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Quantum-like model of brain's functioning: decision making from decoherence

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Abstract

We present a quantum-like model of decision making in games of the Prisoner's Dilemma type. By this model the brain processes information by using representation of mental states in a complex Hilbert space. Driven by the master equation the mental state of a player, say Alice, approaches an equilibrium point in the space of density matrices (representing mental states). This equilibrium state determines Alice's mixed (i.e., probabilistic) strategy. We use a master equation which in quantum physics describes the process of decoherence as the result of interaction with environment. Thus our model is a model of thinking through decoherence of the initially pure mental state. Decoherence is induced by the interaction with memory and the external mental environment. We study (numerically) the dynamics of quantum entropy of Alice's mental state in the process of decision making. We also consider classical entropy corresponding to Alice's choices. We introduce a measure of Alice's diffidence as the difference between classical and quantum entropies of Alice's mental state. We see that (at least in our model example) diffidence decreases (approaching zero) in the process of decision making. Finally, we discuss the problem of neuronal realization of quantum-like dynamics of in the brain; especially roles played by lateral prefrontal cortex or and orbitofrontal cortex.

Keywords: Decision making, dynamics of mental state, quantum-like model, interaction with memory, decoherence, quantum master equation, neuronal paradigm

1. Introduction

The idea that the mathematical formalism of quantum information theory can be used to describe information processes in the brain was elaborated in a series of papers, see [1]–[14]. This approach is based on the fundamental conjecture that the real physical brain developed an ability to represent the probabilistic information in complex linear space, by complex vectors (pure quantum-like mental states) or more generally density operators (mixtures of pure quantum-like mental states).¹ (This paper contains a brief appendix on mathematics of quantum mechanics which may be useful for biologists, see also book [11].)

¹In section 5 we discuss the problem of neuronal realization of quantum-like dynamics of in the brain; especially roles played by lateral prefrontal cortex or and orbitofrontal cortex. We use studies on neuronal correlates observed in the process of decision making [15],

We know well that quantum information processing can be simulated on the classical computer. Therefore our model need not be based on physically quantum brain as, e.g., the models of Homeroff [19], [20] and Penrose [21], [22]. Since the brain has huge computa-

[16]. Of course, these are very preliminary considerations on a possible neuronal realization of the quantum-like dynamics of decision making. However, it is useful to initiate such a discussion as soon as possible to attract attention of neuroscientists and psychologists to the problem of neuronal realization of the quantum-like dynamics of decision making. We also mention that quantum-like probabilistic effects can be found not only in functioning of advanced cognitive systems, but even in cell's biology. For example, we can point to the well known glucose effect on *E. coli* growth; destructive interference of two factors: the presence of lactose and glucose in a *E. coli* cell [17]. It is also well known that a dental epithelial cell grows in a medium as it is (no differentiation). A dental mesenchymal cell grows similarly. However, this two contexts are incompatible, see [18] for statistical data. Their unification induces quantum-like interference.

tional resources (a “neural computer” with 300 billions of processing units), the main reason for usage of the quantum representation of information may be not increasing the speed of computations. The main distinguishing feature of the quantum information representation is a possibility to operate with *superpositions of states*. We explore this feature of quantum information processing. From the very beginning we want to distance from models of cognition based on quantum physical carriers of information. It seems that it is impossible to make these models consistent with the *neuronal paradigm of cognition*. We like to combine quantum information theory with the neuronal paradigm. A possible physical realization of quantum-like processing of information by the brain operating with classical electromagnetic fields was presented in [23].

A few years ago J. Busemeyer et al. [2] noticed that quantum-like models of decision making can be used to explain disjunction effect in experiments of cognitive psychology. In particular, he reproduced statistical data from experiments of the Prisoner’s Dilemma (PD) type obtained by Shafir and Tversky [24], [25], see also [26]. Busemeyer’s quantum-like model of decision making in PD is a psychological analog of the quantum model for the hydrogen atom. Of course, both models are oversimplified and to be applied to more complicated phenomena, they should be essentially modified. However, both express the main distinguishing features of corresponding theories (quantum physics and quantum-like cognitive psychology). Therefore in this paper we proceed with PD. Following to tradition of quantum information theory we call players Alice and Bob, see section 2 for details.

Our aim is to describe the process of decision making in the brain; in particular, interaction with memory. Quantum-like models of decision making were presented in a series of papers [1]–[3], [6], [8]–[14].

Dynamical models of decision making are of the main interest for us. We recall that, in a few papers [1], [2], [12], the process of decision making was described by Schrödinger’s evolution of the mental state. The latter was assumed to be a pure state (mathematically it is represented by a normalized vector of complex Hilbert space). In this paper we shall explore in more detail the idea [14] that decision making has to be represented by a more complicated dynamics which describes the evolution of quantum (or more generally quantum-like state) interacting with an environment. Such dynamics plays an important role in quantum physics. Its fundamental feature is transformation of pure states (described by complex vectors) into mixed states (described by density matrices) – *decoherence*. In our cognitive model

memory is an important part of the “mental environment” which induces decoherence of a pure state, Alice’s state of mind before starting the process of decision making. In this paper we do not model the process of interaction with memory; as often in quantum information theory we represent memory (as well as the external mental environment) as a bath, in our case a “mental bath.” In a future paper we plan to model this process in more detail by using the apparatus of quantum Markov chains, cf. [6].

