frame is [30]

$$\frac{D^2\xi^d}{ds^2} = \tilde{R}_{abc}{}^{d}\frac{dx^c}{ds}\frac{dx^b}{ds}\xi^a, \tag{69}$$

where ξ^a is the separation vector between two test masses [30], i.e.

$$\xi^a \equiv x^a_{m1} - x^a_{m2}, \tag{70}$$

$\frac{D}{ds}$ is the covariant derivative and s the affine parameter along a geodesic [30]. In the Einstein frame the Riemann tensor rescales as [26]

$$\begin{aligned} R_{abc}{}^{d} &= \tilde{R}_{abc}{}^{d} - 2\delta^d_{[a}\bigtriangledown_{b]}\bigtriangledown_c(\ln\sqrt{\tilde{\varphi}}) + \\ &+2g^{de}g_{c[a}\bigtriangledown_{b]}\bigtriangledown_e(\ln\sqrt{\tilde{\varphi}}) - 2\bigtriangledown_{[a}(\ln\sqrt{\tilde{\varphi}})\delta^d_{b]}\bigtriangledown_c(\ln\sqrt{\tilde{\varphi}}) + \\ &+2\bigtriangledown_{[a}(\ln\sqrt{\tilde{\varphi}})g_{b]c}g^{de}\bigtriangledown_e(\ln\sqrt{\tilde{\varphi}}) + 2g_{c[a}\delta^d_{b]}g^{ef}\bigtriangledown_e(\ln\sqrt{\tilde{\varphi}})\bigtriangledown_f(\ln\sqrt{\tilde{\varphi}}). \end{aligned} \tag{71}$$

Eq. (71) has to be put into eq. (69). Using the contraction properties of δ^a_b, the symmetry properties and recalling the normalization condition [26, 30]

$$g_{ac}\frac{dx^a}{ds}\frac{dx^c}{ds} = 1, \tag{72}$$

a bit of algebra gives

$$\frac{D^2\xi^d}{ds^2} = \tilde{R}_{abc}{}^{d}\frac{dx^c}{ds}\frac{dx^b}{ds}\xi^a + k\frac{D}{ds}(\partial^d\tilde{\varphi}) \tag{73}$$

Thus, an extra term of the geodesic deviation equations, which is not present in the Jordan frame, see Eq. (69), is present in the Einstein frame, i.e. the term $k\frac{D}{ds}(\partial^d\tilde{\varphi})$.

4 Using gravitational waves to discriminate

The line element (36) for the scalar component of massless scalar GWs reduces to

$$ds^2 = -dt^2 + dz^2 + [1 + \Phi(t-z)][dx^2 + dy^2], \tag{74}$$

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for a wave propagating in the z direction. In the same way the line element (56) for the scalar component of massive scalar GWs reduces to

$$ds^2 = [1 + \Phi(t - v_G z)](-dt^2 + dz^2 + dx^2 + dy^2). \tag{75}$$

The cases of massive scalar-tensor gravity and $f(R)$ theories are totally equivalent [3, 35, 36, 37, 38]. This is not surprising as it is well known that there is a more general conformal equivalence between scalar-tensor gravity and $f(R)$ theories [3, 35, 36, 37, 38]. In fact, $f(R)$ theories can be conformally reformulated in the Einstein frame by choosing the conformal rescaling in a slight different way, i.e. $e^{2\tilde{\varphi}} = |f'(R)|$ [17, 38].

In the Jordan frame the motion of test masses, which is due to scalar GWs, in the gauge of the local observer is well known [35, 36]. GWs manifest them-self by exerting tidal forces on the test-masses, i.e. the mirror and the beam-splitter in the case of an interferometer [35, 36]. By putting the beam-splitter in the origin of the coordinate system, the components of the separation vector are the coordinates of the mirror. At first order in Φ and h^+ the total motion of the mirrors due to GWs in massless scalar-tensor gravity in the Jordan frame is (scalar mode plus quadrupole modes) [35, 36]

$$\delta x_M(t) = \frac{1}{2} x_{M0} h^+(t) + \frac{1}{2} x_{M0} \Phi(t) \tag{76}$$

and

$$\delta y_M(t) = -\frac{1}{2} y_{M0} h^+(t) + \frac{1}{2} x_{M0} \Phi(t), \tag{77}$$

where x_{M0} and y_{M0} are the initial (unperturbed) coordinates of the mirror.

