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Fault isolation filter and sensors scheduling co-design for networked control systems

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Abstract— This paper addresses the problem of multiple fault detection and isolation under communication constraints. More specifically, we consider the issue of sensor scheduling and fault isolation co-design under limited bandwidth capacity. The proposed isolation filter can be viewed as special structure of the traditional Kalman filter. The sensor scheduling sequence and the proposed filter are built in order to ensure the fault isolability property and noise effect minimization.

I. INTRODUCTION

The study of networked control systems (NCS) is receiving much importance in this recent years. This is mainly due to the several advantages resulting from using a shared real time network through which sensors, actuators and controllers communicate. Compared with classic fault detection (FD) systems, diagnosis over networks can reduce the system wiring, make the system easy to supervise, maintain and increase system agility... etc. Nevertheless, new constraints also arise when sensor information and control information are transmitted over a network. Such constraints can be categorized in five types [5], [6]:
1) Quantization errors in the signals transmitted over the network due to the finite word length of the packets;
2) Packet dropouts caused by the unreliability of the network;
3) Variable sampling/transmission intervals;
4) Variable communication delays;
5) Medium access constraints due to the limited bandwidth and sharing the network by multiple nodes and the fact that only one node is allowed to transmit its packet per transmission
6) Power consumption mainly in wireless networked control systems.

The presence of these network induced effects can degrade the performance of FD systems and implies more robust algorithms to this communication constraints.

It is clear that all of these constraints can exist in a communication network, but only some of them were fully considered in the literature, mainly the induced delay effect, packet loss and sampling influences. The delay issue is considered for example in [1], [2], [3], [5]. In [4], the authors deal with the design of robust FD systems for NCS with large transfer delays, in which it is impossible to totally decouple the fault effects from unknown inputs. An adaptive Kalman filter is proposed in [7] to minimize the effects of the network induced delay on the residual signal. [16] considered the problem of FD for NCS with both delayed inputs and measurements. In [19], [20], the FD system for NCSs with packet dropouts was designed by modeling the NCSs as a Markov jumping linear system (MJLS). The issue of FD with multiple network induced constraints has been considered for example [2], [14], [21]. Note that, to the best of the author knowledge, the issue of fault isolation in networked systems has not been fully investigated nowadays.

In this paper, we will address the problem of multiple fault detection and isolation under communication constraints. More specifically, we will consider medium access constraints. In this case, the shared network can only accommodates a limited number of simultaneous communications between components. In this context, it is only meaningful to specify a fault detection and isolation module in conjunction with a communication policy which indicates the times at which the plants sensors are to be granted medium access. This communication policy is known in the literature as communication sequence. The communication sequence specifies which sensors are able to send information to the detection filter at each time step. Hence, the considered problematic leads naturally to consider a co-design problematic. That is, the design of a fault detection and isolation filter in conjunction with sensor scheduling sequence. The sensor scheduling sequence and the proposed filter are built in order to ensure the fault isolability property and noise effect minimization.

The rest of the paper is organized as follow: section II gives the problem formulation. In section III, we will give our main results. The sensor scheduling problem will be discussed in section IV. A numerical example will be given in section V to illustrate the effectiveness of the proposed co-design method. Finally, we will provide some conclusions and some future research directions in section VI.

II. PROBLEM FORMULATION

Consider the remote system depicted in Fig.1. The state space representation of the plant under actuator and/or com-
Let us introduce the application: \( \mu_k : Z \rightarrow \mathcal{M} = \{1, \ldots, \sigma\} \), that determine at each sample time the corresponding sensors group index. We call this application the switching pattern for the sensor side. In Fig. 1, the signal \( \hat{y}_k \in \mathbb{R}^b \) is related to \( y_k \) by the following relation: \( \hat{y}_k = S(\mu_k)y_k \). The switch matrix \( S(\mu_k) \in \mathbb{R}^{b \times m} \) is used to select the subset of measures that will be sent to the controller at each time step \( k \). This subset is indexed by the values of the switching pattern \( \mu_k \). Considering the band limited effect, the extended plant model is described by

\[
\begin{align*}
{x_{k+1}} & = Ax_k + Bu_k + FY_k + w_k \\
y_k & = Cx_k + v_k \\
\end{align*}
\]  

