Accepted Manuscript

Title: The heterogeneous electrical conductivity structure of the lower mantle

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PII: S0031-9201(10)00156-1
DOI: doi:10.1016/j.pepi.2010.08.002
Reference: PEPI 5321

To appear in: *Physics of the Earth and Planetary Interiors*

Received date: 25-9-2009
Revised date: 2-7-2010
Accepted date: 13-8-2010

Please cite this article as: Tarits, P., Mandéa, M., The heterogeneous electrical conductivity structure of the lower mantle, *Physics of the Earth and Planetary Interiors* (2010), doi:10.1016/j.pepi.2010.08.002

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The heterogeneous electrical conductivity structure of the lower mantle

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Abstract

We developed a new three-dimensional electromagnetic time domain technique to invert 32 years of magnetic monthly mean values (1958-1990) from 120 geomagnetic observatories to image the mid-lower mantle. A preliminary global analysis of the data provided a source field model and an initial one-dimensional (1-D) conductivity profile for the inversion. The 1-D analysis showed that monthly mean magnetic data could resolve the lower mantle conductivity structure in the depth range $\sim 900 - 1400$ km. We run a series of synthetic data inversion to test the approach. The inversion of the true data with different initial conditions and different fractions of the data set was found fairly stable. Regions of low and high conductivity values were identified. The most resistive structures are beneath the Australian region, near Japan, North and Central America while highly conductive structures are identified beneath Eastern Africa and Eurasia. Some of these significant variations in the lower mantle conductivity are well correlated with structures in seismic tomography model. The uncorrelated features suggest

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that temperature and compositional effects known as predominant on both conductivity and velocity could be associated with other processes affecting mainly the conductivity, probably in relation with minor phases.

Key words: lower mantle conductivity, three-dimensional electromagnetic inversion, monthly mean geomagnetic data, lower mantle heterogeneities

1. Introduction

Estimates of the lateral variations of Earth's properties in the lower mantle come mainly from global seismic tomography. While great progress in lower mantle seismology models have been achieved recently (e.g., Romanowicz, 2003), interpretation of the seismic velocity anomalies in terms of thermodynamical and petrological parameters of the mantle is uncertain and needs additional information. The only other mean to directly probe the physical properties of the lower mantle is geomagnetic and geoelectric deep sounding to obtain the electrical conductivity. Conductivity is thermically active and sensitive to small amount of conducting material such as partial melting or water. Very recent progress in global and regional induction studies of the three-dimensional (3-D) earth provides new upper mantle conductivity models that may be quantitatively compared with other geophysical parameters (e.g., Kelbert et al., 2009; Utada et al., 2009). Most of known information on the lower mantle conductivity comes from one-dimensional (1-D) induction studies (Schultz and Larsen, 1990; Semenov and Jozwiak, 1999; Utada et al., 2003) and mineral physics (e.g., Shankland et al., 1993; Katsura et al., 1998; Xu et al., 2000). From these studies a body of evidence has been assembled that suggests that the lower mantle is electrically heterogeneous.

20 A number of aspherical 3-D induction solutions have been developed in the
 21 last decade to address the issue of global heterogeneous mantle conductivity
 22 (see Kuvshinov, 2008, for a review) and a global inverse solution is recently
 23 proposed by Kelbert et al. (2008). Kelbert et al. (2009) are the first to pro-
 24 pose a 3-D conductivity model of the uppermost lower mantle. The progress
 25 has been possible thanks to great improvement in computing systems and to
 26 increasingly larger data source, including satellite data. While the later are
 27 not yet included in 3-D inversion, the observatory data are being compiled
 28 for this purpose.

29 To infer conductivity distribution at lower mantle depth implies the use of
 30 very long reliable time series of geomagnetic and geoelectric data. Geomag-
 31 netic observatory data , geoelectric and geomagnetic time series or recent
 32 satellite magnetic data have been analyzed to obtain reliable geomagnetic
 33 data at the longest possible periods, typically a few months to 1 year (e.g.,
 34 Egbert and Booker, 1982; Fujii and Schultz, 2002; Kuvshinov and Olsen,
 35 2006). At these periods, the external time-varying magnetic field induces
 36 electric currents down to the mid lower mantle that in turn produce an in-
 37 ternal magnetic field observable at the earth surface.

38 The time-varying geomagnetic field at these long periods is actually the
 39 superimposition of two types of sources, the internal secular variation of the
 40 main geomagnetic field by the earth's core dynamics and the interaction,
 41 external to the earth, between the sun and the main geomagnetic field. The
 42 external sources are organized at and within the boundary between the solar
 43 wind and the magnetic field (Courillot and Le Mouel, 1988). When mea-
 44 sured at the earth surface, the external magnetic field may be described with

45 simple geometry (Banks, 1969) namely an uniform continuously varying field
 46 which source is the ring current and two harmonic fields of period 6 months
 47 and 1 year. The field is also modulated by the 11 years period of solar activ-
 48 ity. The external magnetic field induces electric currents in the conducting
 49 earth which in turn produces a magnetic field of internal origin. The later
 50 is a function of electrical conductivity to a maximum depth controlled by
 51 the skin effect. Electromagnetic (EM) induction analysis consists of the de-
 52 termination of the internal induced field and its interpretation in terms of
 53 conductivity structure.

