Joint Time-Frequency Spectrum Sensing for Cognitive Radio

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***Abstract*—In this paper[1], we propose a new concept of spectrum sensing techniques based on a joint time-frequency detection of primary users. In this new approach, we aim at detecting the presence of the PU in frequency and in time as well.**

The proposed technique based on an algebraic detection of the spectrum is compared to one of the most well known tool in time frequency analysis tools: the Wigner Ville Distribution.

Simulation results show how reliable the proposed technique is comparing to classical energy detection in time-frequency plane.

***Index Terms*—Cognitive radio, sensing algorithm, Wigner Ville distribution , algebraic detector, joint time frequency detection.**

I. Introduction

Cognitive Radio (CR) as introduced by Mitola [1] is a self aware and ”intelligent” device that can adapt itself to the Wireless environment changes. Such a device is able to detect the changes in Wireless network to which it is connected and adapt its radio parameters to the new opportunities that are detected. In CR networks, these opportunities are spectrum resources left idle by a licensed user, also called Primary User (PU), and can be exploited by an unlicensed user also called Secondary User (SU).

Many techniques already exist to model the spectrum use: we talk about Black Areas when the PU is present in the transmission sub-band. These areas of the spectrum are so occupied by the PU. Other regions are the Gray Areas where the PU transmission power enables the SU to transmit but with high interference level with the PU. The most secure areas are the White Areas, also called holes, where the SU can transmit with nearly no interference with the PU.

Spectrum Sensing in CR aims in finding the holes in the PU transmission which are the best opportunities to be used by the SU. Many statistical approaches already exist. The easiest to implement and the reference on in complexity is still the Energy Detector (ED). Nevertheless, the ED is highly sensitive to noise and does not perform well in low Signal to Noise Ratio (SNR). Other advanced techniques based on signals modulations and exploiting some of the transmitted signals inner properties were also developed. For instance, the detector that exploits the built-in cyclic properties on a given signal is the Cyclostationary Features Detector (CFD). The CFD do have a great robustness to noise compared to ED but its high complexity is still a consequent draw back.

The paper is organized as follows. After the presentation of the system model in Section II, some key notes are given in Section III.Then, the AD implementation is presented in Section IV. In Section V, the performance evaluation and advantages are described, and a comparison with some statistical approaches is given. Finally, Section VI concludes the paper.

II. System Model

In this section, we describe the system model that will be used throughout this paper. For the radio channel measurement we have chosen to thoroughly investigate the DVB-T primary user system. In this system, the transmitted signal is convolved with a multi-path channel and a Gaussian noise is added. The received signal at time n, denoted by x_n, can be modeled as:

$$x_n = A_n s_n + e_n \tag{1}$$

where A_n being the transmission channel gain, s_n is the transmit signal sent from primary user and e_n is a stationary, Gaussian noise with zero mean. The goal of spectrum sensing is to decide between the following two hypothesis:

$$x_n = \begin{cases} e_n & \mathrm{H}_0 \\ A_n s_n + e_n & \mathrm{H}_1 \end{cases} \tag{2}$$

We assume the sensed sub-band to be unoccupied if it contains only a noise component, as defined in H_0; on the other hand, once there exist primary user signals besides noise in a specific band, as defined in H_1, we say the band is occupied. Let P_F be the probability of false alarm given by:

$$P_F = P(\mathrm{H}_1 \mid \mathrm{H}_0) = P(s_n \text{ is present} \mid \mathrm{H}_0) \tag{3}$$

that is the probability of the spectrum detector having detected a signal under hypothesis H_0, and P_D the probability of detection expressed as:

$$\begin{aligned} P_D &= 1 - P_M = 1 - P(\mathrm{H}_0 \mid \mathrm{H}_1) \\ &= 1 - P(s_n \text{ is absent} \mid \mathrm{H}_1) \end{aligned} \tag{4}$$

the probability of the detector having detected a signal under hypothesis H_1, where P_M indicates the probability of missed

[1]The work reported herein was partially supported by the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement SACRA n ° 249060

detection. We propose comparing the suggested sensing algorithm: time frequency distribution discontinuities detection based on the algebraic detector (AD) to the WVD based detector (WVD). In order to decide on the nature of the received signal, we calculate a threshold for each detector AD, ED. The decision threshold is determined using the required probability of false alarm P_F given by (3). The threshold TH for a given false alarm probability is determined by solving the equation:

$$P_F = P(x_n \text{ is present} \mid \mathrm{H}_0) = 1 - F_{H_0}(TH) \tag{5}$$

where F_{H_0} denote the cumulative distribution function (CDF) under H_0.