(1)

where \( x_k \in \mathbb{R}^n \) is the state vector, \( u_k \) is the control input, \( F = [f_1, f_2, \ldots, f_q] \in \mathbb{R}^{n \times q} \) is the fault distribution matrix, \( Y_k = [Y_k^1, Y_k^2, \ldots, Y_k^q]^T \in \mathbb{R}^q \) is the fault vector and \( y_k \in \mathbb{R}^m \) is the measurement signals vector. We assume that each component of the output vector \( y_i \) represents the sensor \( i \) with \( i \in \{1, 2, \ldots, m\} \). The initial state vector \( x_0 \) process noise \( w_k \) and measurement noise \( v_k \) are uncorrelated, zero mean white Gaussian random processes with \( x_0 \sim N(\bar{x}_0, P_0) \), \( w_k \sim N(0, W) \) and \( v_k \sim N(0, I_m) \) respectively, where \( P_0, W \) and \( R \) are symmetric, positive definite matrices.

The main objective of this paper consists in the design of a fault detection and isolation filter that takes into account the communication constraints induced by the shared communication medium. More specifically, the communication constraint we deal with in this paper is referred to in the literature as a medium access constraint. In this case, the shared network can only accommodate a limited number of simultaneous communications between components. In this context, it is only meaningful to specify a filter in conjunction with a communication policy which indicates the times at which the plants sensors are to be granted medium access. This communication policy is known in the literature as communication sequence [13]. The communication sequence specifies which sensors are able to send information to the filter at each time step.

We will consider that the communication medium connecting the sensors and the residual generator has \( b \) output channels, with

\[
1 \leq b \leq m
\]  

(2)

At any time, only \( b \) of the \( m \) sensors can access these channels to communicate with the residual generator while others must wait.

A. Communication sequence

Suppose that there are \( m \) different sensors and that at each time step \( k \) only \( b < m \) are allowed to transmit messages. We then have \( \sigma = \binom{m}{b} \) possible configurations.
Consider the filter given by (4). The estimation error \( e_k = (x_k - \hat{x}_k) \) and the output residuals \( q_k = (\hat{y}_k - \bar{y}_k) \) dynamics are given by

\[
\begin{align*}
\dot{e}_{k+1} &= (A - K_k C_k) e_k + \dot{F} \hat{x}_k - K_k \hat{v}_k + w_k, \\
q_k &= \varphi e_k + \bar{v}_k
\end{align*}
\] (7)

From superposition principle, it follows that for an additive faults occurring at time instant \( r \) (with \( k > r + s \), the output residuals \( q_k \) can be expressed as:

\[
q_k = \bar{q}_k + \bar{p}_{k,r} \left[ \begin{array}{c}
\hat{n}_T \\
\hat{n}_{k-1} \\
\vdots \\
\hat{n}_{k-s}
\end{array} \right] T
\] (8)

with

\[
\bar{p}_{k,r} = \varphi \mu_k \left[ G_{k-1}, \bar{F} \ldots G_{k-1,k-(r-1)} \bar{F} \ldots G_{k-1} \bar{F} \bar{F} \right]
\]

\[
G_{k-1,k-j} = G_{k-1} G_{k-2} \ldots G_{k-j}
\]

\[
G_k = (A - K_q C_k)
\]

and \( \bar{q}_k \) corresponds to the output residuals for the non faulty case.

Following similar arguments as in [22], the following result is derived.

