Double-linear fuzzy interpolation method

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***Abstract*—In this paper, we present an original fuzzy interpolation method. In contrast to existing approaches, our method is able to always construct an interpolated fuzzy interval without a need of a special step dedicated to the "standardization" of non-viable solutions, which fractures the sense of the interpolation. In fact, these "standardization" steps imply that, for instance, a point obtained from the interpolation of the upper limit (right side) of the fuzzy sets, is used to build the lower limit (left side) of the interpolated conclusion, breaking the underlying hypothesis of (linear) graduality. To achieve the direct interpolation, our method is based on the deviation of the observation from the expected linearly interpolated solution and constrains of the constructed solution between extreme cases. We illustrate and discuss the behavior of our method by comparison to other well-known fuzzy interpolation methods.**

I. Introduction

Interpolation of fuzzy rules has been intensively studied since 1991 with a first paper proposing a method [1] (or more accessible [2]). Since then, many different points of views were adopted: α-cuts approaches (recent ones proposed in [3], [4], [5] or [6]), analogy or similarity based approaches ([7], [8], [9]), logical approaches, among others. The interested reader can find a quasi exhaustive list of references in [10] and [11].

Some methods suppose that fuzzy data are represented only by trapezoidal fuzzy intervals, some others are more general and propose methods applicable to all membership function. Frameworks were proposed to compare different existing methods [10], [12], [13], or to unify them [11], [14]. The latter framework is an interesting theoretical approach enabling to write many methods in a unique way thanks to an analytic approach, but, unfortunately, it is not easy to instantiate the different behaviors in concrete situations.

Most methods agree on the *position* of the interpolated conclusion. Thus differences arise when dealing with the challenge of constructing an adequate shape, with the risk of obtaining a non viable fuzzy set. The reason of the latter, as shown in this paper, is that the shape of the fuzzy sets can *not* be linearly interpolated. Thus, very often a standardization step dedicated to ensure that the obtained membership is actually a viable normalized function is proposed. Unfortunately, as we point out in this paper, these steps break several fundamental underlying hypotheses, as for instance the graduality one. In fact, state of the art "standardization" steps always imply that, for instance, a point obtained from the interpolation of the upper limit (right side) of the fuzzy sets, is used to build the lower limit (left side) of the interpolated conclusion, breaking the underlying hypothesis of (linear) graduality. Moreover, the fact of "swaping" points in a "standardization" steps also implies that the associated uncertainty (which was actually used to identify the concerned points for the interpolation) is ignored. As consequence, for instance, almost uncertain values may be used (via the "standardization" step) to build the totally certain kernel of the interpolated solution.

To achieve a direct estimation of the solution, we perform a double linear interpolation by, first, linearly interpolating the expected shape value based on the known rules and secondly, by linearly interpolating the deviation from that value.

Another major difference of the method described in this paper compared to the state of the art is that it constrain the solutions between two know natural boundaries. In the one extreme, we know that the shape of the solution cannot be more precise than a certain single value and in the other limit we assume that the solution has to fall between the *know* rules. In other words, our solution takes into account, indirectly, the spread between the rules to constrain the solution: a novelty.

Our paper is organized as follows. In the next section we present the notations, before describing our method which is divided in two phases: the interpolation of the position as described in Section III and the interpolation of the shape as described in Section IV. In SectionV, we present some empirical comparisons with known methods putting in contrast the differences and common behaviors. Finally, we conclude and provide some hints about future works.

II. Interpolative Reasoning and Notations

A. Interpolative reasoning general principles

Let us consider two numerical variables X and Y defined on the universe $\mathbf{R}$ of real numbers. Let $\mathbf{F}$ denote the set of fuzzy sets of $\mathbf{R}$. We suppose that we are given fuzzy sets A_i in $\mathbf{F}$, $1 \leq i \leq n$, such that: $A_1 \preceq A_2 ... \preceq A_i \preceq A_{i+1} ... \preceq A_n$, for a given order $\preceq$ on $\mathbf{F}$. We also suppose that we are given fuzzy sets B_i in $\mathbf{F}$, $1 \leq i \leq n$, which are also ordered according to $\preceq$.