In quantum physics interaction of a quantum system with a bath is described by a quantum version of the master equation. Quantum Markovian dynamics given by the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) equation, see e.g. [27] for detail, is the most popular approximation of quantum dynamics in the presence of interaction with a bath. We remind shortly the origin of the GKSL-dynamics. The starting point is that the state of a composite system, a quantum system s combined with a bath, is a pure quantum state, complex vector Ψ , which evolution is described by Schrödinger’s equation. This is an evolution in a Hilbert space of the huge dimension (since a bath has so many degrees of freedom). The existence of the Schrödinger dynamics in the huge Hilbert space has a merely theoretical value. Observers are interested in the dynamics of the state ϕ_s of the quantum system s . The next fundamental assumption in derivation of the GKSL-equation is the Markovness of the evolution, the absence of long term memory effects. It is assumed that interaction with the bath destroys such effects. Thus, the GKSL-evolution is Markovian evolution. Finally, we point to the condition of the factorizability of the initial state of a composite system (a quantum system coupled with a bath), $\Psi = \phi_s \otimes \phi_{\text{bath}}$, where $\otimes$ is the sign of the tensor product. Physically factorization is equivalent to the absence of correlations (at the beginning of evolution; later they are induced by the interaction term of Hamiltonian – the generator of evolution). One of distinguishing features of the evolution under the mentioned assumptions is the existence of one or a few *equilibrium points*. The state of the quantum system s stabilizes to one of such points in the process of evolution; a pure initial state, a complex vector ψ_s , is transformed into a mixed state, a density matrix $\rho_s(t)$. In contrast to the GKSL-evolution, the Schrödinger evolution does not induce stabilization; any solution different from an eigenvector of Hamiltonian will oscillate for ever. Another property of the Schrödinger dynamics is that it always transfers a pure state into a pure state, i.e., a vector into a vector. And we want to obtain mixed states, diagonalized in the basis corresponding to the decision operator. The GKSL-

evolution gives such a possibility.

In the process of decision making the brain selects a pure mental state describing possible decisions of the problem under consideration and drives this state – denote it by ϕ_A (superposition of possible decisions of Alice in her game with Bob). The state of the mental environment is represented by another complex vector, say ϕ_B . In general, this is a huge mental state representing all superpositions in memory and even permanent supply of superpositions created by the brain through its interaction with the environment. However, if Alice is concentrated on her strategy of play with Bob, we can restrict ϕ_B to Alice’s mental image of the possible actions of Bob. In reality ϕ_B belongs to complex Hilbert space of a large dimension. Therefore the standard assumption used in the derivation of the GKSL-equation is fulfilled. (In this paper, we consider a toy model in which ϕ_B is two dimensional, representing superposition of possible actions of Bob created in Alice’s brain. Thus formally one of the most important assumptions of derivation of the GKSL-equation is not fulfilled. However, more detailed analysis shows that, in fact, in quantum physics the dimension of a bath is not crucial. The crucial property of a bath is that it is very stable to fluctuations in the quantum system s interacting with it. This assumption is fulfilled if Alice’s image of possible actions of Bob is sufficiently stable with respect to fluctuations of the state of her possible actions.) The assumption of Markovness of the mental state evolution in decision making is natural. To proceed quickly to a decision, Alice must ignore the history of her reflections on possible actions with respect to Bob.² An input from (long-term) memory or mental environment destroys (working) memory of her reflections. (Working memory does not preserve a long chain of Alice’s reflections.) Finally, we can assume that the initial composite state is factorized, i.e., correlations between Alice’s image of Bob and her possible actions are created in the process of decision making. Under these assumptions we can model the process of decision making by using the GKSL-equation.

The mental state representing possible actions of Alice stabilizes to one of equilibrium points of the GKSL-dynamics. (In the mathematical model stabilization is achieved only in the limit $t \rightarrow \infty$. However, in reality the brain cannot wait too long. We can assume the presence (in the brain) of a threshold ϵ which is used to terminate the process of stabilization of the mental

²Such reflections are processed in her working memory. So, we discuss Markovness of working memory. Of course, in our model long-term memory is not ignored; it is a part of the mental bath.

state to a point of equilibrium, see also Remark 3.) A model equation considered in this paper has a single equilibrium point. Thus Alice elaborates the unique solution (which depends only on the mental environment, in particular, memory). However, in general the GKSL-equation can have a few different equilibrium points. In such a case depending on the initial state of mind Alice can obtain different solutions of the same problem. Such equations with a richer structure of equilibrium points will be studied in one of coming papers.

2. Prisoner’s Dilemma

In the paper of [14], we designed a quantum-like model for decision-making process in two-player games. This section explains briefly how a player in our model decides his own action, cf. also Busemeyer et al. [2].

2.1. Pay-off Table of Two-player Game

Let us consider a two-player game with two strategies. We name two players Alice (“A”) and Bob (“B”). Two strategies which A and B can choose are denoted by “0” and “1”. The following table shows pay-offs assigned to possible four consequences of “ $0_A 0_B$ ”, “ $0_A 1_B$ ”, “ $1_A 0_B$ ” and “ $1_A 1_B$ ”. Here, a , b , c and d denote

$A \backslash B$	0_B	1_B
0_A	$(a a)$	$(b c)$
1_A	$(c b)$	$(d d)$

the values of pay-offs.

For example, a game of prisoner’s dilemma (PD) type is characterized by the relation of

$$c > a > d > b.$$

For the player A, his pay-off will be a or c if the player B chose “0” and b or d if the player B chose “1”. In the both cases, from the relations of $c > a$ and $d > b$, he can obtain larger pay-offs if he choose 1. The situation is same for the player B. Conventional game theory concludes that in PD game a “rational” player, who wants to maximize his own payoff, always chooses “1”.

However, the above discussion does not explain completely the process of decision-making in real player’s mind. Actually, as seen in statistical data in some experiments on so called disjunction effect [24]–[26] real players frequently behave “irrationally”. Our model is an attempt to describe such real player’s behaviors by some mathematical formalism. Our model is a

“quantum-like model” which is derived from basic concepts of quantum mechanics, but not an expansion of conventional game theory.

