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Observer design for state and clinker hardness estimation in cement mill

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Abstract: This paper addresses the problem of state and clinker hardness estimation in a cement mill process. A Takagi-Sugeno model with unmeasurable premise variables is developed for a nonlinear model of a cement mill. Based on this model, a nonlinear observer is proposed in order to estimate the state variables and also the clinker hardness, which is an unknown input of the process. The convergence of the estimation error is studied using the Lyapunov theory and the input-to-state stability (ISS) approach. An optimization problem with LMI constraints is then provided for the synthesis of this observer. Finally, simulation results and some discussions about the effectiveness of the observer are given.

Keywords: Cement mill process, nonlinear systems, Takagi-Sugeno modeling, unknown premise variables, state and unknown input estimation.

1. INTRODUCTION

The general objective of process control in the mineral industry is to optimize the recovery of the valuable minerals, while maintaining the quality of the concentrates delivered to the metal extraction plants. Surveys on the current status and future trends in the automation and control of mineral and metal processing are proposed by Jämsä-Jounela (2001) and Hodouin et al. (2001). As explained in Hodouin (2011), because the processes are strongly disturbed, poorly modeled and difficult to measure, the peripheral tools of the control loop (fault detection and isolation system, data reconciliation procedure, observers, soft sensors, optimizers, model parameter tuners) are as important as the controller itself. In the present paper, we focus on system state and unknown input estimation, which is often a necessary step for automatic control since neither all the state variables nor all the inputs can be measured.

Estimation is often achieved with the use of observers or Kalman filters. Andersen et al. (2006) propose to estimate the coal flow in pulverized coal mills with a Kalman filter using the measurements of combustion air flow led into the furnace and oxygen concentration in the flue gas. The estimation robustness is enhanced with an extended Kalman filter in Cuevas and Cipriano (2008). In Niemczyk and Bendtsen (2011), the state of a coal pulverizer process is estimated from the grinding power consumption and the amount of coal accumulated in the mill by employing a variant of a Luenberger observer for bilinear systems.

Alternative approaches for state estimation consist in designing appropriate data driven models, such as neural networks (NN) and support vector machine (SVM) (Curilem et al., 2011). In Ko and Shang (2011), a time delay NN model is developed to predict the feed particle size of a semi-autogenous grinding mill. Estimation based on NN is also proposed in Makokha and Moys (2012). In McElroy et al. (2009), it is shown how the unique insights provided by a discrete element method model of a rotating drum can be used to create soft-sensor models detecting flow regime. In Acuña and Curilem (2009), a comparison is made between a dynamic NN model and a SVM model for estimating the filling level of the mill for a semi autogenous ore grinding process.

Acoustic methods can also be employed. In Aldrich and Theron (2000), digital acoustic signals are transformed to power spectral densities that can be related to particle size distributions in the mill. Acoustic signal analysis is also used in Andrade-Romero et al. (2011) to characterize the resistance percentage of small-size coarse aggregate in a ball mill. In Tang et al. (2012), a novel soft sensor approach is proposed, based on spectral analysis for mill load estimation.

Complex grinding mill circuits are hard to control due to poor plant models, large external disturbances, uncertainties from internal couplings, and process variables that are difficult to measure. To cope with this difficulty, Olivier et al. (2012) proposes a novel fractional order disturbance observer for a run-of-mine ore milling circuit. Ball mill grinding circuits considered in Chen et al. (2009) encompass lumped disturbances including external disturbances, such as the variations of ore hardness and feed particle size, and internal disturbances, such as model mismatches and coupling effects. A disturbance observer based multi-
variable control scheme is developed to control a two-
input-two-output ball mill grinding circuit. Such an ob-
server is introduced to estimate the disturbances in grind-
ing circuit in Yang et al. (2010) and also for feed-forward
control, in Yang et al. (2011). In Lepore et al. (2007), a
multivariable controller is developed for a mill. As the
particle size distribution inside the mill is not directly
measurable, a receding-horizon observer is designed, using
measurements at the mill exit only.

In this paper, it is proposed to model the cement mill with
a Takagi-Sugeno (T-S) system with unmeasurable premise
variable. The system is described by three state variables,
where only two are accessible to the measure. Moreover,
one input out of two is known. Thus, there is a need
reconstruct, not only the whole state vector, but also
an unknown input. For that purpose, a state and unknown
input observer for T-S system is designed in order that the
state and unknown input estimation errors are bounded.
More precisely the input-to-state (ISS) stability of the
system describing the estimation errors is established.