In the case of massive scalar-tensor gravity and of $f(R)$ theories the total motion of the mirror due to GWs is (scalar mode plus quadrupole modes) [35, 36]

$$\begin{aligned} \delta x_M(t) &= \tfrac{1}{2} x_{M0} h^+(t) + \tfrac{1}{2} x_{M0} \Phi(t) \\ \delta y_M(t) &= -\tfrac{1}{2} y_{M0} h^+(t) + \tfrac{1}{2} y_{M0} \Phi(t) \\ \delta z_M(t) &= -\tfrac{1}{2} m^2 z_{M0} \psi(t), \end{aligned} \tag{78}$$

where [35, 36]

$$\ddot{\psi}(t) \equiv \Phi(t). \tag{79}$$

Note: the most general definition is $\psi(t - v_G z) + a(t - v_G z) + b$, but one assumes only small variations of the positions of the test masses, thus $a = b = 0$ [35, 36]. Then, in the case of massive GWs a longitudinal component is present because of the presence of a small mass m [35, 36]. As the interpretation of Φ is in terms of a wave-packet, solution of the the Klein - Gordon equation (42), it is also

$$\psi(t - v_G z) = -\frac{1}{\omega^2} \Phi(t - v_G z). \tag{80}$$

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Now, let us see what happens in the Einstein frame. Eqs. (2) and (1) can be used to express the linearized rescaled scalar field and the linearized conformal transformation. At first order in Φ it is

$$\tilde{\Phi} = \delta\tilde{\varphi} = \frac{1}{k}\frac{\delta\varphi}{\varphi_0} = \frac{1}{k}\Phi \tag{81}$$

$$\tilde{g}_{ab} = (1 + k\tilde{\Phi})g_{ab}. \tag{82}$$

When the scalar GW passes, it produces an oscillating (linearized) curvature tensor [35, 36], plus an addictive component due to the quantity $k\frac{D}{ds}(\partial^d\tilde{\varphi})$ in Eq. (73). In the gauge of the local observer all the correction due to Christoffell-symbols vanish [30]. The gauge of the local observer is a coordinate system that, at first order in the metric perturbation, moves with the beam splitter and with its proper reference frame [30]. At first order, the coordinate time t is the same as the proper time in this locally inertial gauge [30]. Hence, putting again the beam-splitter in the origin of the coordinate system, from Eqs. (2), (73) and (81) the time evolution of the coordinates of the mirror in the presence of the scalar GWs, is

$$\frac{d^2x^\alpha_M}{dt^2} = \tilde{R}_{0\beta0}{}^\alpha x^\beta_M + k\frac{\partial^2\tilde{\Phi}}{\partial x_\alpha \partial x_\beta}x^\beta_M. \tag{83}$$

In the Einstein frame, using Eq. (82), the line element (74) for massless GWs rescales like

$$ds^2 = (1 + k\tilde{\Phi})[-dt^2 + dz^2] + (1 + 2k\tilde{\Phi})][dx^2 + dy^2]. \tag{84}$$

As it is well known that the linearized Riemann tensor is *gauge invariant* [30], the components $\tilde{R}_{0\beta0}{}^\alpha x^\beta_M$ can be computed directly in the gauge of Eq. (84). From [30] it is:

$$\tilde{R}_{ambn} = \frac{1}{2}\{\partial_m\partial_b h_{an} + \partial_n\partial_a h_{mb} - \partial_a\partial_b h_{mn} - \partial_m\partial_n h_{ab}\}. \tag{85}$$

In the case of eq. (84) one gets (only the non-zero elements will be explicitly written down)

$$\tilde{R}^1_{010} = \tilde{R}^2_{020} = -k\ddot{\tilde{\Phi}}. \tag{86}$$

Then, from Eq. (83), the time evolution of the coordinates of the mirror in the gauge of the local observer is

$$\begin{aligned} \ddot{x}_M &= -k\ddot{\tilde{\Phi}}x_M \\ \ddot{y}_M &= -k\ddot{\tilde{\Phi}}y_M \\ \ddot{z}_M &= -k\ddot{\tilde{\Phi}}z_M, \end{aligned} \tag{87}$$

i.e., for $j = 3$ a third equation is present. Thus, a longitudinal oscillation, which does not exist in the Jordan frame for massless scalar GWs, is present in

