On Convergence of Non-Monotone Series. A Letter.

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Abstract
We hope to bring to a close the analyses of the convergence of real valued series of the form $\Sigma f(n)|n \in N$ for differentiable functions, via the introduction of a theorem as a necessary condition for convergence of such series regardless of the nature of f.

Introduction
The difficulties surrounding the analysis of fourier series for the purpose of establishing everywhere convergent functions has been expressed in a recent paper by SV. Konyagin[1]. Many techniques used in the establishment of convergence are specific to monotone convergent series, as such convergence is difficult to establish in Fourier series. We introduce a new technique for use in such cases.

1. Convergence

Theorem 1.1.

The series $\sum_{1}^{\infty} f(n)|n \in N$ is convergent if and only if the integral

$$\underset{h\to\infty}{Lim}\int_{1}^{h} xf'(x)dx$$

converges.

Proof
A necessary condition for a series of the form $\sum_{1}^{\infty} f(n)$ to be convergent is that $\underset{h\to\infty}{Lim} f(h) = 0$.[1] Using the property stated, we have that $\int_{\infty}^{c} f'(x)dx = f(c)$. From this, we may re write $\sum_{1}^{\infty} f(n)$ as $\sum_{c=1}^{\infty}\int_{c}^{\infty} f'(x)dx$. From the summation, it is easy to see that over every interval of one starting at one, the integrals are $nA(n)$ where n is the n^{th} interval of length one beginning at one and $A(n) = \int_{n}^{n+1} f'(x)dx$.

[1]For an oscillating nonconvergent series, we may subtract k from the integrals that follow in our proof, where k is the limit of the function in concern.

We note now that $\{n \cap x\} = N$, thus $\sum_{\forall n} nA(n)$ is approximately $\int_1^\infty xf'(x)dx$. The difference between the two is smaller than $\int_1^\infty f'(x)dx$, since the sum of the difference between the upper and lower bound integrals $\sum_{\forall n}\int_n^{n+1} f'(x)dx$ and $\sum_{\forall n}\int_{n+1}^{n+2} f'(x)dx$, is : $\int_1^\infty f'(x)dx$. Finally since $\int_1^\infty f'(x)dx = f(1)$, this integral is always convergent and as such will not influence the behaviour of $\int_1^\infty xf'(x)dx$.

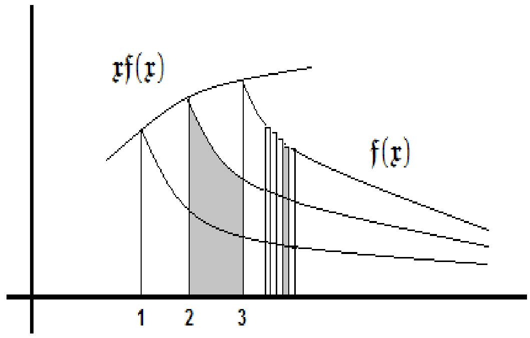

Figure 1:

References.

[1] SV. Konyagin. Almost everywhere convergence and divergence of Fourier series. *Proceedings of the international congress of mathematics,Madrid,Spain.* (2006).