The paper is organized as follows. The section 2 is devoted
to the description of the studied process. In section 3, the
Takagi-Sugeno modeling is presented and applied to the
process. The state and unknown input observer design is
detailed in section 4. Before concluding, simulation results
are provided in section 5.

2. PROCESS DESCRIPTION

The cement mill, represented in figure 1, consists of a
ball mill in closed-loop with a separator. The separator is
driven by its rotational speed v (rpm). The rotating ball
mill is fed with cement clinker at feeding rate u (tons/h), in
which balls perform the breakage of the material particles
by fracture and/or attrition. The output material of the
mill is transfered to the separator which separates the
material into the finished product flow yf (tons/h) and the
recycled flow yr (tons/h), which is recirculated to the
mill inlet.

![Cement mill process](image)

The mathematical model of the process is described by
three differential equations (see Grognard et al. (2001)).
The first two equations (1) and (2) describe how the input
flow rate of the separator is divided into the overflow
and the underflow depending on the separation function.
Equation (3) corresponds to the material conservation in
the ball mill while expressing the time evolution of the
load inside the mill.

\[
T_f \dot{y}_f(t) = -y_f(t) + (1 - \alpha(v))\varphi(w(t), d(t)) \\
T_r \dot{y}_r(t) = -y_r(t) + \alpha(v)\varphi(w(t), d(t)) \\
\dot{w}(t) = -\varphi(w(t), d(t)) + y_f(t) + u(t)
\]

where

\[
\varphi(w(t), d(t)) = p_1 w(t) \exp(-p_2 d(t) w(t)) \\
\alpha(v) = p_3 v^3(t) + p_4 v^4(t) + p_5 v^5(t)
\]

and \(T_f\), \(T_r\) (h) are time constants, \(w(t)\) is the amount of material in the mill (mill load) and \(d(t)\) is the clinker
hardness. Here, the separation function defined by \(\alpha(v)\) depends on the rotation speed of the separator and ball
mill outflow rate is defined by the function \(\varphi(w(t), d(t))\)
which is related to its load and the hardness of the
material. The system state equation is defined by

\[
x(t) = [y_f(t) \ y_r(t) \ w(t)]^T
\]

The measured outputs are the finished product \(y_f(t)\) and
the recycled flow \(y_r(t)\). Then, the system is described by
the following state equations

\[
\dot{x}(t) = f(x(t), d(t), v(t)) + Bu(t) \\
y(t) = Cx(t)
\]

where

\[
B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
\]

\[
f(x(t), d(t), v(t)) = \begin{bmatrix} \frac{1}{T_f} \left( (1 - \alpha(v)) \varphi(x_3(t), d(t)) - x_1(t) \right) \\ \frac{1}{T_r} \left( \alpha(v) \varphi(x_3(t), d(t)) - x_2(t) \right) \\ x_2(t) - \varphi(x_3(t), d(t)) \end{bmatrix}
\]

in which the hardness \(d(t)\) is unknown.

3. EXACT TAKAGI-SUGENO MODEL

The Takagi-Sugeno (T-S) modeling, introduced by Takagi
and Sugeno (1985), allows to represent the behavior of a
nonlinear system (i.e. \(\dot{x}(t) = f(x(t), u(t))\)) by the
interpolation of a set of linear sub-models. Each sub-model
contributes to the global behavior of the nonlinear system
through a particular weighting function \(\mu_i(z(t))\). The T-S
structure is given by

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))(A_i x(t) + B_i u(t))
\]

where \(A_i \in \mathbb{R}^{n \times n}\) and \(B_i \in \mathbb{R}^{n \times m}\) are known matrices, \(r\)
being the number of sub-models. The weighting functions
\(\mu_i(z(t))\) depend on the premise variable \(z(t)\) which can
be measurable (as the input or the output of the system)
or non measurable variables (as the state of the system).
These functions verify the so-called convex sum property

\[
\sum_{i=1}^{r} \mu_i(z(t)) = 1, \quad 0 \leq \mu_i(z(t)) \leq 1, \quad i = 1, ..., r
\]