The context of study concerns sparse fuzzy rule-based systems where fuzzy rules are of the type : (R_i): "if X is A_i then Y is B_i". The sparsity of the system means that the premises of the rules do not cover the input space $\mathbf{F}$ and there exist inputs A_* such that $\exists i / A_i \preceq A_* \preceq A_{i+1}$.

The aim of a fuzzy interpolation method is to provide the conclusion corresponding to the observation A_* by considering only the two rules R_i and R_{i+1} when $A_i \preceq A_* \preceq A_{i+1}$.

B. Notations and hypotheses

Our approach is based on three fundamental hypothesis. The first hypothesis lies in the fact that Y has a *gradual* *behavior* with regard to X. The second hypothesis is that there is a *gradual behavior* for the space of forms. This hypothesis translates the intuition that if an observation is smaller (i.e more precise) than the premisses, then the conclusion should be smaller than the known conclusions. The third hypothesis is that the interpolated conclusion has to be *between* the conclusions of the adjacent rules. Intuitively, we know that on the extremes, rules 1 and 2 apply, and thus, if something is observed in the middle, the conclusion should also be in the middle.

Moreover, we require that for $A_i \preceq A_{i+1}$ the order $\preceq$ verifies no value of the support (respectively kernel) of A_{i+1} is smaller than any value of the support ((respectively kernel) of A_i and no value of the support (respectively kernel) of A_i is greater than any value of the support (respectively kernel) of A_{i+1}.

In this paper we focus on trapezoidal fuzzy sets. We choose to describe such a fuzzy set $A_i = [a_{i1}, a_{i2}, a_{i3}, a_{i4}]$, with the following four parameters. For a visual illustration see Fig. 1:

- its position defined as the center of its kernel:

$$A_i^P = \frac{a_{i2} + a_{i3}}{2} \quad (1)$$

- its certain values range characterized by the kernel's amplitude left and right from its center, as computed by:

$$\overline{A}_i^L = \overline{A}_i^R = \frac{a_{i3} - a_{i2}}{2} \quad (2)$$

- the extend of the uncertainty on the left and on the right, defined by:

$$\underline{A}_i^L = a_{i2} - a_{i1} \quad (3)$$

$$\underline{A}_i^R = a_{i4} - a_{i3} \quad (4)$$

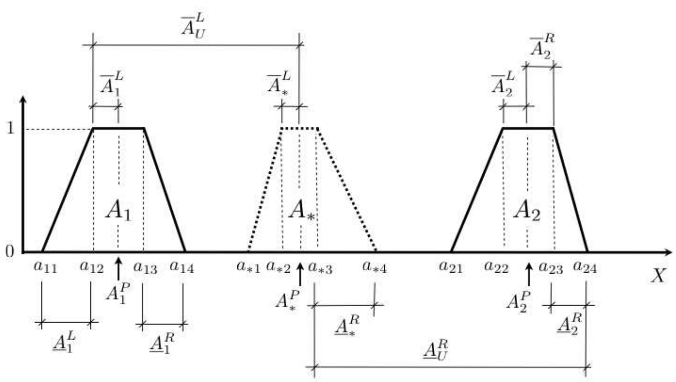

Fig. 1. A trapezoidal fuzzy set $A_i = [a_{i1}, a_{i2}, a_{i3}, a_{i4}]$ has a position A_i^P. Its kernel's form is described by its left and right length: $\overline{A}_i^L = \overline{A}_i^R$. $\underline{A}_i^L$ (and $\underline{A}_i^R$) describe its left (and right) uncertainties. We claim that in an interpolative reasoning problem it is necessary to know the range of values between the known rules to constraint the solutio. Thus, we define the global uncertainty for the kernel on the left $\overline{A}_U^L$ and on the right $\overline{A}_U^R$. Analogously $\underline{A}_U^R$ defines the global uncertainty right, ranging from the largest possible value of the left premise A_2 to the largest certain value of the observation A_*.