Remark 1. (Rationality) The conventional decision making theory is based on the notion of rationality which is closely related to the *Savage sure thing principle* (SSP) [29] (one of the basic principles of modern economics) having a large impact to cognitive psychology and cognitive science, see [12], [11] for popular presentations for biologists and psychologists. This principle matches well our intuition. This is a general principle of *consistency of our decisions*. If, e.g., Noboru plans to buy some shares under the condition that an event A happens and at the same time he plans to buy these shares even under the condition that A does not happen, then it seems natural for him to buy them even if there is no information whether A happened or not. This principle of consistency of decision making is related to the very foundation of classical probability theory, *the formula of total probability*, see [2], [12], [11], [9] for details. The formula of total probability is the basis of the Bayesian approach to decision making. Therefore violation of SPP (which is equivalent to violation of the formula of total probability) implies impossibility to use the Bayesian scheme of decision making. In game theory the approach developed by von Neumann and Morgenstern [30] is also closely related to SSP. In particular, the use of the mini-max principle and the Nash equilibrium are characteristic for “rational players”, rational in the sense of SSP. We remark that SSP and the von Neumann-Morgenstern [30] approach to game theory had been criticized from the first days of their creation. Many bright thinkers pointed to inadequacy of SSP-rationality to real brain’s functioning.³ However, a better mathematical model had not been created. In particular, we can point to a recent detailed analysis of SSP-rationality [31]. The authors of this paper rightly couple SSP-rationality with the Bayesian approach to decision making. They criticized the Bayesian scheme of decision making pointing to numerous experiments in cognitive science and psychology demonstrating inapplicability of this scheme. However, the conclusion of the authors of [31] – it is impossible to create a general formal mathematical model of decision making – is not acceptable for us. We generalize the conventional Bayesian scheme of decision making by operating with a priori probabilistic amplitudes, instead of Bayesian a priori probabilities. Thus, opposite to [31], we claim that, in spite of violations of

³Thus the critique of SSP-rationality is not the main aim of our paper.

SSP, the notion of rationality can be formalized.

Remark 2. (Quantum games) Theory of quantum games is an important part of quantum information theory, see, e.g., [32], [33]. In such a game theory quantum carriers of information, e.g., photons, are used as a source of randomness. Quantum game theory is typically coupled to quantum *physics*. In contrast, we consider games played by “classical” macroscopic players. We do not assume that any really quantum source of randomness is involved in a game.⁴ Therefore, opposite to the traditional quantum game theory, we do not assume that Alice and Bob e.g. share entangled photons. This is the basic *interpretational difference* between “physical quantum game theory” and our quantum-like game theory. This difference in interpretations plays a crucial role in application of quantum mathematics to real games. Opposite to e.g. [33], we consider the real PD and not a quantum PD-game which is based on sharing of entangled quantum systems by “prisoners.” (Experts in “physical quantum game theory” typically stress its role in future world of quantum technologies.) Our quantum-like game approach has violation of the formula of total probability as the point of departure. So, we do not speculate what kind of gambling would be popular after invention of quantum computers and communication. Finally, we point to an interesting publication of Cheon and Tsutsui [34] which may serve as a bridge between the traditional theory of quantum games and our quantum-like game theory. They rightly stressed the role of non-factorizability of probabilities in generation of nonclassical strategies. The condition of non-factorizability can be formulated without the usage of the formalism of quantum mechanics. There is no need to couple this condition to entanglement of states of quantum carriers of information. Therefore the Cheon-Tsutsui approach can be considered as quantum-like. We now turn to the mathematical structures of quantum and quantum-like game theories. If one forget about interpretation differences⁵, then quantum and quantum-like games are based on the same mathematics – the mathematical formalism of quantum mechanics. The main problem of the “ordinary quantum game theory” is that it considers physical quantum systems as sources of randomness. Therefore all actions on these systems which are permitted by quantum mechanics can be used in a quantum game. All possible unitary trans-

⁴Although we do not reject completely physical quantum models of brain’s functioning, e.g., based on entanglement in brain microtubules, we proceed without coupling to such models.

⁵I guess that for adherents of the conventional interpretation of quantum mechanics it would be impossible.

formations are in usage as game strategies. Roughly speaking in this framework it is possible to obtain everything. In contrast, quantum-like games under consideration are coupled to brain's functioning (at least to psychological functions). This stimulates *dynamical approach* to decision making, i.e., selection of strategies through quantum-like mental dynamics, see Busemeyer et al. [2], [12], Khrennikov [1], [11]. In [14] we made a new step: in the process of decision making the final state is determined not by a unitary evolution, but by a decoherence process described by the quantum master equation. This is an important extension of the mathematical formalism of quantum game theory. One of the main distinguishing features of the "quantum(-like) decoherence game theory" is non-invertibility of actions of players. Different initial states can produce the same output. This feature of our model matches well information processing by (macroscopic) biological systems. (Unitarity is not present in the macroworld. Therefore any model based on application of the quantum formalism to macrosystems has to contain a mechanism of violation of linearity; decoherence is the most natural one.)

2.2. Complex Hilbert Space Representation of Player's Mind

We present a model for the decision-making process in two-players games.⁶ We focus on player A's mind. In principle, the player A is not informed of which action the player B chooses. The player A will be conscious of two potentials of B's action simultaneously, and then he can not deny either of these potentials. In our model, this indeterminacy the player A holds is described by the following quantum superposition

$$|\phi_B\rangle = \alpha |0_B\rangle + \beta |1_B\rangle \in \mathbb{C}^2. \quad (1)$$

The values of α and β are related to degrees of consciousness to B's actions (so to say, these are complex probabilistic amplitudes of A's intentions that B may make decisions 0 or 1, respectively). We call this complex vector $|\phi_B\rangle$ a *prediction state vector*. (In accordance with the formalism of quantum mechanics, we assume $|\alpha|^2 + |\beta|^2 = 1$.)