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the Einstein frame. By using the perturbation method [30, 35, 36] the solutions are:

$$\delta x_M(t) = x_{M0} k\tilde{\Phi}(t)$$

$$\delta y_M(t) = y_{M0} k\tilde{\Phi}(t) \tag{88}$$

$$\delta z_M(t) = z_{M0} k\tilde{\Phi}(t).$$

In this way, the longitudinal oscillation makes the total oscillations of the mirror of the interferometer perfectly isotropic in the Einstein frame. The third longitudinal oscillation exists as the theory is non-metric in the Einstein frame.

For a sake of completeness, let us add to Eqs. (88) the motion of the mirrors due to the ordinary quadrupole modes [39]. As we have shown in Subsection 2.3 that the quadrupole modes are conformal invariants, in the Einstein frame the motion of the mirrors due to quadrupole modes remains unchanged. Hence, we get the total motion:

$$\delta x_M(t) = \tfrac{1}{2} x_{M0} h^+ + x_{M0} k\tilde{\Phi}(t)$$

$$\delta y_M(t) = -\tfrac{1}{2} y_{M0} h^+ + y_{M0} k\tilde{\Phi}(t) \tag{89}$$

$$\delta z_M(t) = z_{M0} k\tilde{\Phi}(t).$$

Now, let us discuss the massive case. Using again eq. (82), at first order in $\tilde{\Phi}$, in the Einstein frame Eq. (75) rescales as

$$ds^2 = (1 + 2k\tilde{\Phi})(-dt^2 + dz^2 + dx^2 + dy^2). \tag{90}$$

Taking into account Eq. (42) that, under the transformation (81) remains unaltered, i.e. $\Box\tilde{\Phi} = m^2\tilde{\Phi}$, and by considering that, from Eqs. (80) and (81) it is

$$\tilde{\psi} = \frac{1}{k}\psi, \tag{91}$$

Eq. (85) gives

$$\tilde{R}^1_{010} = \tilde{R}^2_{020} = -k\ddot{\tilde{\Phi}}, \qquad \tilde{R}^3_{030} = km\ddot{\tilde{\psi}}. \tag{92}$$

To obtain the time evolution of the coordinates of the mirror, one has to consider the extra term in Eq. (83) too. In this case, as the scalar field depends from $t - v_G z$, at the end it is

$$\ddot{x}_M = k\ddot{\tilde{\Phi}} x_M$$

$$\ddot{y}_M = k\ddot{\tilde{\Phi}} y_M \tag{93}$$

$$\ddot{z}_M = k(v_G^2\ddot{\tilde{\Phi}} - m^2\ddot{\tilde{\psi}}) z_M,$$

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Recalling that $m = \sqrt{(1 - v_G^2)}\omega$ [35, 36] and using Eqs. (80) and (91) the perturbation method gives the solutions

$$\delta x_M(t) = k x_{M0} \tilde{\Phi}(t)$$

$$\delta y_M(t) = k y_{M0} \tilde{\Phi}(t) \tag{94}$$

$$\delta z_M(t) = k z_{M0} \tilde{\Phi}(t),$$

which are exactly the same of the massless case (88). In fact, even if the non-metric longitudinal motion is different with respect to the massless case, in the massive case there is also a metric longitudinal motion. Thus, the sum of the non-metric longitudinal motion and of the metric longitudinal motion in the massive case results equal to the total non-metric longitudinal motion in the massless case. In the massless case the longitudinal motion is totally non-metric. However, even if the motion of the mirror is the same for massless and massive scalar GWs in the Einstein frame, in principle, careful analyses of coincidences between various detectors could permit to discriminate between massless and massive cases because in the massless case the speed of the GW is exactly the speed of light, while in the massive case the speed of the GW is the group velocity v_G, lower than the speed of light.