**Proposition 1. (Fault isolability condition)** Under the condition \( \text{rank}(\varphi \mu_k) = q \), the solutions of the algebraic constraints: \( (A - K_k C_k) \Psi = 0 \) can be parameterized as \( K_k = \omega \Pi_{\mu_k} + \bar{K}_k \Sigma_{\mu_k} \) with

\[
\Sigma_{\mu_k} = \alpha_{\mu_k} (I_m - D_{\mu_k} \Pi_{\mu_k}, \Pi_{\mu_k} = D_{\mu_k}^T \omega \Pi_{\mu_k} = A \Psi \) (9)
\]

where \( \bar{K}_k \in \mathbb{R}^{n,b-q} \) is the reduced gain describing the remaining freedom of design, \( D_{\mu_k} \) is the generalized inverse or pseudo-inverse of \( D_{\mu_k} \) and \( \alpha_{\mu_k} \) is an arbitrary matrix determined so that matrix \( \Sigma_{\mu_k} \) is of full rows rank.

Under these conditions, the output residuals \( q_k \) can then be expressed as:

\[
q_k = \bar{q}_k + D_{\mu_k} \left[ \begin{array}{c}
\hat{n}_{k-1}^T \\
\hat{n}_{k-2}^T \\
\vdots \\
\hat{n}_{k-s}^T
\end{array} \right] T
\] (10)

**Remark 1.** In the result given above, it is important to recall that the matrices \( \varphi \mu_k \) depend on the switching pattern \( \mu_k \), \( \mu_k \) being a design parameter, it follows that the switching patterns that contain sequences which violate the rank condition in Proposition 1 have to be excluded. Hence, let us define the set of admissible switching patterns \( \Xi^* \) given by

\[
\Xi^* = \{ \mu_k : \Xi \rightarrow \mathcal{M}^* \subseteq \mathcal{M} \}
\] (11)

where \( \mathcal{M}^* \) contains the indices corresponding to sensor configurations (truncated by corresponding matrices \( \varphi \mu_k \)) that verify the rank condition \( \text{rank}(\varphi \mu_k) = q \).

Based on the development above, the fault isolation filter can be designed by computing the free parameter \( \bar{K}_k \) so that the trace of covariance matrix \( \bar{P}_k = E(\bar{q}_k \bar{q}_k^T) \) is minimized.

**Proposition 2. (Fault isolation filter design)** For a fixed switching pattern \( \mu_k \in \Xi^* \), The proposed fault detection filter described by the following relations:

\[
\dot{x}_{k+1} = A \hat{x}_k + Bu_k + \omega q_k + \bar{K}_k \bar{y}_k
\] (12)

\[
\bar{P}_k = (\bar{A}_{\mu_k} - \bar{K}_k \bar{C}_{\mu_k}) \bar{P}_k (\bar{A}_{\mu_k} - \bar{K}_k \bar{C}_{\mu_k})^T + \bar{K}_k \bar{V}_{\mu_k} \bar{K}_k^T + \bar{W}_{\mu_k} = \phi_{\mu_k} (P_k)
\] (13)

\[
\bar{K}_{\mu_k} = \bar{A}_{\mu_k} + \bar{P}_k \bar{C}_{\mu_k} \bar{C}_{\mu_k} \bar{P}_k \bar{C}_{\mu_k} + \bar{V}_{\mu_k} \bar{V}_{\mu_k}^{-1}
\] (14)

with

\[
\bar{A}_{\mu_k} = A - \omega \Pi_{\mu_k} \varphi \mu_k
\]

\[
\bar{C}_{\mu_k} = \Sigma_{\mu_k} \varphi \mu_k
\]

\[
\bar{V}_{\mu_k} = \Sigma_{\mu_k} \Sigma_{\mu_k}^T
\]

\[
\bar{W}_{\mu_k} = W + \omega \Pi_{\mu_k} \Pi_{\mu_k}^T \omega^T
\]

where

\[
\bar{y}_k = \Sigma_{\mu_k} (\tilde{y}_k - \varphi \mu_k \tilde{x}_k)
\]

\[
\bar{q}_k = \Pi_{\mu_k} (\tilde{y}_k - \varphi \mu_k \tilde{x}_k)
\] (20)

have the following properties

- \( \bar{y}_k \) is decoupled from the faults
- \( \bar{q}_k \) satisfy the relation

**Proof.** The proof of this proposition follows similar arguments as for the proof of Theorem 3.1 in [22].

One can see that the evolution of the covariance matrix given by the Riccati equation (13) depends on the initial covariance matrix \( P_0 \) and the switching pattern given by \( \mu_k \). Hence, in addition to the isolability condition (see Remark 1), the scheduling strategy can be generated to optimize the covariance matrix evolution. This point will be further exposed in the next section.