Since it is an appealing mean to tackle nonlinear systems,
a considerable amount of work is devoted to the stability
analysis, the state estimation, the diagnosis and the
control of T-S systems, see the reference book by Tanaka
and Wang (2001) or the more recent one by Lendek et al.
(2010). It is important to note that most of the previ-
ous works are dedicated to T-S systems with measurable
premise variables.
It is known that if the nonlinearities of the system are bounded, the sector nonlinearity approach allows to derive an exact re-writing of any nonlinear system under a quasi-LPV (Linear Parameter-Varying) form and then under a T-S form (Tanaka and Wang, 2001). The main steps of this derivation are now presented.

Assume that the clinker hardness \( d(t) \) is different from zero at all time, which is a realistic assumption, and since the function \( f(x(t), d(t), v(t)) \) is Lipschitz and satisfies the property \( f(0, 0, v(t)) = 0 \), the system (6) can be re-written in quasi-LPV form as follows

\[
\dot{x}(t) = \left( \begin{array}{ccc}
-\frac{1}{T_f} & 0 & 0 \\
0 & -\frac{1}{T_f} & 0 \\
0 & 0 & 1
\end{array} \right) x(t) + \left( \begin{array}{c}
z_1(x(t), d(t)) \\
0 \\
0
\end{array} \right) x(t) + \left( \begin{array}{c}
-\frac{z_2(x(t), d(t), v(t))}{T_f} \\
\frac{z_2(x(t), d(t), v(t))}{T_f} \\
0
\end{array} \right) d(t)
\]

where the variables \( z_1(x(t), d(t)) \) and \( z_2(x(t), d(t), v(t)) \), later selected as premise variables, are defined by

\[
z_1(x(t), d(t)) = p_1 \exp(-p_2 d(t)x_3(t))
\]

\[
z_2(x(t), d(t), v(t)) = \alpha(v(t))p_1 x_3(t) \exp(-p_2 d(t)x_3(t))
\]

One can see that the premise variables depend on the unmeasurable state variables \( x_3(t) \) and \( d(t) \) and on the measured input \( v(t) \). For the sake of brevity, the premise variable are denoted by \( z(t) \) in the remaining of the paper, but it must be kept in mind that they depend on the system state. Due to their physical meaning, \( d(t) \) and \( z(t) \) are bounded, and so are the premise variables

\[
z_{1\min} \leq z_1(x(t), d(t)) \leq z_{1\max} \quad z_{2\min} \leq z_2(x(t), d(t), v(t)) \leq z_{2\max}
\]

(11)

Since they are bounded, the premise variables can be written as

\[
z_1(t) = F_1^0(z_1(t))z_{1\max} + F_1^1(z_1(t))z_{1\min}
\]

(12)

\[
z_2(t) = F_2^0(z_2(t))z_{2\max} + F_2^1(z_2(t))z_{2\min}
\]

(13)

where the functions \( F_1^0, F_1^1, F_2^0 \) and \( F_2^1 \) are defined by

\[
F_1^0(z_1(t)) = \frac{z_1(t) - z_{1\min}}{z_{1\max} - z_{1\min}}, \quad F_1^1(z_1(t)) = \frac{z_{1\max} - z_1(t)}{z_{1\max} - z_{1\min}}
\]

\[
F_2^0(z_2(t)) = \frac{z_2(t) - z_{2\min}}{z_{2\max} - z_{2\min}}, \quad F_2^1(z_2(t)) = \frac{z_{2\max} - z_2(t)}{z_{2\max} - z_{2\min}}
\]

and satisfy the following property for \( i = 1, 2 \) and \( j = 0, 1 \)

\[
0 \leq F_i^j(z_i(t)) \leq 1, \quad \sum_{j=0}^1 F_i^j(z_i(t)) = 1
\]

(14)

With this last property, and defining the weighting functions \( \mu_i(z(t)) \) by

\[
\mu_1(z) = F_1^0(z_1)F_2^0(z_2), \quad \mu_3(z) = F_1^1(z_1)F_2^0(z_2)
\]

\[
\mu_2(z) = F_1^0(z_1)F_2^1(z_2), \quad \mu_4(z) = F_1^1(z_1)F_2^1(z_2)
\]

the equivalent T-S model of (1) is given, without loss of information, by

\[
\dot{x}(t) = \sum_{i=1}^4 \mu_i(z(t)) (A_i x(t) + Bu(t) + E_i d(t))
\]