54 We analyzed 32 years of monthly mean values of the geomagnetic field
 55 recorded at 120 geomagnetic observatories to obtain a new conductivity
 56 model of the mid-lower mantle. The strategy and methodology presented
 57 in this study were designed to analyze and model the observatory data to
 58 recover the lateral variation of lower mantle conductivity. The approach fol-
 59 lowed three steps. First, the data were globally analyzed to obtain a robust
 60 estimate of the external source field time series. Then, we carried out a 1-D
 61 inversion of the response function between the internal field and its domi-
 62 nant external dipole source. The 1-D conductivity profile was analyzed to
 63 infer the depth resolution of the data set and used as an initial model in
 64 the subsequent 3-D inversion. Finally, we ran a new 3D inversion scheme to
 65 derive the mantle electrical conductivity in the depth range acknowledged
 66 by the 1-D resolution analysis. The 3-D inversion was carried out in time
 67 and space domain. The rational to use a time and space domain approach
 68 compared to the usual spectral and spherical harmonic (SH) domain tech-
 69 niques was twofold. First, data are of various quality and duration. Several

70 observatories have been running for less than 32 years. Furthermore while
 71 an observatory is in operation, the control of the baseline to maintain con-
 72 tinuity over time may have been uneven, creating spurious behavior in the
 73 data. Second, the geographical distribution of the observatories (Figure 1)
 74 limits considerably our ability to recover accurately the SH spectral coeffi-
 75 cients related to mantle heterogeneity. The approach proposed here limits
 76 the spatial global analysis and attempts to match the actual time series at
 77 each observatory.

78 **2. The source field and the 1-D earth**

79 *2.1. Data analysis*

80 We assembled a new global data set of geomagnetic observatory monthly
 81 mean values (Table 1) from 1958-1990 (Figure 2). The data sources are the
 82 IAGA yearbooks and the national observatory archives. The data spatial
 83 distribution is given in Figure 1. The geomagnetic main field and secular
 84 variation model of Jackson et al. (2000) was removed from data. Spurious
 85 effects in the time series are detected by time differencing the field (TDF).
 86 The TDF is also a high-pass filter that weights down long period residuals
 87 of the secular variation as well as the longest external source field such as
 88 the 11 years sun cycle. The TDF data file were edited. We inspected all
 89 the files to remove data with any spurious behavior. The editing was first
 90 carried out by hand with plot control. In general, we flagged any suspicious
 91 behavior in terms of continuity and lack of homogeneity in a given time series
 92 component compared to the other components and to observatories nearby
 93 (when existing). Whenever possible, we controlled the coherence between

94 a given component at an edited observatory with the same component at
 95 close observatories with no flagged values. At the end of the editing process,
 96 the final time series at some observatories were reduced and included gaps.
 97 Finally, three observatories (las, sba, crp, see Table 1) were removed from
 98 the database because of unclear and maybe spurious time changes in their
 99 time series.

100 In the period range of our time series, the external magnetic field may
 101 be described to a good approximation by a small number of SH coefficients
 102 (Banks, 1969). The continuum field and the 6 months harmonic field may
 103 be described by the first (degree $l = 1$) SH while the 12 months SH field is
 104 well described by the second spherical harmonic (degree $l = 2$). While higher
 105 odd spherical harmonic terms are also expected, the corresponding energy
 106 seems to decrease rapidly with the SH degree (Courillot and Le Mouel,
 107 1988). The skewness in the geographic distribution of the observatories is a
 108 major limiting factor in the derivation of the SH coefficients, particularly the
 109 internal field resulting from the mantle heterogeneities. Only the lowest order
 110 terms are in general accurately recovered because of the simple geometry of
 111 the source field. So we limited the SH analysis to degrees and orders up to
 112 3.

113 Because the dominant dipole part of the geomagnetic field is tilted with
 114 respect to the geographic equator, it is best to analyse the data in the geo-
 115 magnetic coordinate system. We used the 1960-1990 IGRF dipole to rotate
 116 the observatory locations and the horizontal magnetic components in this
 117 reference frame.

118 The global analysis approach takes stock of the simple geometry of the

119 source field to model the external field and the corresponding internal induced
 120 field. At and above the earth surface, the magnetic field is curl free and derive
 121 from a scalar potential $V = V_e + V_i$ (V_e external and V_i internal).

$$V(r, \theta, \varphi, t) = a \sum_{l,m}^{lmax_e} (r/a)^l V_{el}^m(t) Y_l^m + a \sum_{l,m}^{lmax_i} (a/r)^{l+1} V_{il}^m(t) Y_l^m \quad (1)$$

122 where V_l^m are the SH coefficients of V . The function $Y_l^m(\theta, \varphi)$ is the spherical
 123 harmonic of degree l and order m at geomagnetic colatitude θ and longitude
 124 φ . The coefficient a is the earth radius and r the radius. The maximum
 125 degree of SH expansion $lmax_i$ is equal to $lmax_e$ for a spherically symmetric
 126 1-D earth and larger for the 3-D case. The field $\vec{B}$ is obtained by taking the
 127 gradient of (1)