**IV. FINITE HORIZON OPTIMAL SCHEDULING**

The problem addressed here is to choose which \( b \) sensors should operate at each time-step to minimize a function of the error covariance of the state estimation at each time step. Defining the scheduling strategy \( s_N \) equivalent to define the values of \( \mu_k \) for each \( k = 0, \ldots, N - 1 \), or equivalently:

\[
\hat{S}_N = \left[ \begin{array}{c}
\mu_0 \\
\mu_1 \\
\vdots \\
\mu_{N-1}
\end{array} \right]
\]

Let \( \mathcal{J}_N = \mathcal{J}^N \) be the set of all possible \( N \)-horizon scheduling strategies and let \( \mathcal{J}_N^{\mu_k} \) be the set of all admissible \( N \)-horizon scheduling strategies (see Remark 1). The problem of optimal scheduling is formulated as

\[
\min_{s_N \in \mathcal{J}_N^{\mu_k}} \mathcal{J}(s_N)
\] (22)
where
\[
\mathcal{J}(s_N) = \sum_{i=1}^{N} \text{tr}(P_i) = \sum_{i=0}^{N-1} \text{tr}(\Phi_i(P_i)) \quad \text{and} \quad \mu_i = s_N(i).
\]

**The search algorithm:**
Search algorithms are used for solving optimization problems (22). The trivial way of solution is to perform all possible scheduling cases. This enumerating method is only tractable for relatively short time horizons. It requires much resources in memory and computational time for longer estimation horizons. To overcome this limitation, we will use in this paper a pruning technique proposed in [18]. As showed in [18], the proposed algorithm can significantly reduce the computation complexity. Before proceeding, we will first recall some definitions to ease the reading of the paper.

**Definition 3. (Characteristic sets)** Let \( \{\mathcal{H}_k\}_{k=0}^{N} \) be defined as the characteristic sets as they completely characterize the objective function. Each set is of the form \((\bar{P}, \gamma) \in \mathcal{A} \times \mathbb{R}_+^p\), \(\mathcal{A}\) being the set of all symmetric positive semidefinite matrices) and is generated recursively by

\[
\mathcal{H}_{k+1} = \pi_{\mathcal{H}^*}(\mathcal{H}_k) \quad \text{from} \quad \mathcal{H}_0 = \{(\bar{P}_0, \text{tr}(\bar{P}_0))\}
\]

with

\[
\pi_{\mathcal{H}^*}(\mathcal{H}) = \{(\phi(\bar{P}), \gamma + \text{tr}(\phi(\bar{P}))): \forall i \in \mathcal{A}^*, \forall (\bar{P}, \gamma) \in \mathcal{H}\}
\]

Note that the above definition differs from the original one in [18] by using \( \mathcal{A}^* \) instead of \( \mathcal{A} \). This is due to the fault isolability constraints in our context.

The sets \( \mathcal{H}_k, k = 1, \cdots, N \), express the covariance of the estimate and the objective cost at every time-step under every possible sensor schedule. Let \( \mathcal{H}_k(i) \) be the \( i \)-th element of \( \mathcal{H}_k \), \( \bar{P}(i) \) and \( \gamma(i) \) be the covariance matrix and objective cost corresponding to \( \mathcal{H}_k(i) \) and \( \mathcal{H}^{\mathcal{A}} \) the set of all ordered sequences of admissible (in terms of isolability constraint) sensor schedules of length \( k \), \( \lambda(\bar{P}(i)) \in \mathcal{H}^{\mathcal{A}} \) be the optimal sensor schedule corresponding to the covariance matrix \( \bar{P}(i) \) and \( \lambda^* \) be the optimal sensor schedule for the problem.