(15)

where it can readily be checked that the functions \( \mu_i(z(t)) \) satisfy (9) and the matrices are defined by

\[
A_1 = \left( \begin{array}{ccc}
-\frac{1}{T_f} & 0 & -z_{1\max} \\
0 & -\frac{1}{T_f} & 0 \\
0 & 0 & 1
\end{array} \right), \quad A_3 = \left( \begin{array}{ccc}
-\frac{1}{T_f} & 0 & -z_{1\min} \\
0 & -\frac{1}{T_f} & 0 \\
0 & 0 & 1
\end{array} \right)
\]

\[
E_1 = \left( \begin{array}{c}
z_{1\max} \\
z_{1\min} \\
0
\end{array} \right), \quad E_2 = \left( \begin{array}{c}
z_{1\min} \\
z_{1\max} \\
0
\end{array} \right), \quad B = \left( \begin{array}{c}
0 \\
0 \\
1
\end{array} \right)
\]

3.1 Computation of the bounds \( z_{1\max} \) and \( z_{1\min} \)

The chosen premise variables \( z_i, i = 1, 2 \) (11) are bounded by \( z_{1\max} \) and \( z_{1\min} \). The computation of these bounds is performed by using the sector nonlinearity approach (see chapter 14 of Tanaka and Wang (2001)). The variation analysis of each premise variable leads to obtain the corresponding bounds of the nonlinear sector, as shown in the figure 2 for the variable \( z_1 \).

Fig. 2. Sector nonlinearity approach

The obtained exact T-S model (15) is valid in a compact set of the state space. This set can be enlarged to take into account a larger operating region by modifying the parameters \( z_{1\max} \) and \( z_{1\min} \). Furthermore, taking into account realistic situations, the values of these bounds allow satisfying observability condition of each sub-model of the T-S model.

4. OBSERVER DESIGN FOR STATE AND CLINKER HARDNESS ESTIMATION

In this section, an observer is synthesize for estimating the unmeasured system state \( w(t) \) and the unknown input \( d(t) \) from only the knowledge of the flow rates \( y_1(t) \) and \( y_2(t) \). It is known that the clinker hardness \( d(t) \) may be considered as constant during limited periods of time and due to the physical meaning of the state variables, the following nonrestrictive assumption can be made

**Assumption 1.** In the remaining it is supposed that

- the state \( x(t) \) is bounded;
- the unknown input \( d(t) \) is constant \((i.e., \dot{d}(t) = 0)\).

Under this realistic assumption, the proposed proportional-integral observer with gains \( L_i \) and \( K_i \) takes the following form

\[
\dot{x}(t) = \sum_{i=1}^4 \mu_i(z(t)) (A_i x(t) + Bu(t) + E_i d(t))
\]
The aim is to design an observer for the system and unknown input estimation error, defined by Theorem 1. The state and unknown input estimation error, defined by \(e_a(t) = x_a(t) - \hat{x}_a(t)\), obeys the following differential equation

\[
\dot{e}_a(t) = \sum_{i=1}^{4} \mu_i(\hat{z}(t)) (A_i \hat{x}(t) + Bu(t) + E_i \hat{d}(t) + L_i (y(t) - \hat{y}(t)))
\]

where \(\hat{x}, \hat{d}\) and \(\hat{z}\) respectively denote the estimates of \(x, d\) and \(z\). Let us define the augmented state by \(x_a(t) = [x(t) \; d^T(t)]^T\) and denote its estimate by \(\hat{x}_a\). The state and unknown input estimation error, defined by \(e_a(t) = x_a(t) - \hat{x}_a(t)\), obeys the following differential equation

\[
\dot{e}_a(t) = \sum_{i=1}^{4} \mu_i(\hat{z}(t)) (A_i - M_i C) e_a(t) + \Delta(t)
\]

where

\[
A_i = \begin{pmatrix} A_i & E_i \\ 0 & 0 \end{pmatrix}, \quad M_i = \begin{pmatrix} L_i \\ K_i \end{pmatrix}, \quad C = (C \; 0),
\]

\[
\Delta(t) = \sum_{i=1}^{4} (\mu_i(z(t)) - \mu_i(\hat{z}(t))) A_i x_a(t)
\]

The aim is to design an observer for the system, such that the state and unknown input estimation errors remain bounded. Thus the definition of input to state stability (ISS) is needed.