128 At each time step (one month) of the time series from 1958-1990, the
 129 potentials V_e and V_i were fitted to the observatory data in geomagnetic coor-
 130 dinates using Eq. 1. We tested various SH expansion models ($lmax = 1 - 3$
 131 for the external field and $lmax = 1 - 7$ for the internal field). The system
 132 (1) for the magnetic field $\vec{B}$ was solved with a damped weighted robust least
 133 square method that minimizes:

$$M = \sum_{No} W_o \Sigma^{-2} |\vec{B}_o - \vec{B}_c|^2 + \lambda \sum_{l,m} l |V_{el}^m|^2 + (l+1) |V_{il}^m|^2 \quad (2)$$

134 The subscripts o and c stand for observed and computed values. The first
 135 term in the right hand side of the equation is the misfit function. The weights
 136 W_o were obtained using the formalism proposed by Mochizuki et al. (1997) to
 137 make the observatory distribution uniform on average. The value Σ normal-
 138 izes the field values to non-dimensional data and is taken arbitrarily equal

139 to $1/nT$. The second term is the magnetic energy times λ , the damping
140 parameter.

141 In spite of gaps in the time series either because the observatory was not
142 running or because data were discarded, at any given time step there were
143 data available from about simultaneous 60 observatories in the early years
144 up to 115 in the latest years. We also tested the influence of the auroral
145 observatories (geomagnetic latitudes larger than $\pm 65^\circ$ deg). We found that
146 $l_{max} = 3$ provided a good fit on average to all observatories including the
147 auroral's. In contrast, at periods shorter than 2 months Fujii and Schultz
148 (2002) observe a strong auroral current effect down to rather low latitudes.
149 We observe that this effect seems to vanish at our longer periods. The dom-
150 inant zonal SH coefficients ($l = 1 - 3$) are shown in Figure 3. As expected,
151 the ring current signal and the 6 months period field is dominant on P_1^0 while
152 the 12 months period field is dominating the P_2^0 geometry. Note that these
153 coefficients are for the TDF's so the 11 years sun cycle is only marginally
154 visible.

155 2.2. The one-dimensional earth

156 In spherically symmetric approximation the ratio $Q = V_i/V_e$ in the fre-
157 quency domain and at a given SH degree and order is the 1-D earth response
158 function (e.g. Banks, 1969). The value Q as a function of frequency (or pe-
159 riod) is used to obtain a 1-D conductivity profile of the earth. The dominant
160 P_1^0 term is used because of its high signal over noise ratio. We obtained Q
161 from the processing of the internal and external P_1^0 time series with a stan-
162 dard robust approach (Chave and Thompson, 1999). Prior to processing the
163 periods 6 and 12 months were filtered out. The Q values were converted into

the classical induction length C (e.g. Olsen, 1999). The C values are listed in Table 2.

We used a non-linear inversion scheme (Hautot et al., 2000) adapted for a spherical symmetric solution to infer the a posteriori pdf on the conductivity model that best fit the data. The fit to the data is measured with a misfit function M of the form:

$$M = \sum_{Nf} \Sigma^{-2} |C_o - C_c|^2 + \lambda \sum_{i,j} [\log(\sigma_i/\sigma_j)]^2 \quad (3)$$

where the subscripts o and c stand for observed and computed. The first term in the right hand side of the equation is a χ^2 misfit. The value Σ is the standard deviation (Table 2). The regularisation (second term) damped by the parameter λ is based on the minimisation of the conductivity contrast between two different layers i and j .

The absolute minimum of χ^2 is $\chi^2 = 12.2$ for this set of data and was obtained for a Dirac distribution of conductances (Parker, 1980) (Figure 4). In our inversion, the earth was parametrized with a series of homogeneous shells to find the depth range resolved by the data. A datum (here Q or C) is representative of a certain depth approximatively given by the real part of C (Weidelt, 1972). We tested this estimation with models with different numbers of layers between 0-2400 km in depth. We observed that the data set were mostly sensitive to the conductivity in the depth range $\sim 1000 - 1400$ km. An example of an a posteriori pdf is presented in Figure 4a for a model with 5 layers. For this model the minimum misfit is 13.8. The conductivity is well resolved only between $\sim 1000 - 1400$ km with a value close to ~ 7 S/m. The data are weakly or not sensitive to the upper mantle conductivity.

187 Between 660 – 1000 km, the conductivity may have any values less than
 188 $\sim 0.1 - 0.6$ S/m and below ~ 1400 km any value less than $\sim 3 - 10$ S/m.
 189 The deepest part of the mantle is conductive but its conductivity is not
 190 resolved either.

191 While the purpose of this analysis is mainly to prepare the subsequent
 192 3-D inversion of the data, it is worth noting the large average conductivity
 193 value ($\sim 6 - 7$ S/m) in the depth range $\sim 900 - 1500$ km. This value is
 194 in general agreement with other global studies (see Kuvshinov and Olsen,
 195 2006, and references herein) (Figure 4b). The conductivity value is also in
 196 agreement with the estimate of the mean mantle conductivity obtained by
 197 Manda et al. (1999) deduced from the study of the diffusion through the
 198 mantle of the geomagnetic secular variation impulses. Depending on the
 199 thermodynamical conditions of the lower mantle (mineralogy, temperature,
 200 iron content, minor phase), mineral physics studies lead to large ranges of
 201 conductivity values (Verhoeven et al., 2008). Our estimates would suggest a
 202 hot mantle and/or large iron content.