The player A who is getting to choose the action "0" will be conscious of two consequences of " $0_A 0_B$ " and " $0_A 1_B$ " with probability amplitudes of α and β . This situation is described with a vector from $\mathbb{C}^2 \otimes \mathbb{C}^2$ given

by

$$\begin{aligned} |\Phi_{0_A}\rangle &= \alpha |0_A 0_B\rangle + \beta |0_A 1_B\rangle \\ &= |0_A\rangle \otimes |\phi_B\rangle \end{aligned} \quad (2)$$

Similarly,

$$|\Phi_{1_A}\rangle = |1_A\rangle \otimes |\phi_B\rangle, \quad (3)$$

is given for the situation such that A is getting to choose "1". By using these state vectors $|\Phi_{0_A}\rangle$ and $|\Phi_{1_A}\rangle$, we define the following vector:

$$|\Psi_A\rangle = x |\Phi_{0_A}\rangle + y |\Phi_{1_A}\rangle \in \mathbb{C}^2 \otimes \mathbb{C}^2, \quad (4)$$

($|x|^2 + |y|^2 = 1$). This state vector describes A's intentions to act. We call it a *mental state vector* (of A's intentions to act).

The player A's brain in this mental state chooses his own action probabilistically. His decision is described as "quantum measurements" of projectors corresponding to the vectors $|\Phi_{0_A}\rangle$ or $|\Phi_{1_A}\rangle$ on the state $|\Psi_A\rangle$. (Probabilities of "0" and "1" are given by $P(0_A) = P_{0_A} = |x|^2$ and $P(1_A) = P_{1_A} = |y|^2$.)

In our model, the decision-making process is described as a dynamics changing $|x|^2$ and $|y|^2$, and its dynamics has an *equilibrium solution*. Such stabilization of the mental state explains the following psychological activity in the player's mind: The player has two psychological tendencies, the one to choose 0 and the one to choose 1. Degrees of these two opposite tendencies change in his mind, and they become stable with balancing. (Fluctuations die and the definite probabilistic picture of the situation is created in A's mind).

The most simple dynamics of the stabilization of probabilities, the equations like chemical equilibration can be used in cognitive modelling:

$$\begin{aligned} \frac{d}{dt} P_{0_A} &= -k P_{0_A} + \tilde{k} P_{1_A}, \\ \frac{d}{dt} P_{1_A} &= k P_{0_A} - \tilde{k} P_{1_A}. \end{aligned} \quad (5)$$

The parameter $k(\tilde{k})$ corresponds to the velocity of the reaction from 0_A to 1_A (from 1_A to 0_A), and in the equilibrium state, the probabilities P_{0_A} and P_{1_A} are given as

$$P_{0_A}^E = \frac{\tilde{k}}{k + \tilde{k}}, \quad P_{1_A}^E = \frac{k}{k + \tilde{k}}. \quad (6)$$

The differential equations (5) are not described as a quantum dynamics. In fact, they give only a part of the complete system of quantum dynamical equations, the dynamics of the diagonal terms of the density matrix. We shall complete the system (5) and obtain a quantum dynamics; a system of differential equation for elements of the density matrix of the mental state, (12).

⁶At the first stage it repeats the model of Busemeyer et al [2]. However, we restrict consideration to factorized initial state. This factorization will provide a possibility to apply decoherence dynamics.

2.3. Amplitudes of Velocities of Random Transitions in Alice's Mind

As we have seen from Eq. (5), player's tendency to choose 1 or 0 is proportional to the velocity k or $\tilde{k}$, and these parameters determine the stability solution (6). The choice of k and $\tilde{k}$ is a very important issue in our model. We assume, their values are determined through comparison of possible consequences, $0_A 0_B$, $0_A 1_B$, $1_A 0_B$ and $1_A 1_B$. The player in our model will consider the following four kinds of comparisons:

$$\begin{aligned} 0_A 0_B &\stackrel{k_1}{\rightleftharpoons} 1_A 0_B, \quad 0_A 1_B \stackrel{k_2}{\rightleftharpoons} 1_A 1_B, \\ &\quad \tilde{k}_1 \quad \quad \quad \tilde{k}_2 \\ 0_A 1_B &\stackrel{k_3}{\rightleftharpoons} 1_A 0_B, \quad 0_A 0_B \stackrel{k_4}{\rightleftharpoons} 1_A 1_B. \end{aligned} \quad (7)$$

These comparisons are represented like the conditions of chemical equilibrium, each of which is specified by the reaction velocities, k_i and $\tilde{k}_i$.

The velocities k and $\tilde{k}$ should have the forms reflecting effects of the four comparisons and interferences between them. In order to define these velocities in the appropriate forms, we introduce complex numbers μ and $\tilde{\mu}$, which determine k and $\tilde{k}$ by

$$k = |\mu|^2, \quad \tilde{k} = |\tilde{\mu}|^2,$$

and define these μ and $\tilde{\mu}$ (see [14] for details) as

$$\begin{aligned} \mu &\equiv |\alpha|^2 \mu_1 + |\beta|^2 \mu_2 + \alpha \beta^* \mu_3 + \alpha^* \beta \mu_4 \\ \tilde{\mu} &\equiv |\alpha|^2 \tilde{\mu}_1 + |\beta|^2 \tilde{\mu}_2 + \alpha^* \beta \tilde{\mu}_3 + \alpha \beta^* \tilde{\mu}_4. \end{aligned} \quad (8)$$

Here $\mu_{i=1,2,3,4}$ and $\tilde{\mu}_{i=1,2,3,4}$ are complex numbers satisfying $|\mu_i|^2 = k_i$, $|\tilde{\mu}_i|^2 = \tilde{k}_i$ for given k_i and $\tilde{k}_i$. We point out that already here we introduced *complex amplitudes*. These are amplitudes of velocities of fluctuations (in Alice's brain) between various possibilities. Thus, although the system of equations (5) looks as purely classical dynamics, the quantum-like structure of decision making is already incorporated in it, through these complex amplitudes. The system of equations (5) depends on the prediction state, i.e., the state of possible Bob's actions which was created in Alice's brain. This is an example of so called *adaptive dynamics* [28]. In principle, adaptive dynamics in combination with complex amplitudes of velocities is sufficient to reproduce statistical data obtained in experiments on the disjunction effect [24], [25], [26]. As was mentioned, this dynamics can be represented as a quantum-like dynamics of density operator, (12). The use of the quantum-like form

for dynamical equations simplifies essentially the formalism and provides a possibility to use standard mathematical methods of quantum mechanics and quantum information theory. However, this is not the main reason to apply the quantum-like representation of dynamical equations. The main reason is that we proceed under the basic assumption that the brain (as a macroscopic neuronal system) really uses the vector representation of probabilities.