Again, let us add to Eqs. (94) the motion of the mirrors due to the ordinary quadrupole modes [39]. We obtain the total motion

$$\delta x_M(t) = \tfrac{1}{2} x_{M0} h^+ + k x_{M0} \tilde{\Phi}(t)$$

$$\delta y_M(t) = -\tfrac{1}{2} y_{M0} h^+ + k y_{M0} \tilde{\Phi}(t) \tag{95}$$

$$\delta z_M(t) = k z_{M0} \tilde{\Phi}(t).$$

Now, let us explain why we are claiming that the GW astronomy will be the definitive test for the "Einstein frame versus Jordan frame" controversy. In principle, if advanced projects on the detection of GWs will improve their sensitivity allowing to perform a GW astronomy, one will only have to look which is the motion of the mirror in respect to the beam splitter of an interferometer in the locally inertial coordinate system in order to understand which is the physical frame of observations. If such a motion will be governed by Eqs. (76) and (77) for massless scalar waves or by Eqs. (78) for massive scalar waves, one will conclude that the physical frame of observations is the Jordan frame. If the motion of the mirror is governed by Eqs. (89) for massless scalar GWs which are equal to Eqs. (95) for massive scalar GWs one will conclude that the physical frame of observations is the Einstein frame.

On the other hand, such signals will be quite weak. Thus, in order for the analysis to be useful in practice, we have to provide a specific application of the proposed method [39]. In particular, we have to compare the trajectories in both of the frames and determine the experimental sensitivity required to distinguish them. We have also to compare with the sensitivities of ongoing and

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future experiments [39]. To make this, we consider an astrophysical event which produces GWs and which can, in principle, help to simplify the problem. In Subsection 2.2 we discussed two potential sources of potential detectable scalar radiation:

1. mixed binary systems like BH-NS, BH-WD, or NS-WD;

2. the gravitational collapse of quasi-spherical astrophysical objects.

The second source looks propitious because in such a case the energy emitted by quadrupole modes can be neglected [41] (in the sense that the monopole modes largely exceed the quadrupole ones. In fact, if the collapse is completely spherical, the quadrupole modes are totally removed [30]). In that case, only the motion of the test masses due to the scalar component has to be analysed. Hence, the motion of the test masses in the Jordan frame is given by

$$\delta x_M(t) = \frac{1}{2} x_{M0} \Phi(t) \tag{96}$$

and

$$\delta y_M(t) = \frac{1}{2} x_{M0} \Phi(t), \tag{97}$$

for massless GWs and by

$$\begin{aligned} \delta x_M(t) &= \tfrac{1}{2} x_{M0} \Phi(t) \\ \delta y_M(t) &= \tfrac{1}{2} y_{M0} \Phi(t) \\ \delta z_M(t) &= -\tfrac{1}{2} m^2 z_{M0} \psi(t), \end{aligned} \tag{98}$$

for massive GWs, while Eqs. (88) for massless GWs and Eqs. (94) for massive GWs govern the motion of the test masses in the Einstein frame. Thus, the problem is simpler. The authors of [41] analysed the interesting case of the formation of a neutron star through a gravitational collapse. In that case, they found that a collapse occurring closer than 10 kpc from us (half of our Galaxy) needs a sensitivity of $3 * 10^{-23} \sqrt{Hz}$ at 800 Hz (which is the characteristic frequency of such events) to potential detect the strain which is generated by the scalar component in the arms of LIGO.

At the present time, the sensitivity of LIGO at about 800 Hz is $10^{-22} \sqrt{Hz}$ while the sensitivity of the Enhanced LIGO Goal is predicted to be $8*10^{-22} \sqrt{Hz}$ at 800 Hz [1]. Then, for a potential realization of the test proposed in this paper, we have to hope in Advanced LIGO Baseline High Frequency and/or in Advanced LIGO Baseline Broadband. In fact, the sensitivity of these two advanced configuration is predicted to be $6 * 10^{-23} \sqrt{Hz}$ at 800 Hz [1]. If such a sensitivity will be really achieved, it will be possible to distinguish the different trajectories of the mirror in the two frames.