203 **3. Three-dimensional inversion**

204 We have developed (Tarits et al., 1992) a forward 3-D spherical solver for
 205 induction to study the detectability of 3-D mantle conductivity (e.g., Tarits,
 206 1994; Grammatika and Tarits, 2002). This solver was now implemented in an
 207 inversion algorithm adapted from the 3-D plane wave inverse solution of Hau-
 208 tot et al. (2000). The forward solver is in the frequency domain and calculates
 209 the electromagnetic (EM) field for an external (or internal) magnetic source
 210 input by its SH coefficients at a given frequency. The earth is parametrised

211 by a stack of shells which may be either homogeneous or laterally heteroge-
 212 neous. The stack is limited downward by a homogeneous spherical core. In
 213 each heterogeneous shell, the electrical conductivity is given on a 2-D spher-
 214 ical grid. The conductivity is always in the space domain while the EM field
 215 is transformed forward and backward between space and SH domains in the
 216 numerical solution. The computed field is provided in the SH domain.

217 The inversion is based on the non-linear minimization of a misfit function
 218 M between the observed magnetic field and the computed magnetic field time
 219 series at each observatory plus a damping term to smooth the conductivity
 220 contrasts. The computed field is the response function of the conductivity
 221 model to the observed source field obtained before and used as forcing terms
 222 in the 3-D forward solver. The inversion technique minimizes M with a
 223 steepest descent approach adapted from the work of Beiner (1973). The
 224 gradients with respect to the parameters are not needed with this algorithm
 225 at a cost of a large number of calls to the forward solver. The number of calls
 226 is at least 10-20 times the number of parameters seeked in the inversion.

227 The misfit function M writes now:

$$M = \sum_{No} \sum_t W_o \Sigma^{-2} |\vec{B}_o - \vec{B}_c|^2 + \lambda \sum_{i,j} [\log(\sigma_i/\sigma_j)]^2 \quad (4)$$

228 The term $\vec{B}$ is the vector magnetic field (o for observation and c for calcu-
 229 lated). The weight W_o are the same as in (2). The value Σ is the standard
 230 deviation taken here equal to $1nT$. The sum is over the number of obser-
 231 vatory No and all time steps t . The regularisation term is the sum squared
 232 of all the log-conductivity contrasts in the model. The subscripts i and j
 233 designate the conductivity value in either cell or layer i or j . The damping

parameter is adjusted so that the damped regularisation is smaller than the misfit. As a result, while the models obtained are not necessarily smooth, they do not exhibit spurious large conductivity contrasts.

One call to calculate the quantity M consists of the calculations of the induced field for all source field SH coefficients at all frequencies needed to restore the magnetic field time series at the earth surface. It is added to the external field and the total field is Fourier and Legendre transformed back into time and space domains at all observatory locations to calculate M . Note that all the calculations are carried out in the geomagnetic reference frame. The total number of data is:

$$NT = 3 \sum_i^{N_o} N_t(i) \quad (5)$$

where $N_t(i)$ is the number of data points in the time series at observatory i . The value of NT ranges from 48,000 – 110,000 depending on the type of tests done on data inversion.

In order to obtain some benchmark for the approach proposed, we carried out a series of runs with synthetic data. The conductivity model considered was derived from the 1-D analysis presented before. A total of 4 layers over a homogeneous core were considered (Table 3). Only layer 3 is heterogeneous.

We designed a conductivity model with a checkerboard structure in layer 3 with the lowest conductivity values equal to the 1-D conductivity value (6.1 S/m) divided by $\sqrt{10}$ and the highest value equal to that 1-D conductivity times $\sqrt{10}$. The conductivity in layer 1,2 and 4 are those listed in Table 3. We simulated the magnetic field time series at all observatories and added 1 nT of noise. The source field is the one described in the previous section. The data

257 were synthesised on a grid much finer than the grid used for the inversion.
 258 We ran tests with different grids, different mesh sizes for the checkerboard
 259 and different inversion grids. Here we present a selection of results with the
 260 following characteristics. The data were synthesized on a 48x24 grid and
 261 the EM field computed with a maximum SH degree and order of 17. The
 262 checkerboard model has 16x8 meshes (Figure 5). The inversion grid is either
 263 the same size (Figure 5) or has 24x12 meshes (Figure 6). For the inversion
 264 the maximum SH degree and order was 9.

265 In the first example (Figure 5), the inversion grid is identical to the
 266 checkerboard model. The resulting inverse solution is very good despite
 267 the uneven distribution of data over the map. The polar regions however
 268 are not well recovered. During the inversion process (not shown here), the
 269 conductivity model was recovered progressively from the equatorial region
 270 to the polar regions in relation with the strength of the source field. In
 271 the second example presented in this study, the inverse grid was finer than
 272 the checkerboard model. As a result, the inverse grid did not coincide with
 273 the checkerboard's. The inverse solution in Figure 6 is therefore much more
 274 patchy than in the previous example. Again, the model recovery was driven
 275 by the equatorial-polar trend during the inversion. The final model in Figure
 276 6 has not reproduced the polar regions at all. The areas with data are re-
 277 covered with obviously some spreading of the conductivity values where the
 278 inverse grid meshes are over the boundary of two checkerboard meshes. In
 279 general, the conductivity contrast are well recovered when data are over such
 280 a boundary. The model has overshoot values in resistive regions.