**Definition 4. (Algebraic redundancy)** [18] A pair \((\bar{P}, \gamma) \in \mathcal{H}\) is called algebraically redundant with respect to \( \mathcal{H} \setminus \{(\bar{P}, \gamma)\}, \) if there exist nonnegative constants \( \{\alpha_l\}_{l=1}^{l-1} \) such that

\[
\sum_{i=1}^{l-1} \alpha_i = 1 \quad \text{and} \quad \begin{bmatrix} \bar{P} & 0 \\ \gamma & 0 \end{bmatrix} \geq \sum_{i=1}^{l-1} \alpha_i \begin{bmatrix} \bar{P}(i) & 0 \\ 0 & \gamma(i) \end{bmatrix}
\]

where \( l = \text{card}(\mathcal{H}) \) and \( \{(\bar{P}(i), \gamma(i))\}_{i=1}^{l-1} \) is an enumeration of \( \mathcal{H} \setminus \{(\bar{P}, \gamma)\} \).

The following theorem provides a condition which characterizes the branches that can be pruned without eliminating the optimal solution of the sensor scheduling problem.

**Theorem 1.** [18] If the pair \((\bar{P}, \gamma) \in \mathcal{H}_k\) is algebraically redundant, then the branch and all of its descendants can be pruned without eliminating the optimal solution from the search tree.

We are now in position to describe the sensor scheduling algorithm. Before doing this, let us recall the notion of equivalent subset of the search tree. This one is defined as a set that still contains the optimal sensor schedule after pruning, the pruning being realized using Theorem 1. The computation of the equivalent subsets is done via Algorithm 1 in [18] The sensor scheduling algorithm is given as follows

**Algorithm 2. Sensor scheduling for a finite horizon**

i) \( \mathcal{H}_0 = \{(\bar{P}_0, \text{tr}(\bar{P}_0))\} \)

ii) for \( k = 1, \cdots, N \), do

- \( \mathcal{H}_k = \pi_{\mathcal{H}^*}(\mathcal{H}_{k-1}) \)
- Perform Algorithm 1 in [18] with \( \mathcal{H}_k \)

end for

iii) \( \lambda^* = \arg \min_{j \in 1, \cdots, \text{card}(\mathcal{H}_{k})} \mathcal{J}(\lambda(\mathcal{H}_k(j))) \)

**Remark 2.** In [18], the authors proposed a suboptimal solution which consists in approximating the search tree by pruning branches which are numerically redundant. To this end, they use the notion of ε-redundancy. As pointed out by the authors, the ε-redundancy concept can typically eliminate many more branches of the search tree leading to less complexity problems.

V. ILLUSTRATIVE EXAMPLE

We consider the following discrete-time system

\[
A = \begin{bmatrix} 0.2 & 1 & 0 & 0.2 \\ 0 & 0.1 & 1 & 0.4 \\ 0 & 0 & 0.4 & 1 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}
\]

\[
C = \begin{bmatrix} 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \quad V = I
\]

\[
W = \begin{bmatrix} 0.89 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0.58 \end{bmatrix}
\]

The fault associated to the first column of the matrix \( F \) occurs at time instant \( r_1 = 50 \), with \( \eta^1_k = 10 \sin(0.1k) \), while the second fault (associated to the second column of \( F \)) occurs at time \( r_2 = 120 \) with \( \eta^2_k = 5 \).

We use the suboptimal version of Algorithm 2 (see Remark 2) to compute the suboptimal sensor schedule. Figure 2 shows the reduced output residuals \( q_k = [q_k^1 \ q_k^2 \ q_k^3]^{T} \), in the case of using the suboptimal sensor schedule sequence and an arbitrary periodic schedule, respectively. One can
specifically, we have considered the issue of sensor scheduling and isolation under communication constraints. More noise effect minimization. Future directions of research will include the infinite horizon case and extension to online scheduling techniques.

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