III. Interpolating the position

The linear hypothesis for the position states that the position of the observation A_* and the premises A_1 and A_2 are in a linear relationship with coefficient α and that the interpolated conclusion B_* and the conclusions B_1 and B_2 are also in a linear relationship with the *same* coefficient α. Formally, we have:

$$A_*^P = \alpha \cdot A_1^P + (1-\alpha) \cdot A_2^P \quad (5)$$

$$B_*^P = \alpha \cdot B_1^P + (1-\alpha) \cdot B_2^P \quad (6)$$

Now, using these two equations we can easily compute the position of the interpolated conclusion B_*. First, based on equation 5, we obtain α:

$$\alpha = \frac{A_2^P - A_*^P}{A_2^P - A_1^P} \quad (7)$$

Second, using that value in equation 5, we obtain the position B_*^P.

IV. Interpolating the shape

On Fig. 2 we observe that the shape of observation A_* is not necessarily in a linear relationship with the shapes of the premises A_1 and A_2, when considered in the universe of description X (which coincides with position). One could argue that this lack of linearity depends upon the way the shapes are measured, but in fact for any non trivial measure it is easy to imagine a case breaking the linearity.

In fact, a simple way to achieve this is by, first computing the expected shape value (linearly) and then building a fuzzy set observation A_* (the counterexample), which has a different shape value. This counterexample exists because the measure is assumed not trivial.

Moreover, this non linearity applies not only to any general description of the shape but also to any shape measure based on a length descriptor. Thus, any α-cut based method will suffer from this non-linearity. This fundamental limitation has also been been observed in [15].

One of the direct consequences is that any method attempting to linearly interpolate the shape (based on the position) will produce degenerated shapes, as pointed out in [7]. The more recent methods avoid ill solutions, by using heuristics that wisely choose among a set of points the ones providing a viable solution. Unfortunately these heuristics are solely designed to avoid degenerative solutions, ignoring any interpolative argumentation.

To overcome the above fundamental reality, we propose to compute the value of the interpolated conclusion in a two step interpolation: first linearly interpolate the *expected shape values* (between the premises and between the conclusions) and then linearly interpolate the *shape deviation* between the expected shape and a limit case, which follows our fundamental assumptions. De facto, our two limit cases correspond, in the one extreme, to zero (assuming that any shape measure is positive), and in the other extreme, to the shape measure of the global uncertainty. The latter corresponds to the assumption, mentioned in Section II-B, that the interpolated conclusion must be between the known conclusions.

As a result, the solutions are constrained to a reasonable range, corresponding to the limit cases, in which they linearly evolve. The shapes of the premises and conclusions influence *only indirectly* the solution via the double linear interpolation: first the linear interpolation of the expected value and second the linear interpolation of the deviation.

A. Describing the shape of a trapezoidal fuzzy set

The shape of a fuzzy set, and in particular of a trapezoidal one, can be described in numerous ways. For instance it could be described by a single value, as the surface (or integral of the membership function), which summarizes the global spread of uncertainty. But more refined ways, with several parameters, can also be imagined. In this paper we choose to describe the shape of a trapezoidal fuzzy set with the four parameters $\overline{A}_i^L$, $\overline{A}_i^R$, $\underline{A}_i^L$ and $\underline{A}_i^R$, defined in Section II-B. This description focuses on the amplitudes, and symmetry, of the kernel and the support sets.

Since these are all length-descriptors, the double linear interpolation required by the nature of the problem, as described above, has to be applied. In the following we describe how this is performed, analogously in the four cases.

B. Interpolating the kernel's length

In order to interpolate the kernel's shape, we compute two double linear interpolations: one for the left lengths $\overline{A}_i^L$ and one for the right ones $\overline{A}_i^R$. Since the calculations are identical, in the following we only describe the left case.