281 The synthetic models were designed to test the ability of the time and

space domain algorithm to recover conductivity structures in the depth range 900-1400 km with synthesized observatory data. The tests run with various checkerboard sizes and inversion grid led to similar results. With coincidental grids, the model was always well recovered. The inversion with a grid that encompassed checkerboard mesh borders led to smeared structures which nevertheless agree reasonably with the original conductivity distribution in areas with data. Finally we observed that the resistive regions were in general too resistive, an overshoot resulting from the sharp conductivity contrast between two checkerboard meshes.

4. Inversion of the observatory data

The inversion was initiated with a starting conductivity model identical to the initial model used in the study with synthetics. Again only layer 3 between 900 and 1400 km was heterogeneous. The analysis with the synthetic data suggested to use a 24x12 inversion grid and a maximum SH degree and order of 9. The parameters for the inversion are the conductivity in the 288 cells. Optionally, the conductivity in layer 1, 2 and 4 as well as their depth could be included in the inversion.

We carried out a series of inversion with different initial models and different sets of data. In all inversions, the regularisation term in (3) was set to about 1-10 % of the misfit term. In all trials, we observed that the auroral observatories could not be well fitted in spite of the presence of the 6 months and 1 year source terms. As a result, we removed 5 observatories with geomagnetic latitudes higher than $\pm 75^\circ$ deg (Table 1). Note that the data set is in the geomagnetic reference frame for the inversion.

306 All runs led to similar patterns in the conductivity structure. A typi-
 307 cal run is presented in Figure 7. The 1-D initial model explained a large
 308 fraction of the time series, in particular the North component X which car-
 309 ries a dominant source field. The inversion was stopped when no significant
 310 changes were observed, roughly after ~ 5000 calls to the forward solver. The
 311 misfit decreases monotonically with regular jumps when a general new con-
 312 ductivity pattern is obtained. The occurrence of this pattern is visible in the
 313 regularisation parameter. This pattern is obtained after all directions for all
 314 parameters have been explored and the steepest slope found for the misfit
 315 function M . We found the Y component the most difficult to fit (Figure
 316 8). In the equatorial region, the Y amplitude is quite large compared to the
 317 model response. There might be some source field residual. The X com-
 318 ponent is in principle not very discriminatory for the conductivity structure
 319 because the source field is dominant in this component. But it is in general
 320 very accurately measured and is therefore very useful. Both components Z
 321 and Y carry a large fraction of the induced signal and are therefore discrimi-
 322 natory although Z is in general more noisy than the horizontal components.
 323 These observations are summarized in Figure 9. The values presented in
 324 Figure 9 are the differences between the *rms* of the final solution and the
 325 *rms* of the starting 1-D solution for each observatory. The *rms* is defined
 326 by:

$$rms = \sqrt{\frac{3}{NT} \sum_{No} \sum_t \Sigma^{-2} |\vec{B}_o - \vec{B}_c|^2} \quad (6)$$

327 for each component X, Y, Z . Most of the *rms* decrease is observed in Z while
 328 in several stations X may increase, probably in relation with uncertainties
 329 in the source field. It is difficult to estimate the impact of the source field

errors in the results. One possibility is to use different parts of the data time series because the source field is calculated independently at each time step. The results discussed below suggested that this impact might be small.

We present two cases with different runs. The first case (Figure 10) results in models with two different initial 1-D conductivity values in layer 3 (6.1 S/m and 1.8 S/m). In the second case (Figure 11) we split the data in two. One set was from 1958-1974 and the second set was from 1975 to 1990. Both inversion started with a value of 1.8 S/m in layer 3. The models in Figures 10-11 are presented without any interpolation. We also tested (not shown here) the influence of the conductivity values in layer 1, 2 and 4 as well as the thickness of layer 3 with a run in which these parameters were included in the inversion. The conductivity values in those layers were unstable and varied according to the pattern in Figure 4. The thickness changed by about 50-100 km without change in the layer 3 conductivity structure.

In case 1 (Figure 10), we observed that the largest scale structures were always reproduced with slightly different conductivity values. The conductivities were somehow dependent on the choice of the initial values. The final model with an initial conductivity value of 1.8 S/m in layer 3 is overall more resistive than the final model with an initial value of 6.1 S/m. We have averaged the two models (Figure 10). In a given mesh, the conductivity value $\bar{\sigma}$ is the arithmetic mean of the conductivity from both models. We calculated the relative difference Δ in each mesh of the grid between both models as follow:

$$\Delta = |\sigma_1 - \sigma_2|/\bar{\sigma} \quad (7)$$

where σ_1 and σ_2 are the conductivity value in one mesh for each model.

354 These relative differences (Figure 10) are the largest in the polar region
 355 and in areas with no data. The pattern in the relative difference map is
 356 however complicated. Some regions with no data inside but with data on
 357 the border have weak differences while regions with some data inside have
 358 significant values. These differences are probably caused by several factors,
 359 the maximum SH degree cut off, the distribution of observatories as well as
 360 a moderate level of regularisation in the misfit function M in equation (3).