**Definition 1.** (Sontag and Wang (1995)) The system is said to be ISS if there exists a \(\mathcal{KL}\) function \(\beta: \mathbb{R} \times \mathbb{R} \to \mathbb{R}\) and a \(\mathcal{K}\) function \(\sigma\) such that, for each input \(\Delta(t)\) satisfying \(\|\Delta(t)\|_\infty < \infty\) and each initial condition \(e_a(0) \in \mathbb{R}^3\), the trajectory of (17) associated with \(e_a(0)\) and \(\Delta(t)\) satisfies

\[
\|e_a(t)\| \leq \beta(\|e_a(0)\|, t) + \alpha(\|\Delta(t)\|_\infty), \forall t
\]

From Assumption 1 and the fact that the functions \(\mu_i\) are bounded, the term \(\Delta(t)\) is bounded. Indeed, the system is stable which provides bounded states for bounded input \(\delta_f\). Under bounded perturbation term \(\Delta(t)\), the observer (16) is synthesized by solving the optimization problem under LMI constraints given in Theorem 1.

**Theorem 1.** Under the Assumption 1, if there exists a symmetric and positive definite matrix \(P\), gain matrices \(K_i\) and a positive scalar \(\bar{\gamma}\) solution to the following optimization problem, \(i = 1, \ldots, 4\)

\[
\min_{P, K_i, \bar{\gamma}} \bar{\gamma}
\]

under the following LMI constraints

\[
\begin{pmatrix} A_i^T P + PA_i - K_i C + C^T K_i^T + I & P \\ P & -P I \end{pmatrix} \preceq 0
\]

\[
\begin{pmatrix} \bar{\gamma} I & P \\ P & \geq 0 \end{pmatrix} \preceq 0
\]

\[
P \preceq \bar{\gamma} I
\]

then the error dynamics (17) is ISS with respect to \(\Delta(t)\) and \(e_a(t)\) satisfies the following inequality

\[
\|e_a(t)\| \leq \bar{\gamma} (e^{-\bar{\gamma}t} \|e_a(0)\| + \sqrt{\bar{\gamma}} \Delta_{\infty})
\]

The gains of the observer are computed by

\[
M_i = P^{-1} K_i
\]

**Proof.** Assume that the LMIs (21) are feasible. Let us consider the vector

\[
\xi(t) = [e_a^T(t) \; \Delta^T(t)]^T
\]

Pre- and post multiplying (21) by \(\xi(t)\) and \(\xi^T(t)\) respectively and with \(\gamma = \bar{\gamma}^2\) and \(\Phi_i = A_i - M_i C\), the following is obtained

\[
\begin{align*}
& e_a^T(t) (\Phi_i^T P + P \Phi_i) e_a(t) + e_a^T(t) P \Delta(t) \\
& + \Delta^T(t) P e_a(t) + e_a^T(t) \gamma^2 \Delta^T(t) \Delta(t) < 0
\end{align*}
\]

Since \(0 \leq \mu_i(\cdot) \leq 1\), multiplying (26) by \(\mu_i(\hat{z}(t))\), and summing the obtained four equations, one obtains

\[
\sum_{i=1}^{4} \mu_i(\hat{z}(t)) (e_a^T(t) (\Phi_i^T P + P \Phi_i) e_a(t) + e_a^T(t) P \Delta(t) \\
& + \Delta^T(t) P e_a(t) < -e_a^T(t) e_a(t) + \gamma^2 \Delta^T(t) \Delta(t)
\]

which is equivalent to

\[
V(t) = e_a^T(t) e_a(t) + \gamma^2 \Delta^T(t) \Delta(t)
\]

where

\[
V(t) = \bar{\gamma}^2 e_a^T(t) P e_a(t) + P > 0
\]

From (29), it obviously follows that

\[
\lambda_{\min}(P) \|e_a(t)\|^2 \leq V(t) \leq \lambda_{\max}(P) \|e_a(t)\|^2
\]

Consequently, if (21) holds, the time derivative of \(V(t)\) is bounded as follows

\[
\dot{V}(t) \leq -\frac{1}{\lambda_{\max}(P)} V(t) + 2 \|\Delta(t)\|_2
\]