III. Wigner Ville Distribution

The Wigner Ville Distribution is a time-frequency energy distribution which is particularly interesting an is defined as:

$$W_x(t,f) = \int_{-\infty}^{+\infty} x(t+\tau/2)\ x^*(t-\tau/2)\ e^{-j2\pi f\tau}\ d\tau, \tag{6}$$

This distribution satisfies a large number of desirable mathematical properties. In particular, the WVD is always real-valued, it preserves time and frequency shifts and satisfies the marginal properties.

An interpretation of this expression can be found in terms of probability density: expression (6) is the Fourier transform of an acceptable form of characteristic function for the distribution of the energy.

Because of the quadratic nature of the WVD, its sampling has to be done with care. Let us write it as follows:

$$W_x(t,f) = 2\int_{-\infty}^{+\infty} x(t+\tau)\ x^*(t-\tau)\ e^{-j4\pi f\tau}\ d\tau$$

If we sample x with a period T_e, write $x[n] = x(nT_e)$, and evaluate the WVD at the sampling points nT_e in time, we obtain a discrete-time continuous-frequency expression of it:

$$W_x[n,f] = 2\ T_e \sum_k x[n+k]\ x^*[n-k]\ e^{-j4\pi fkT_e}$$

As this expression is periodic in frequency with period $\frac{1}{2\ T_e}$ (contrary to period $\frac{1}{T_e}$ obtained for the Fourier transform of a signal sampled at the Nyquist rate), the discrete version of the WVD may be affected by a spectral aliasing, in particular if the signal x is real-valued and sampled at the Nyquist rate. Two alternatives to this problem can be found. The first one consists in oversampling the signal by a factor of at least 2, and the second one in using the analytic signal. Indeed, as its bandwidth is half the one of the real signal, the aliasing will not take place in the useful spectral domain $[0, 1/2]$ of this signal. This second solution presents another advantage: since the spectral domain is divided by two, the number of components in the time-frequency plane is also divided by two. Consequently, the number of interference terms decreases significantly.

IV. Algebraic Detection Techniques

Before talking about the AD as a spectrum sensing approach, it is to be noticed that the signal feeding this sensing algorithm is the he Short-Time Fourier Transform STFT defined by :

$$F_x(t,f;\omega) = \int_{-\infty}^{+\infty} x(u)\ \omega^*(u-n)\ e^{-j2\pi fu}\ du$$

where $\omega(t)$ is a *short time analysis window* As far as the proposed technique is concerned, the short time analysis window $\omega(t)$ is a simple sliding one. The discrete version of the STFT is given by:

$$F_x(n,f;\omega) = \sum_{i=-\infty}^{+\infty} x(i)\ \omega^*(i-n)\ e^{-j2\pi fiT_e}$$

The AD is a new approach based on the advances lead in the fields of differential algebra and operational calculus. In this method, the primary user's presence is rather casted as a change point detection in its transmission spectrum [7] [6]. In this approach, the mathematical representation of the amplitude spectrum of the received signal is assumed to be a piecewise N^{th} polynomial signal expressed as following:

$$F_x(n,f) = \sum_{i=1}^{K} \chi_i[f_{i-1}, f_i](f) p_i(n, f-f_{i-1}) + E_n(f) \tag{7}$$

where $\chi_i[f_{i-1}, f_i]$ is the characteristic function, $(p_i)_{i\epsilon[1,K]}$ is a polynomial series that is assumed N^{th} order polynomials and $E_n(f)$ is the additive corrupting noise and n represents the discrete time.

Let $S(n,f)$ the clean version of the received signal given by:

$$S(n,f) = \Sigma_{i=1}^{K} \chi_i[f_{i-1}, f_i](f) p_i(n, f-f_{i-1}) \tag{8}$$

And let b, the frequency band, given such as in each interval $I_b = [f_{i-1}, f_i] = [\nu, \nu+b]$, $\nu \geq 0$ one and only one change point occurs in the interval I_b. Denoting $S_\nu(t,f) = S(t, f+\nu)$,$f\epsilon[0,b]$ for the restriction of the signal in the interval I_b and redefine the change point relatively to I_b say f_ν given by:

$$\begin{array}{ll} f_\nu = 0 & \text{if } S_\nu \text{ is continuous} \\ 0 < f_\nu \leq b & \text{otherwise} \end{array}$$

The primary user presence on a sensed sub-band is equivalent to find $0 < f_\nu \leq b$ on this band. The AD gives the opportunity to build a whole family of detectors for spectrum sensing, depending on a given model order N. Depending on this model order, we can show that performance of the AD is increasing as the order N increases.