361 In case 2 (Figure 11), the final model with the 1958-1974 data set is the
 362 result of a smaller number of observatories (60-90) and shorter time series
 363 than for the 1974-1990 data set (80-110 observatories). The model is also
 364 smoother than the model obtained with the 1974-1990 data set although all
 365 the main features are observed in both models. One likely contribution to
 366 the differences between the 1958-1974 and 1975-1990 models comes from the
 367 uncertainty in the source field model used. The comparison between these
 368 models and the full data set (1958-1990) model in Figure 11 showed that
 369 both 1958-1974 and 1975-1990 data contributed.

370 5. Discussion

371 On the basis of the analysis carried out with synthetic data, we attempted
 372 to draw some conclusions about the resolution and uncertainty in the lower
 373 mantle (LM) conductivity model we obtained. Despite the uneven distri-
 374 bution of observatories, the synthetic tests showed that LM conductivity
 375 contrasts were correctly recovered. The conductivity structures were grid de-
 376 pendent and smeared over the actual LM bodies. Furthermore, the resistive
 377 regions tended to be undershoot. On the bright side, the stability observed

378 in the various inversions of the real data carried out with different settings
 379 suggested that at least the large scale structures might be robust features of
 380 the model. The models in Figure 12 are a smooth version of the model in
 381 Figure 10. We expanded the conductivity values in SH and we recalculated
 382 the conductivity distribution for different ranges of SH degrees and orders to
 383 outline the structures that dominated at long wavelength. There are several
 384 salient features in this model. We observe areas of low conductivity beneath
 385 the Australian region, Western Africa, near Japan, North and Central Amer-
 386 ica. High conductivity is observed in Eastern Africa, South-East Asia and
 387 Eurasia. The low conductivity values observed in the Pacific ocean seems
 388 resolved while the N-S extension of the Western Africa low conductivity re-
 389 gion is probably an artefact caused by the very small number of observatories
 390 along the African coast.

391 The conductivity value obtained from the inversion of induction data is
 392 in principle absolute although not necessarily well resolved. The value is rep-
 393 resentative of the conduction mechanism in the lower mantle. There is some
 394 sense to discuss the conductivity distribution in terms of high and low values
 395 compared to a mean 1 S/m (Figure 12). A number of 1-D global studies
 396 (Kuvshinov and Olsen, 2006) shows a value of 1-2 S/m for the mean con-
 397 ductivity in the 1000-1500 km range. Mineral physics studies, using average
 398 mantle temperature and composition conditions, show similar values (e.g.,
 399 Xu et al., 2000).

400 The resistive regions in the model are likely to show values too low if
 401 we referred to the synthetic analysis. We do not really know by how much
 402 the value is too low. It could be a factor of 2 or 3 according to the results

403 presented in Figure 6. In contrast, the synthetic analysis showed that the
 404 conductivity in high conductivity regions (Figure 6) should be of the right
 405 value. Some of the conductivity values are large (more than 10 S/m) which
 406 agreed with the 1-D conductivity profile obtained with the global earth re-
 407 sponse of Table 2. Eventually, what might probably be the most robust
 408 parts of the models in Figure 12 are the high and low conductivity regions
 409 but their exact size, shape and conductivity could not be very well defined.
 410 The worldwide variability of our conductivity values are in agreement with
 411 the few deep 1-D conductivity profiles obtained with different type of EM
 412 data (Egbert and Booker, 1982; Schultz et al., 1993; Semenov and Jozwiak,
 413 1999; Utada et al., 2003). We observed significant discrepancies between our
 414 model and the conductivity distribution proposed by Kelbert et al. (2009)
 415 in the same depth range. Their model shows a weaker conductivity contrast
 416 (1 log unit or less) than our model. This result is not surprising given the
 417 widely different approach used, in particular for the smoothing parameteri-
 418 zation and the fact that our technique may produce conductivity too low in
 419 resistive areas. More problematic is the absence of some of our prominent
 420 features in Kelbert et al.'s model. For instance, the Australian low does not
 421 exist neither does the Japan low. In contrast, our models are in good agree-
 422 ment from Southern Europe to Middle East as well as Eurasia. One possible
 423 explanation lays in the short period data set (maximum period 100 days)
 424 used by Kelbert et al. (2009). Their longest period is approximatively our
 425 shortest period (Table 2). The short period data ought to be only marginally
 426 sensitive to structures at depths more than 1200 km which may explain our
 427 agreement in areas with many high quality observatories.

428 Lateral variation of seismic velocity and electrical conductivity result from
 429 temperature and compositional changes in the mantle in relation with global
 430 geodynamics. Kelbert et al. (2009) observe some degree of correlation be-
 431 tween conductivity and seismic tomography models in the upper mantle tran-
 432 sition zone. Good correlation or more precisely anti-correlation is expected
 433 when temperature variations are the leading process in a dry mantle (Shank-
 434 land et al., 1993; Utada et al., 2009; Verhoeven et al., 2008). When temper-
 435 ature increases, the velocity decreases while the conductivity increases.