2.4. Quantum-like Model of Irrational Behavior

The parameters $(k_i, \tilde{k}_i)$ introduced in the previous subsection specify the player's four kinds of comparisons, see Eq. (7). It is natural that these comparisons depend on a given game, namely its pay-off table like $(a, b, c$

$A \setminus B$	0_B	1_B
0_A	$(a \setminus a)$	$(b \setminus c)$
1_A	$(c \setminus b)$	$(d \setminus d)$

and d in the above table mean values of pay-offs.) The most simple relation of pay-offs and parameters $(k_i, \tilde{k}_i)$ can be obtained via depending on magnitude relation between values of pay-off. In the case of PD-type game, the relation of pay-offs is $c > a > d > b$, and then, k_i and $\tilde{k}_i$ are given as

$$\begin{aligned} k_1 &= 1, \quad k_2 = 1, \quad k_3 = 1, \quad k_4 = 0, \\ \tilde{k}_1 &= 0, \quad \tilde{k}_2 = 0, \quad \tilde{k}_3 = 0, \quad \tilde{k}_4 = 1. \end{aligned} \quad (9)$$

Such setting is simple, but not real. The real player's decision-making will depend on differences between pay-offs, not only magnitude relations. That is, the following setting will be more realistic

$$\begin{aligned} k_1 &= f_1(|a - c|), \quad k_2 = f_2(|b - d|), \quad k_3 = f_3(|b - c|), \quad k_4 = 0; \\ \tilde{k}_1 &= 0, \quad \tilde{k}_2 = 0, \quad \tilde{k}_3 = 0, \quad \tilde{k}_4 = \tilde{f}_4(|a - d|). \end{aligned} \quad (10)$$

The functions $f_i(x)$ are assumed to be monotone increasing functions.

Under the settings of k_i and $\tilde{k}_i$ of (9) or (10), the probability P_{0A}^E of Eq. (6) is non-zero as a result. Thus, our model explains that the player A generally has potential to make the "irrational" choice of 0 in the PD-game. The reason for this result is that the parameter of $\tilde{k}_4$ is non-zero. The $\tilde{k}_4$ represents the degree of tendency to choose 0 which occurs from the comparison between consequences of $0_A 0_B$ and $1_A 1_B$. It should be noted that such comparison is not considered in classical game theory.

3. Dynamics of Alice's Mental State, Density Matrix

In our model, the dynamics of probabilities corresponding to the mental state is specified by the differential equations Eq. (5):

$$\begin{aligned}\frac{d}{dt}P_{0_A} &= -kP_{0_A} + \tilde{k}P_{1_A}, \\ \frac{d}{dt}P_{1_A} &= kP_{0_A} - \tilde{k}P_{1_A}.\end{aligned}$$

Here, $P_{0_A} = |x|^2$ and $P_{1_A} = |y|^2$. (x and $y \in \mathbb{C}$ are coefficients of the mental state $|\Psi\rangle = x|0_A\rangle \otimes |\phi_B\rangle + y|1_A\rangle \otimes |\phi_B\rangle = x|\Phi_{0_A}\rangle + y|\Phi_{1_A}\rangle$, where $|\phi_B\rangle = \alpha|0_B\rangle + \beta|1_B\rangle$, $\alpha, \beta \in \mathbb{C}$ is the prediction vector defined in Eq. (1). The parameters k and $\tilde{k}$ are defined in (8).

In this section, we complete the dynamics Eq. (5) to a quantum dynamics of the density matrix $\rho = |\psi\rangle\langle\psi|$,

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}. \quad (11)$$

corresponding to the mental state $|\psi\rangle = x|0_A\rangle + y|1_A\rangle$ describing the superposition of A's decisions. Details of derivation (based on discrete quantum Markov chains) can be found in [14]. Here we present the final dynamical equation:

$$\begin{aligned}\frac{d}{dt}\rho_{00}(t) &= -|\mu_0|^2\rho_{00} + |\mu_1|^2\rho_{11}, \\ \frac{d}{dt}\rho_{01}(t) &= -\frac{1}{2}\rho_{01} + \mu_0^*\mu_1\rho_{10}, \\ \frac{d}{dt}\rho_{10}(t) &= -\frac{1}{2}\rho_{10} + \mu_0\mu_1^*\rho_{01}, \\ \frac{d}{dt}\rho_{11}(t) &= |\mu_0|^2\rho_{00} - |\mu_1|^2\rho_{11},\end{aligned} \quad (12)$$

where

$$|\mu_0|^2 = \frac{|\mu|^2}{|\mu|^2 + |\tilde{\mu}|^2}, \quad |\mu_1|^2 = \frac{|\tilde{\mu}|^2}{|\mu|^2 + |\tilde{\mu}|^2}.$$

It is clear that the equations for diagonal parts correspond to the equations (5) essentially.⁷

⁷We state again that the dynamics (12) is an oversimplified version of the GKSL-equation. In general, the equations for the diagonal terms contain the off-diagonal terms and vice versa. Surprisingly enough, already the equation (12) describes the process of decision making which is essentially different from the classical Markovian decision making. We shall see that the presence of the off-diagonal terms decreases the entropy of the mental state. In some way off-diagonal terms monitor the interaction with memory, the mental bath, in the process of decision making. In the quantum-like regime the brain uses less memory in the process of decision making, i.e., it performs a more optimal selection of information for decision making.