For the first linear interpolation, we estimate the linearly expected shapes $\overline{A}_E^L$ and $\overline{B}_E^L$ using the kernels' left lengths of the premises $\overline{A}_1^L$ and $\overline{A}_2^L$, and of the conclusions' $\overline{B}_1^L$ and $\overline{B}_2^L$. As illustrated on Fig. 2 and 3, using simple mathematics we obtain :

$$\overline{A}_E^L = \left(\overline{A}_1^L - \overline{A}_2^L\right) \cdot \left(\frac{A_2^P - A_*^P}{A_2^P - A_1^P}\right) + \overline{A}_2^L \tag{8}$$

$$\overline{B}_E^L = \left(\overline{B}_1^L - \overline{B}_2^L\right) \cdot \left(\frac{B_2^P - B_*^P}{B_2^P - B_1^P}\right) + \overline{B}_2^L \tag{9}$$

Then, for the second interpolation, we linearly calculate the deviation of the observed kernel $\overline{A}_*^L$ from the expected kernel $\overline{A}_E^L$. As we will see below two scenarios appear depending on the relative magnitude of $\overline{A}_*^L$ with respect to $\overline{A}_E^L$. It is noteworthy to observe that both scenarios can not be integrated in a single linear transformation.

1) Smaller-shape deviation: Let us assume that $\overline{A}_*^L < \overline{A}_E^L$. In this case the observation is more precise than the expected interpolation. Knowing that any shape measure, and in particular one based on the length, is always positive (or equals to 0), using the linear hypothesis for the deviation we have:

$$\overline{A}_*^L = \beta_L \cdot \overline{A}_E^L + (1 - \beta_L) \cdot 0 = \beta_L \cdot \overline{A}_E^L \tag{10}$$

$$\overline{B}_*^L = \beta_L \cdot \overline{B}_E^L + (1 - \beta_L) \cdot 0 = \beta_L \cdot \overline{B}_E^L \tag{11}$$

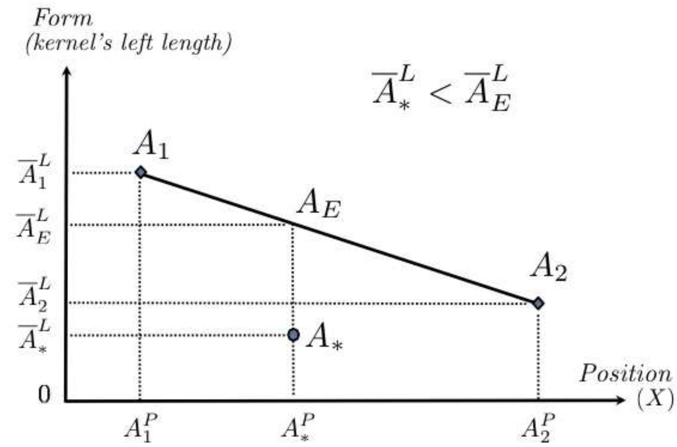

Fig. 2. The shape description of observation A_* is not necessarily in a linear relationship with the shape descritions of the premises A_1 and A_2.

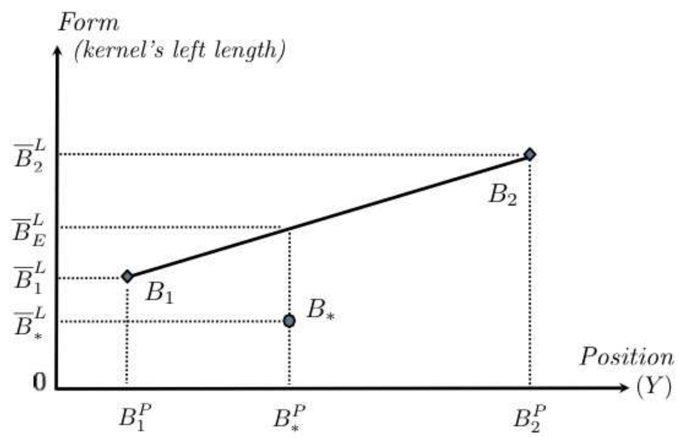

Fig. 3. The shape description of the interpolated conclusion B_* should be in the same linear deviation from the expected shape value, as is the case for the premises.

Thus, in order to obtain the interpolated conclusion's kernel left length, we use Equation 10 to obtain:

$$\beta_L = \frac{\overline{A}_*^L}{\overline{A}_E^L} \tag{12}$$

Which is then used in Equation 11 to obtain the length $\overline{B}_*^L$. The same exact equations, replacing L by R, can be used for the right description of the kernels and, thus, obtain $\overline{B}_*^R$.