436 Several tomography models of the mid-mantle are available (e.g. Becker
 437 and Boschi, 2002; Romanowicz, 2003). Becker and Boschi (2002) have syn-
 438 thetized the common large scale features observed in P-wave and S-wave
 439 models into mean models reproduced in Figure 13 at 1220 km depth. Here
 440 we only show the longest wavelengths in the models ($l = 1-9$) for comparison
 441 with the conductivity model. For this comparison, we selected the model with
 442 the SH expansion form $l = 1-7$ (Figure 12) because the most abrupt conduc-
 443 tivity contrasts are filtered out. Qualitatively, our mid-mantle conductivity
 444 model (Figure 13) shows some large scale common low conductivity-high
 445 velocity features with the tomography models, particularly beneath North
 446 and Central America, partly beneath South Australia and to a less extent in
 447 South Africa, Japan, India and the Eastern part of Europe and North Africa.
 448 Common high conductivity - low velocity regions are observed as well in East
 449 Africa, part of Asia and Pacific though the latter region was poorly constraint
 450 by a few observatories. Among the regions with reverse trends (i.e. high con-
 451 ductivity - high velocity or vice versa), we note the Indonesian region, Central
 452 Australia as well as the Western part of Europe and North Africa. A word

of caution is necessary. For instance, the quasi orthogonal trend observed in the South Atlantic near Africa (NW-SE in the conductivity model and NE-SW in the tomography models) may be the simple result of no magnetic observatory in the South Atlantic. In contrast, the trends observed in areas with observatories are probably robust. Hence, no correlation could be an indication of processes that act differently on conductivity and seismic velocity. A weak correlation between high conductivity and seismic velocity may be attributed for instance to water effect (Koyama et al., 2006).

We are getting close to obtain conductivity models that could be quantitatively compared to seismic data as well as geodynamic models. Some more work is still necessary to ascertain the full robustness of the features observed in Figures 12-13. There is also the need to estimate as precisely as possible the uncertainty in the conductivity parameters. Last but not least, we should be able to provide some description of the conductivity variation with depth if the data allowed it. The obvious step forward to address these questions is to update our database with observatory data from 1991-present.

6. Conclusion

We analyzed 32 years of magnetic monthly mean values from a worldwide set of geomagnetic observatory to infer the lower mantle electrical conductivity. Data were processed and inverted in the time and space domain and a vertically averaged laterally heterogeneous conductivity model in the depth range 900-1400 km was obtained. Large scale structures were revealed. Conductivity values vary by more than one order of magnitude between resistive regions and conductive domains. Several inversions were run on synthetic

477 and real data to explore the stability of the inversion and to obtain clues
 478 about model resolution. The conductivity changes in the lower mantle may
 479 be partly due to temperature and iron content changes when correlated to
 480 tomography models. Otherwise, other processes should be involved. The in-
 481 terpretation is limited by the lack of mineral physics studies of conductivity
 482 enhancement of lower mantle minerals by minor phases such as water or car-
 483 bonates. Eventually, the robustness of our findings should be tested further
 484 with long data time series including the magnetic monthly mean values from
 485 1991-present. However, while the most recent data are of improved quality
 486 in terms of measurement precision and baseline reliability, there are fewer
 487 observatories than before and major threats exist to close some of them. Fi-
 488 nally, many more years of satellite data are necessary to obtain geomagnetic
 489 fields at periods long enough to probe the deep mantle.

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Table 1: List of geomagnetic observatory: IAGA code and geographical location.

Obs.	Lat	Long	Obs.	Lat	Long	Obs.	Lat	Long
aaa	43.15	76.55	aae	9.03	38.76	abg	18.64	72.87
agn	43.74	280.73	aia	-65.25	295.74	ale	82.50	297.65
aml	-43.15	172.72	ams	-37.83	77.57	ann	11.37	79.68
api	-13.81	188.23	aql	42.38	13.32	ars	56.43	58.43
bdv	49.08	14.02	bel	51.84	20.79	bfe	55.63	11.67
bji	40.04	116.18	blc	64.33	263.97	bng	4.44	18.57
bou	40.14	254.76	brw	71.32	203.38	bsl	40.50	339.26
cbb	69.20	255.00	clf	48.02	2.27	cmo	64.86	212.16
cnb	-35.31	149.36	cnh	44.05	125.21	coi	40.22	351.58
crp	10.44	275.09	czt	-46.43	51.87	dlr	29.49	259.08
dob	62.07	9.12	dou	50.10	4.59	drv	-66.67	140.01
ebr	40.82	0.49	esk	55.32	356.80	eyr	-43.42	172.35
fcc	58.67	265.91	frd	38.21	282.63	frn	37.10	240.30
fuq	5.47	286.26	fur	48.17	11.28	gdh	69.25	306.47
gln	49.63	262.90	gna	-31.78	115.95	gua	13.58	144.87
gzh	23.09	113.34	had	50.99	355.52	hbk	-25.88	27.71
her	-34.42	19.23	hlp	54.61	18.82	hon	21.32	202.00
hrb	47.87	18.19	hua	-12.05	284.67	hyd	17.41	78.56
irt	52.17	104.45	isk	29.07	38.32	kak	36.23	140.19
kiv	50.72	30.30	kny	31.42	130.88	kod	10.23	77.46
krc	24.95	67.14	las	-35.01	302.31	ler	60.13	358.82
lnn	59.95	30.70	lov	59.35	17.83	lrv	64.18	338.30
lvv	49.90	23.75	lzh	36.09	103.85	mab	50.30	5.68
maw	-67.61	62.88	mbc	76.20	240.60	mbo	14.39	343.04
mcq	-54.50	158.95	mea	54.62	246.67	miz	39.01	141.08
mmb	43.91	144.19	mmk	68.25	33.08	mnk	54.50	27.88
mos	55.47	37.31	naq	61.10	314.80	nck	47.63	16.72
new	48.26	242.88	ngk	52.07	12.68	nur	60.51	24.66
nvs	55.03	82.90	ode	46.78	30.88	ott	45.40	284.45
paf	-49.35	70.26	pag	42.22	24.18	pbq	55.28	282.26
pmg	-9.41	147.15	pod	61.60	90.00	ppt	-17.57	210.43
res	74.70	265.10	sab	30.36	77.80	sba	-77.85	166.78
sit	57.06	224.68	sjg	18.11	293.85	sod	67.37	26.63
spt	39.55	4.35	ssh	31.10	121.19	stj	47.60	307.32
sua	44.68	26.25	tfs	42.09	44.71	thl	77.48	290.83
thy	46.90	17.89	too	-37.53	145.47	trd	8.48	76.95
trw	-43.25	294.68	tsu	-19.22	17.70	tuc	32.25	247.17
val	51.93	349.75	vic	48.52	236.58	vss	-22.40	316.35
whn	30.53	114.56	wik	48.26	16.32	wit	52.81	6.67
wng	53.74	9.07	yak	62.02	129.72	ykc	62.48	245.52