In this dynamics, any initial state of $\rho(0)$ approaches the unique equilibrium state ρ_E ;

$$\rho_E = \begin{pmatrix} |\mu_0|^2 & 0 \\ 0 & |\mu_1|^2 \end{pmatrix}, \quad (13)$$

see [14] for calculations.

The presence of nondiagonal terms in $\rho(t)$ for finite t does not play any role in the asymptotic limit $t \rightarrow \infty$. However, it modifies essentially the information structure of the evolving mental state. We shall explain this point in more detail.

4. Dynamics of Entropy in the Process of Decision Making

As usual in quantum information theory, we can consider von Neumann entropy, see, e.g., [27], of the quantum (mixed) state $\rho(t)$ and study its dynamics, $S(t) = -\text{Tr}(\rho(t) \ln \rho(t))$. This dynamics depends nontrivially on the initial state $\rho(t_0)$ as well as amplitudes of velocities of fluctuations between various possibilities in Alice's brain. In our model Alice's initial state of mind is always pure, this is

$$|\phi_A\rangle = x_0|0_A\rangle + y_0|1_A\rangle, \quad (14)$$

as $|x_0|^2 + |y_0|^2 = 1$. The state $|\Psi_A\rangle$, see (4), can be written as

$$|\Psi_A\rangle = |\phi_A\rangle \otimes |\phi_B\rangle. \quad (15)$$

(We state again that the predictions state $|\phi_B\rangle$ providing the quantum-like representation of Alice's image of possible Bob's actions has been already used to determine dynamics of $\rho(t)$, see (8).) The density matrix $\rho(t_0)$ corresponding to the pure state $|\phi_A\rangle$ has the form

$$\rho(t_0) = \begin{pmatrix} |x_0|^2 & x_0 y_0^* \\ x_0^* y_0 & |y_0|^2 \end{pmatrix}, \quad (16)$$

We can always assume that x_0 is a real number, $0 \leq x_0 \leq 1$, and

$$y_0 = \sqrt{1 - x_0^2} e^{i\theta_{A0}}.$$

We shall not play with all parameters involved in complex amplitudes μ_0 and μ_1 . We just select μ_0 as a real parameter and

$$\mu_1 = \sqrt{1 - \mu_0^2} e^{i\theta_\mu}.$$

Figure 1 contains graphs of quantum entropy $S_1(t), S_2(t), S_3(t)$ corresponding to three different choices of parameters:

S_1 , solid line : $x_0 = 0.33, \mu_0 = 0.85, \theta_{A0} = 0.1, \theta_\mu = 0.2$;

S_2 , dashed : $x_0 = 0.315, \mu_0 = 0.625, \theta_{A0} = 0.1, \theta_\mu = 0.2$; S_3 , dotted : $x_0 = 0.99, \mu_0 = 0.925, \theta_{A0} = 0.1, \theta_\mu = 0.2$.

(So, phases are the same for all graphs, we play only with amplitudes.) These graphs show that behavior of quantum entropy depends essentially on parameters, i.e., the initial state of Alice's mind and velocities of fluctuations between various possibilities.

Figure 1: Dynamics of quantum entropy

There are two main forms of graphs:

1). Entropy monotonically increases by stabilizing to a constant value; the first stage of the process of decision making is characterized by very quick increasing of entropy, then entropy increases very slowly; for large t , it varies negligibly, see the graph of S_1 .

We see that the first period of decision making is characterized by attraction of a large amount of new information (mainly from memory, but from the external environment as well); entropy increases very quickly. Then, when the processing of this information is more or less finished, entropy stabilizes (becoming a constant) by approaching the equilibrium point.

2). Entropy increases very quickly by approaching a local maximum, then it decreases and, finally, stabilizes. Typically, on the way to stabilization it has a local minimum and then it increases again.

In both cases, at the initial stage of decision making entropy increases as the result of interaction with "environment" (in particular, memory), then (for some clusters of parameters) it decreases and uncertainty is minimized, see S_2, S_3 , but then (before the final step of decision making) it increases again. Sometimes this final increasing of entropy is negligibly small, see the graph of S_3 ; sometimes entropy increases visibly, see S_2 .

The dynamics of quantum entropy represents the dynamics of uncertainty in Alice's decision state, entropy of choices between strategies. However, it does not describe the dynamics of Alice's diffidence to make the decision at this point t . We are looking for a quantity

which can be considered as a measure of Alice's diffidence.

First of all we introduce classical entropy corresponding to the diagonal terms in the density matrix $\rho(t)$. This is entropy for Alice's choice between two possible decisions if she were ignoring the off-diagonal terms. The presence of these terms is a consequence the quantum-like representation of information in the brain. The complete mental state of Alice is a pure state (of a huge dimension); the density matrix of Alice's decision state is extracted from this state. The quantum-like dynamics of this density matrix induces not only dynamics of the diagonal terms, but even off-diagonal ones. We set

$$SCL(t) = -(\rho_{00}(t) \ln \rho_{00}(t) + \rho_{11}(t) \ln \rho_{11}(t)),$$

the corresponding graphs are represented on Figure 2.

Figure 2: Dynamics of classical entropy

We see that shapes of the graphs for classical entropy are similar to shapes of corresponding graphs for quantum entropy. However (!), quantum entropy is always less than classical entropy. Thus the quantum-like processing of information induces less uncertainty. In some way, the quantum-like representation in e.g. memory provides a possibility to use a smaller database to make a decision. We can speculate that these off-diagonal terms are responsible for Alice's diffidence.

Let us introduce the diffidence-quantity

$$D(t) = SCL(t) - S(t).$$

It is positive and, as we see from the graphs on Figure 3, diffidence decreases and it approaches zero when Alice's decision state approaches the point of equilibrium.