The proposed algorithm is implemented as a filter banc witch composed of N filters mounted in a parallel way. The impulse response of each filter is:

$$h_{k+1}(f) = \begin{cases} \frac{(f^l(b-f)^{N+k})^{(k)}}{(l-1)!}, 0 < f < b \\ 0, otherwise \end{cases} \tag{9}$$

where $k\epsilon[0..N-1]$ and l is chosen such as $l > 2 \times N$. The proposed expression of $h_{k+1}\rfloor_{k\epsilon[0..N-1]}$ was determined by modeling the spectrum by a piecewise regular signal in frequency domain and casting the problem of spectrum sensing as a change point detection in the primary user transmission [7]. Finally, in each stage of the filter bank, we compute the following equation:

$$\varphi_{k+1}(n,f) = \int_0^{+\infty} h_{k+1}(\nu).F_x(n,f-\nu).d\nu \qquad (10)$$

In order to infer whether the primary user is present in its sub-band, a decision function is computed as following:

$$Df(n,f) = \| \prod_{k=0}^{N-1} \varphi_{k+1} \| \qquad (11)$$

The decision is made by computing the threshold TH through a Monte Carlo simulation satisfying a given probability of false alarm.

V. Performance Evaluation

For simulation results, the choice of the DVB-T primary user system is justified by the fact that most of the primary user systems utilize the OFDM modulation format. This choice is done in the context of the European research project SENDORA [12]. The channel models implemented is AWGN. The simulation scenarios are generated by using different combinations of parameters given in Table I.

Bandwidth	8MHz
Mode	2K
Guard interval	1/4
Channel models	AWGN
Frequency-flat	Single path
Sensing time	1.25ms

TABLE I
THE TRANSMITTED DVB-T PRIMARY USER SIGNAL PARAMETERS

Figures (1) and (2) are the plots of the joint time frequency analysis for the AD and WVD respectively at SNR=0dB.

It is clearly shown through these figures how accurate the location of energy is for the proposed algorithm. Figures (3) and (4) are the plots of the joint time frequency analysis for the AD and WVD respectively at SNR=-15dB. Figure (4) shows that no signal can be detected at such an SNR using the WVD, whereas for the proposed approach we can still distinguish the presence of PU in its subband.

Finally, we plot the $P_{detection}$ Vs SNR for the two detectors (Fig.5) for SNR ranging from -40 dB to 0 dB and at P_F= 0.05.

VI. Conclusion

In this paper, we presented a new approach for spectrum sensing. The implementation of such a technique is easy and the proposed algorithm takes in consideration the noise effect in its inner structure which gives good, accurate and reliable results even in low SNRs. In comparison with the reference time frequency analysis tool, the proposed joint detection out performs the WVD.