Table 2: One-dimensional C-response for the P_0^1 geometry (geomagnetic dipole). Period (T) is in months.

T (mo.)	Re(C) km	Im(C) km	Σ km
21.3	1926.8	-632.3	119.0
16.0	1713.5	-580.7	238.5
10.7	1693.2	-1124.6	446.6
8.0	1523.2	-400.3	111.0
5.3	1469.4	-370.8	73.5
4.0	1164.7	-408.2	52.3
2.7	1112.9	-210.3	58.9

Table 3: Conductivity model used to synthesize the synthetic data. Layer 3 is heterogeneous (het.) .

Layer	Conductivity (S/m)	Depth range (km)
1	0.01	0-660
2	0.1	660-900
3	het.	900-1400
4	3.3	1400-2400
Core	25	-

Figure 1: Worldwide distribution of geomagnetic observatories used in this study.

Figure 2: Data duration for each observatory (horizontal black line).

Figure 3: Time series of the zonal ($l = 1-3$) spherical harmonic coefficients of the external (black) and internal (red) magnetic potentials obtained from the global SH analysis of the monthly mean data.

Figure 4: (a) Conductivity profile and depth resolution for the data set in Table 2. For each layer, the graph shows the χ^2 misfit against the conductivity values in this layer. The ordinate axis is χ^2 divided by its minimum value. (b) Conductivity profiles versus depth. The shaded area corresponds to the values in (a). In black a 1-D profile obtained for a 7 layers model. The Dirac conductances are shown in green, the model by Kuvshinov and Olsen (2006) in red and by Constable and Constable (2004) in blue.

Figure 5: Checkboard model. Bottom: original conductivity structure; Top: inverse solution. The open circles are the position of the sites where the synthetic data were synthesized. See text for details.

Figure 6: Checkboard model. Bottom: original conductivity structure; Top: inverse solution. The open circles are the position of the sites where the synthetic data were synthesized. See text for details.

Figure 7: Evolution of the misfit function with respect to the number of calls to the forward solver. The weighted misfit is in black and the regularisation term in red. The units are arbitrary.

Figure 8: Example of fit for three observatories. For each observatory, X is the north magnetic component, Y the east magnetic and Z the vertical component positive downward. The horizontal field is in the geomagnetic reference frame. The data are in black and the model in red.

Figure 9: Global distribution of *rms* differences at all observatories and each component between the final inverse solution and the 1-D solution. See text for the definition of *rms*.

Figure 10: Top: Conductivity map for layer 3. This model is the average of two models obtained separately with two different 1-D starting values; Bottom: map of the relative conductivity difference (Δ) between the two models. The Δ value is the absolute value of the difference between two conductivities in one mesh divided by the mean value in that mesh. The open circles are the observatories used in the inversion and the crosses the observatories discarded (see text). The maps are in the geomagnetic reference frame.

Figure 11: Maps of conductivity models obtained with data from 1958-1974 with 97 observatories (top), from 1975-1990 (middle) with 119 observatories and for the whole time series (1958-1990). For all 3 models, the 1-D initial conductivity value was 1.8 S/m. The maps are in the geomagnetic reference frame.

Figure 12: Conductivity maps at different wavelength. The wave length decreases from top to bottom with a maximum SH degree of $l = 5$ (top), $l = 7$ (middle), $l = 9$ (bottom). The maps are in the geomagnetic reference frame.

Figure 13: The upper map is the conductivity model from $l=0-7$ from Figure 12. The model was rotated into geographical coordinates and resampled for plotting. The two bottom maps are the tomography models for P wave and S-wave from (Becker and Boschi, 2002) at depth 1220 km recalculated at $l = 1 - 9$.