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For a sake of completeness [39], we recall that in the case of standard general relativity the scalar mode is not present. In that case, the motion of test masses is governed by [30]

$$\delta x_M(t) = \frac{1}{2} x_{M0} h^+(t) \tag{99}$$

and

$$\delta y_M(t) = -\frac{1}{2} x_{M0} h^+(t). \tag{100}$$

In the case of scalar-tensor gravity, it will be very important to understand if a longitudinal component will be present. Such a longitudinal component will be fundamental in order to discriminate between the two frames. If it will be absent and the motion of the mirror will be governed by the transverse eqs. (96) and (97) we will conclude that we are in presence of massless scalar GWs and the physical frame is the Jordan frame. On the other hand, if it will be present we have two possibility. If it will be perfectly isotropic with respect the two transverse oscillations, i.e. the motion of the mirror will be governed by Eqs. (88) or Eqs. (94), we will conclude that the physical frame is the Einstein frame. If it will not be perfectly isotropic with respect the two transverse oscillations, i.e. the motion of the mirror will be governed by Eqs. (98), we will conclude that we are in presence of massive scalar GWs and the physical frame is the Jordan frame.

Let us resume the situation by including a Table with 5 rows and 3 columns [39]. In the first column we include the 5 models to be distinuished (general relativity, massless-Jordan, massive-Jordan, massless-Einstein, massive-Einstein), in the second column we include the corresponding motion of the mirror and in the third column the polarizations and the corresponding symmetry properties of the trajectories [39].

general relativity	$\delta x_M(t) = \frac{1}{2} x_{M0} h^+(t)$ $\delta y_M(t) = -\frac{1}{2} x_{M0} h^+(t)$	transverse motion, only h^+ polarization
massless-Jordan	$\delta x_M(t) = \frac{1}{2} x_{M0} h^+(t) + \frac{1}{2} x_{M0} \Phi(t)$ $\delta y_M(t) = -\frac{1}{2} x_{M0} h^+(t) + \frac{1}{2} x_{M0} \Phi(t)$	transverse motion, h^+ polarization and Φ polarization
massive-Jordan	$\delta x_M(t) = \frac{1}{2} x_{M0} h^+(t) + \frac{1}{2} x_{M0} \Phi(t)$ $\delta y_M(t) = -\frac{1}{2} x_{M0} h^+(t) + \frac{1}{2} y_{M0} \Phi(t)$ $\delta z_M(t) = -\frac{1}{2} m^2 z_{M0} \psi(t)$	transverse and longitudinal motion, h^+ polarization and Φ polarization, no-isotropy between transverse and longitudinal motion due to the scalar component
massless-Einstein	$\delta x_M(t) = \frac{1}{2} x_{M0} h^+ + k x_{M0} \tilde{\Phi}(t)$ $\delta y_M(t) = -\frac{1}{2} y_{M0} h^+ + k y_{M0} \tilde{\Phi}(t)$ $\delta z_M(t) = k z_{M0} \tilde{\Phi}(t)$	transverse and longitudinal motion, h^+ polarization and Φ polarization, the oscillations due to the scalar component are perfectly isotropic

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massive-Einstein	$\delta x_M(t) = \frac{1}{2} x_{M0} h^+ + k x_{M0} \tilde{\Phi}(t)$ $\delta y_M(t) = -\frac{1}{2} y_{M0} h^+ + k y_{M0} \tilde{\Phi}(t)$ $\delta z_M(t) = k z_{M0} \tilde{\Phi}(t)$	transverse and longitudinal motion, h^+ polarization and Φ polarization, the oscillations due to the scalar component are perfectly isotropic

Clearly, this is a simple analysis which could be improved by the realization of a consistent GW astronomy that, by using coincidences between various detectors and by further improving the sensitivity of the detectors, could, in principle, enable a better analysis of the signals that we have discussed.

5 Conclusion remarks

Resuming, in this paper we have shown that the GW astronomy will permit to solve a captivating issue of gravitation, i.e. it will be the definitive test for the famous "Einstein frame versus Jordan frame" controversy. In fact, the author has shown that the motion of test masses in the field of a scalar GW is different in the two frames, thus, if a consistent GW astronomy will be realized, an eventual detection of scalar GWs will permit to discriminate among the two frames.

In this way, direct evidences from observations will solve in an ultimate way the famous and long history of the "Einstein frame versus Jordan frame" controversy.

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