Remark 3. As it was rightly pointed by a referee of this paper, "if a decision is made when the difference reaches a threshold, then the decision time is deterministic. This does not explain why is there a distribution of decision times across from the same person." We can speculate that the diffidence threshold ϵ_{DF} is not rigidly determined by brain's architecture of a concrete person,

Figure 3: Dynamics of diffidence

but it can fluctuate depending on context. It is natural to assume that ϵ_{DF} has the Gaussian distribution. In this model decision times are random, since the difference threshold is random.⁸

5. On experimental verification of dynamics of mental quantum-like state; coupling to the brain architecture

One of the referees of this paper asked a important question:

“How to measure or observe the dynamics of quantum entropy and difference during decision making in PD, in neurobiological, behavioral or psychological experiments?”

In principle, the formalism of quantum information theory provides a possibility to reconstruct a density matrix on the basis of statistical data obtained through a series of measurements of a family of incompatible observables – *quantum tomography*. Since observations are incompatible, we are not able to perform them on the same biological system: human person, animal, nor even insect. Therefore the state reconstruction (at each instance of time) will demand a large statistical ensemble of biological systems participating in experiments. In theory the family of incompatible observables depend on continuous parameters; in reality we discretize these parameters. However, to obtain a good approximation, the discretization has to be sufficiently fine. Thus even for one instance of time the experiment demands a huge group of participants. Moreover, members of such a group should behave homogeneously. It is not clear whether it would be possible to perform experiments on monitoring of mental state dynamics in PD experiments with people. We speculate that it may be possible to design PD-type games for simpler biological systems:

⁸A similar problem was discussed in [12].

animals? may be even insects? In the later case we have a better chance to create samples which behave homogeneously. This is an interesting problem addressed to psychologists and cognitive scientists.

However, it is too early to concentrate studies on the aforementioned problem. The following fundamental problem is essentially more important. In conventional quantum tomography we have to be able to measure linear combinations of incompatible observables. For example, let $\hat{p}$ and $\hat{q}$ denote the momentum and position operators, respectively. We also have to measure rotations of the type $\hat{z}_\theta = \cos \theta \hat{q} + \sin \theta \hat{p}$, $\theta \in [0, 2\pi)$. In quantum physics such measurement can be reduced to rotation of the state. And nowadays the technology of quantum state manipulation is well developed, so there is no problem to perform a state rotation.⁹ We have to develop a similar technology of mental state manipulation in biology. At the moment it is too early to speculate about a possibility of realization of this exciting project. In biology the situation is more complicated than in physics: there is no (or at least it is not yet known) coupling between the structure of space and mental states. For example the polarization vector is still meaningful in quantum physics¹⁰. Therefore we can use the space-representation of classical physics to prepare quantum systems in special states of polarization.

Another important question is about the *neuronal architecture* of quantum-like dynamics of decision making. By our model the dynamics described by the quantum master equation is realized on neuronal level. It describes interaction of the mental state encoding possible decisions with mental environment. As was remarked in [16]: “An optimal action, namely an action that is most likely to yield the most desirable outcome, often changes according to the state of the animals environment. Therefore, if a sensory stimulus informs the animal of a change in the state of its environment, the animal needs to store such information until it produces an appropriate action or until this information can be combined with another stimulus to determine the new state of the environment.” In that paper it was pointed to the *lat-*

⁹We point to the tremendous development of quantum technologies of state manipulation during the last ten years. This development was stimulated by large interest to quantum computing and cryptography. At the very beginning of quantum mechanics there was a huge gap between theoretical consideration involving all possible quantum states and possibilities for experimental realization of these states.

¹⁰The situation is more delicate. In quantum physics the polarization cannot be defined in the same way as in classical electrodynamics. If the polarization vector were well defined, it would serve as a hidden variable. As a consequence of Bell’s theorem, the latter does not exist. (This viewpoint has not been commonly accepted, see e.g. [11]). In any event the operator of projection of polarization to any direction is well defined.

609 *eral prefrontal cortex* as responsible for the state repre- 651
 610 sentation of the mental environment. We may speculate 652
 611 that the linear operator of the quantum master equation, 653
 612 see appendix, is stored in this domain of brain. (In the 654
 613 real situation this operator is time dependent.) 655

614 In [15], [16] it was also remarked that lesions which 656
 615 encompass the *orbitofrontal cortex* in human patients as 657
 616 well as psychiatric conditions as substance abuse and 658
 617 frontotemporal dementia may crucially change the abil- 659
 618 ity to represent the *statistical interdependence* between 660
 619 the values of alternative options [16]. Knowledge of 661
 620 such interdependence might allow the decision makers 662
 621 to switch to a better choice immediately, as soon as 663
 622 they experience the reduction of values from a partic- 664
 623 ular choice. We may speculate that the quantum-like 665
 624 representation of the mental state (by a density matrix) 666
 625 is performed in the orbitofrontal cortex. The density 667
 626 matrix describes correlations between alternative op- 668
 627 tions; it seems that lesions in the *orbitofrontal cortex* 669
 628 and aforementioned psychiatric conditions destroy (at 670
 629 least partially) the density matrix. 671

630 **Conclusive remarks.** We elaborated a quantum-like 672
 631 model of decision making based on interaction between 673
 632 the decision-state, i.e., the state carrying probabilities 674
 633 for possible choices, and the state of memory (or more 675
 634 general mental environment). We can speculate that 676
 635 decoherence may be a key element of transition from 677
 636 quantum unconsciousness (in which mind is represented 678
 637 by vector superpositions) to classical consciousness (in 679
 638 which the mind is represented by classical probabilistic 680
 639 mixtures).¹¹

640 6. Appendix: Mathematical formalism of quantum 681 641 mechanics 682

642 The mathematical formalism of quantum mechanics 683
 643 describes states of systems and observables, see book 684
 644 [11] for a simple representation of quantum mathemat- 685
 645 ics for biologists and psychologists. (The quantum- 686
 646 like approach is based on the observation that “quantum 687
 647 mathematics” can be applied outside quantum physics.) 688