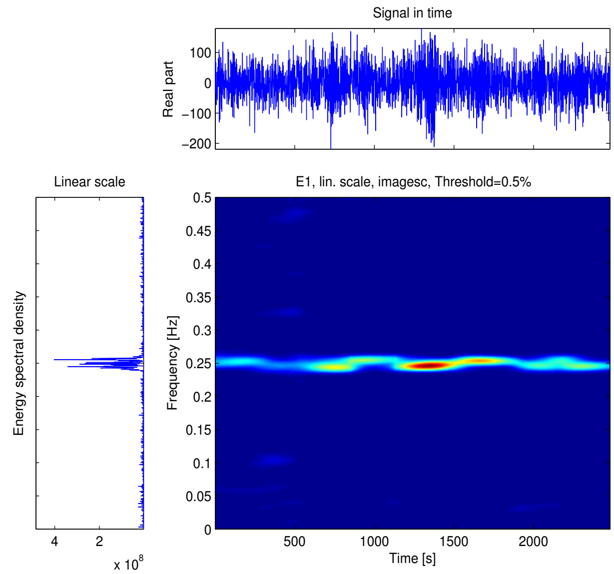

Fig. 1. Time Frequency representation of the DVBT signal at SNR=0dB using AD

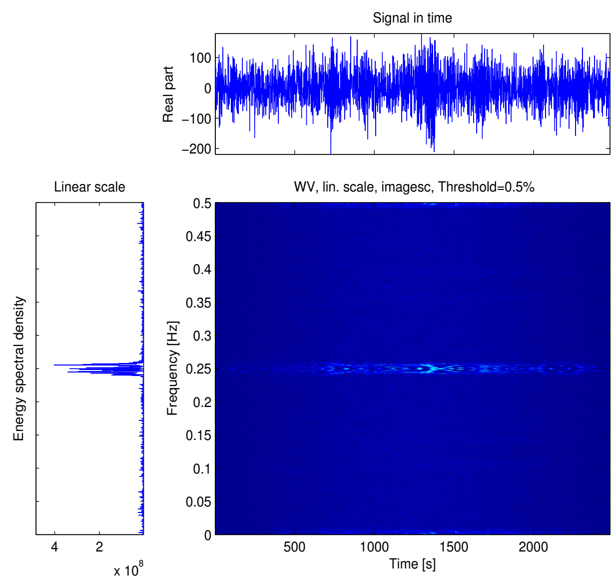

Fig. 2. Time Frequency representation of the DVBT signal at SNR=0dB using WVD

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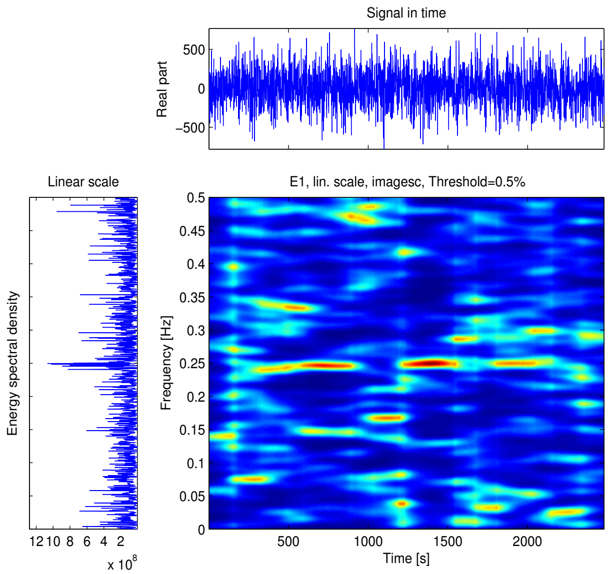

Fig. 3. Time Frequency representation of the DVBT signal at SNR=-15dB using AD

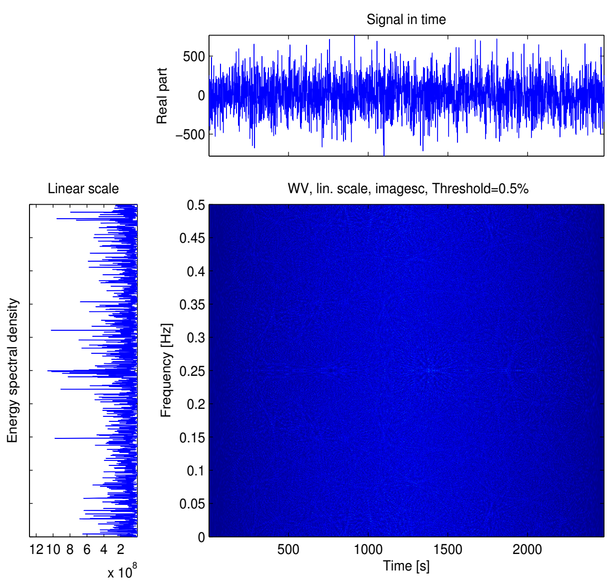

Fig. 4. Time Frequency representation of the DVBT signal at SNR=-15dB using WVD

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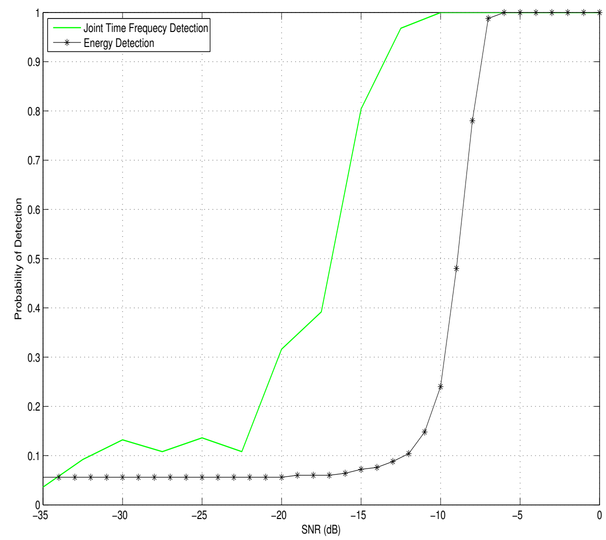

Fig. 5. Probability of detection Vs SNR at constant false alarm rate = 0.05

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