Multiplying both sides of the previous inequality by \(e_{\min}^{-1} \Delta_{\infty}\), and integrating from 0 to \(t\), it follows

\[
V(t) \leq \int_0^t e^{-\lambda_{\min}(P) s} \|\Delta(s)\|_2 ds
\]

Defining \(\Delta_{\infty}\) the upper bound of the euclidean norm of \(\Delta(t)\) (i.e. \(\|\Delta(t)\| \leq \Delta_{\infty}, \forall t\)), it follows

\[
V(t) \leq \int_0^t e^{-\lambda_{\min}(P) s} \|\Delta_{\infty}\|^2 ds
\]

Finally, from (30) and since \(\sqrt{a+b} \leq \sqrt{a} + \sqrt{b}, \forall a, b \in \mathbb{R}^+\), one obtains

\[
\|e_a(t)\| \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \left( e^{-\lambda_{\min}(P)t} \|e_a(0)\| + \sqrt{\bar{\gamma}} \Delta_{\infty} \right)
\]

This inequality implies that if \(\Delta(t) \equiv 0\) then \(e_a(t) \to 0\) when \(t \to \infty\). Moreover, in the presence of the perturbation \(\Delta(t)\), the error \(\|e_a(t)\|\) is bounded by \(\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \Delta_{\infty}\) when \(t \to \infty\). The inequality (34) establishes the ISS of (17).

Note that the size of the convergence set \(D\) depends on the selected matrix \(P\) and the parameter \(\gamma\). The set \(D\) should be made as small as possible to ensure a good accuracy of convergence. The choice of \(\gamma\) and \(P\) providing a small set of convergence is not obvious because the problem is not convex. In the next, a technique is proposed to transform the non convex problem into a convex one expressed by LMI constraints.

Firstly, from (33), it obviously follows that

\[
\lambda_{\max}(P) \leq \bar{\gamma}
\]

Secondly, with a Schur complement, (22), is equivalent to

\[
\bar{\gamma} I \preceq P^{-1}
\]

which is equivalent to

\[
\lambda_{\max}(P^{-1}) \leq \bar{\gamma}
\]
and also to
\[ \frac{1}{\lambda_{\text{min}}(P)} \leq \tilde{\gamma} \]  
(38)

Finally, (22) and (23) imply
\[ \sqrt{\frac{\lambda_{\text{max}}(P)}{\lambda_{\text{min}}(P)}} \leq \tilde{\gamma} \]  
(39)

which ends the proof.

5. SIMULATION RESULTS

In this section, obtained simulation results are provided with some discussions about the proposed observer performances. The observer is designed by using the matrices \( A_i, E_i \) and \( C \), where the bounds of the premise variables are \( z_{\text{min}} = 2, z_{\text{max}} = 20 \) and \( \bar{z}_{\text{min}} = 90, \bar{z}_{\text{max}} = 440 \). By solving the optimization problem given in the Theorem 1, the obtained observer gains are

\[
L_1 = \begin{pmatrix} 714.64 & -71.73 \\ 158.57 & 46.26 \\ 709.83 & -43.59 \end{pmatrix},
L_2 = \begin{pmatrix} 694.71 & 8.54 \\ 166.59 & 14.25 \\ 695.33 & 15.00 \end{pmatrix},
\]

\[
L_3 = \begin{pmatrix} 198.63 & -74.72 \\ 29.72 & 45.50 \\ 179.15 & -46.66 \end{pmatrix}, L_4 = \begin{pmatrix} 178.22 & 5.68 \\ 37.62 & 13.53 \\ 164.20 & 12.06 \end{pmatrix},
\]

\[
K_1 = 10^3 \times (-2.54 \ 10.17), K_2 = 10^3 \times (-0.52 \ 2.08)
\]

\[
K_3 = 10^3 \times (-2.54 \ 10.17), K_4 = 10^3 \times (-0.52 \ 2.08)
\]

The Lyapunov matrix \( P \) is

\[
P = \begin{pmatrix} 0.044 & -0.002 & -0.042 & 0.0001 \\ -0.002 & 0.051 & -0.010 & -0.0003 \\ -0.042 & -0.010 & 0.053 & 0 \\ 0.0001 & -0.0003 & 0 & 0.045 \end{pmatrix}
\]