648 The basic mathematical structure of quantum me- 689
 649 chanics is a complex Hilbert space: a linear space over 690
 650 complex numbers H (i.e., it is possible to form linear 691

combinations of vectors with complex coefficients) en-
 651 dowed with a Hermitian bilinear form mapping a pair
 652 of vectors $\psi_1, \psi_2 \in H$ into a complex number denoted
 653 $\langle \psi_1 | \psi_2 \rangle$ (we use Dirac’s notation which is typical for
 654 quantum information theory). We shall be interested
 655 in complex vectors normalized by one, i.e., $\psi \in H$
 656 such that $\|\psi\|^2 = \langle \psi | \psi \rangle = 1$. Such vectors encode so
 657 called *pure states* of quantum systems. Normalization
 658 by one is crucial for the probabilistic interpretation of
 659 pure states. Observables (e.g., the energy-observable or
 660 the position observable) are encoded by self-adjoint op-
 661 erators. 662

The theory is especially simple in the finite dimen-
 663 sional case (which is typically considered in quantum
 664 information theory). Here $H = \mathbf{C}^n$ is the Cartesian
 665 product of n -copies of the set of complex numbers $\mathbf{C}$.
 666 Hence, a pure state $\phi = (z_1, \dots, z_n)$, where $\|\phi\|^2 =$
 667 $|z_1|^2 + \dots + |z_n|^2 = 1$. Observables are given (in an or-
 668 thonormal basis) by Hermitian matrices $A = (a_{ij})$; here
 669 $\bar{a}_{ij} = a_{ji}$. In the Dirac notation the matrix elements are
 670 written as $\langle \phi_1 | A | \phi_2 \rangle$. Coordinates of a state vector are
 671 interpreted as probabilities (Born’s interpretation of a
 672 pure quantum states [11]). One can say that the quan-
 673 tum formalism provides a geometric representation of
 674 probabilities which is based on the Euclidean distance. 675

Dynamics of a pure state is described by the
 Schrödinger equation

$$i \frac{d\psi}{dt}(t) = \mathcal{H}\psi(t), \psi(0) = \psi_0, \quad (17)$$

676 where the operator $\mathcal{H}$ is the generator of evolution 687
 677 (Hamiltonian, the operator of energy). We remark that 688
 678 Schrödinger dynamics is deterministic. By fixing the 689
 679 initial state we can find the state of a quantum system 690
 680 at any instance of time. The fundamental problem of 691
 681 quantum foundations is that this state does not deter-
 682 mine values of observables. Measurements induce ran-
 683 domness. 684

We remark that each pure state ψ determines a self-
 685 adjoint operator, the projector onto this state; $\rho \equiv |\psi\rangle\langle\psi|$
 686 (the last symbol is simply the Dirac notation): $\rho\phi =$
 687 $\langle\phi|\psi\rangle\psi$. We recall the basic properties of ρ_ψ :

- 688 a) it is positively defined, i.e., $\langle\phi|\rho|\phi\rangle \geq 0$ for any ϕ ;
- 689 b) it is Hermitian;
- 690 c) its trace (the sum of diagonal elements) equals to 691
 one.

The Schrödinger dynamics for pure states (vectors)
 can be rewritten as dynamics for corresponding opera-
 tors:

$$i \frac{d\rho}{dt}(t) = [\mathcal{H}, \rho(t)], \rho(0) = \rho_0, \quad (18)$$

¹¹ Thus, mind is always probabilistic, but it has different representa-
 tions in consciousness and unconsciousness. The process of interac-
 tion of the decision-state with memory can be described by the same
 model as decoherence of a pure state of a quantum system interact-
 ing with an environment, a bath; memory is a kind of mental bath for
 decision states.

where $[\mathcal{H}, \rho] = \mathcal{H}\rho - \rho\mathcal{H}$ is the commutator of operators.

Consider now a statistical mixture (in the classical sense) of a few projection operators ρ_i corresponding to pure states ψ_i with weights $p_i \geq 0$, $\sum p_i = 1$,

$$\rho = p_1\rho_1 + \dots + p_n\rho_n. \quad (19)$$

Each operator of this form satisfies conditions a)–c) and vice versa. Denote the class of all operators with properties a)–c) by the symbol $\mathcal{D}(H)$. This is the space of states of quantum systems. Its elements (called *density operators*) can be interpreted as statistical mixtures of pure states. In general a density operator can be represented in the form (19) in many ways. There is one special expansion corresponding to eigenvectors of ρ . The density operator corresponding to a pure state can be characterized in the following way: in the basis of eigenvectors, its matrix has only one nonzero element (equal to one), i.e., up to a permutation of eigenvectors:

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad (20)$$

where the blocks of zeros have the corresponding sizes. However, this takes place only in the basis of eigenvectors, cf. (16).

Dynamics of a quantum state is described by the equation (18), *von Neumann equation*. This dynamical model can be used only in the absence of interaction of a quantum system with an environment, a bath. If such interaction is essential (so a system cannot be considered as isolated), von Neumann dynamics (18) has to be modified and additional terms have to be included in it. The basic postulate of quantum theory is that the state dynamics is *linear*. Therefore modified dynamics has the form

$$i\frac{d\rho}{dt}(t) = [\mathcal{H}, \rho(t)] + L(\rho), \rho(0) = \rho_0, \quad (21)$$

where L is a linear operator. This operator has to be chosen in such a way that starting with $\rho_0 \in \mathcal{D}(H)$, we shall obtain a trajectory $t \rightarrow \rho(t)$ in $\mathcal{D}(H)$. The corresponding conditions on linear systems were formulated by Gorini, Kossakowski, Sudarshan, and Lindblad, see, e.g., [27] for details.

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>A novel model of quantum-like processing of mental information is presented.

>This model matches the neuronal paradigm of cognitive science.

>Classical electromagnetic signals provide the basis for quantum-like representation.

>Signals are induced by joint activity of neurons.

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