The norm of the state estimation error \( ||e(t)|| \) converges to the region with a size defined by \( \tilde{\gamma}^2 \Delta_{\infty} = 0.0316 \times \Delta_{\infty} \). In the steady state, the computation of \( \Delta_{\infty} \) gives values less than 0.2, so the obtained bound of the error \( ||e(t)|| \) is 0.0316 \times \Delta_{\infty} = 0.0063; in the other hand the computation of \( ||e(t)|| \) gives values less than \( 5 \times 10^{-4} \times 0.0063 \) which confirms the Theorem 1. The initial conditions of the system are \( x(0) = [50 \ 50 \ 50]^T \) and those of the observer are \( \hat{x}(0) = [30 \ 30 \ 30]^T \) and \( z(0) = 0.5 \). The feeding rate (control input) \( u \) (tons/h) is computed from a PI controller as follows

\[
u(t) = -x_{2\text{ref}} + k_1(x_{3\text{ref}} - x_3(t)) + x_c(t)
\]

(40)

\[
\dot{x}_c(t) = k_2(x_{3\text{ref}}(t) - x_3(t))
\]

(41)

where the variables \( x_2(t) \) and \( x_3(t) \) are controlled in order to track the reference trajectories \( x_{2\text{ref}} \) and \( x_{3\text{ref}} \). The parameters of the controller are fixed as follows \( k_1 = 15 \) and \( k_2 = 30 \). The obtained control input is shown in the figure 3 (top). The weighting functions \( \mu_i(.) \) of the T-S model are depicted in the figure 3 (top) and one can see that, since the system is nonlinear, all the sub-models are activated at each time.

The estimated states are depicted in the figure 4 while the state estimation errors are shown in the figure 5 (top) and the clinker hardness estimation is given in the figure 5 (bottom). Given this figure, it should be noted that when the hardness of the ore is increasing, then the output rate decreases, which preserves the physical meaning of such a process. In addition, in the case of noised measurements with centered noise signal in the range \([-10, 10]\), the observer is, also, able to provide acceptable estimations of the state vector \( x(t) \) and the hardness \( d(t) \) as shown in the figure 6. One can note that even if the hardness \( d(t) \) is time varying, an acceptable estimation is provided by the observer. The transient phenomenon in estimation of \( d(t) \) is due to the fact that at \( t = 0 \) the values \( d(0) \) and \( \hat{d}(0) \) are different and the imaginary parts of observer poles are large. The magnitude of the overshoot in this transient can be reduced by pole clustering in LMI region which allows to reduce the imaginary part of the eigenvalues of the observer.
The goal of this paper is to design an observer for mill load and clinker hardness estimation in cement mill process with only the knowledge of the feeding $u$ (tons/h), the tailings $y_t$ (tons/h) and the finished product rate $y_f$ (tons/h). A nonlinear mathematical model of the process dynamics is considered and transformed with sector nonlinearity transformation into a Takagi-Sugeno model with unmeasurable premise variables. Using the Lyapunov theory and the input-to-state stability (ISS), the state and unknown input estimation is turned into an optimization problem under LMI constraints. Simulations results are provided and show that the ISS ensures an accurate estimation of the state and clinker hardness, even with noised measurements.

Work is underway to extend the proposed state reconstruction when one takes into account, in addition to flow rates, particle size distributions of the product throughout the separation-grinding loop.

6. CONCLUSION

The goal of this paper is to design an observer for mill load and clinker hardness estimation in cement mill process with only the knowledge of the feeding $u$ (tons/h), the tailings $y_t$ (tons/h) and the finished product rate $y_f$ (tons/h). A nonlinear mathematical model of the process dynamics is considered and transformed with sector nonlinearity transformation into a Takagi-Sugeno model with unmeasurable premise variables. Using the Lyapunov theory and the input-to-state stability (ISS), the state and unknown input estimation is turned into an optimization problem under LMI constraints. Simulations results are provided and show that the ISS ensures an accurate estimation of the state and clinker hardness, even with noised measurements.

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REFERENCES


Niemczyk, P. and Bendtsen, J. (2011). Improved coal grinding and fuel flow control in thermal power plants. In 18th IFAC World Congress, Milano, Italy.