2) Larger-shape deviation: It may happen that the observation is less precise than the linearly expected value: $\overline{A}_*^L \geq \overline{A}_E^L$. In that case Equation 10 do not apply, since we have a deviation that implies an increase of the shape and therefore can not be constrained, by a lower boundary, as in a reduction scenario. In other words, since the length of the observation is larger than expected, we would like the length of the conclusion to be *also* larger than the expected linearly interpolated conclusion $\overline{B}_E^L$. In addition, the third of hypothesis mentioned in Section II-B implies that its range can not be larger that the gap between the rules. In the case of the kernel's

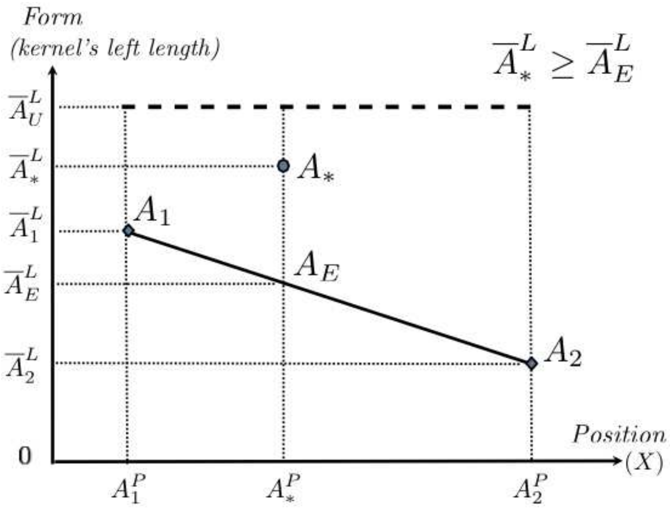

Fig. 4. The shape description of observation A_*, as for instance here the left kernel length $\overline{A}_*^L$, may be larger than the linearly expected value $\overline{A}_E^L$. We propose to constrain the solution by linearly interpolating between the linearly expected shape and the left kernel uncertainty $\overline{A}_U^L$.

left length calculations the uncertainty left between the kernel's premises estimated by:

$$\overline{A}_U^L = A_*^P - a_{12} \tag{13}$$

as shown on Fig. 1.

Consequently, assuming a linear hypothesis for the deviation, we obtain:

$$\overline{A}_*^L = \beta_L \cdot \overline{A}_E^L + (1 - \beta_L) \cdot \overline{A}_U^L \tag{14}$$

$$\overline{B}_*^L = \beta_L \cdot \overline{B}_E^L + (1 - \beta_L) \cdot \overline{B}_U^L \tag{15}$$

The same way as with the smaller shape reduction, in order to obtain the conclusion's kernel-left-length, we use Equation 14 to obtain:

$$\beta_L = \left(\frac{\overline{A}_U^L - \overline{A}_*^L}{\overline{A}_U^L - \overline{A}_E^L} \right) \tag{16}$$

which is then used in Equation 15 to obtain the length $\overline{B}_*^L$. Again, by replacing in the equations L by R we know how to obtain $\overline{B}_*^R$.

C. Interpolating the left and right uncertainties

At this point we know the interpolated conclusion's fuzzy set position (Section III) and its kernel's shape (Section IV-B). Now we interpolate, analogously as for the kernel's lengths, the left and right uncertainties. We first interpolate the expected forms, here only for the right uncertainties:

$$\underline{A}_E^R = \left(\underline{A}_1^R - \underline{A}_2^R\right) \cdot \left(\frac{a_{23} - a_{*3}}{a_{23} - a_{13}} \right) + \underline{A}_2^R \tag{17}$$

$$\underline{B}_E^R = \left(\underline{B}_1^R - \underline{B}_2^R\right) \cdot \left(\frac{b_{23} - b_{*3}}{b_{23} - b_{13}} \right) + \underline{B}_2^R \tag{18}$$

where $b_{*3} = B_*^P + \overline{B}_*^R$. Notice that we anchor, as for the kernels, the length description to the point where the interval starts.

Now, in the same way as for the kernels, we have two scenarios depending of the the relative value of the expected length and the observed length.

If $\underline{A}_*^R < \underline{B}_E^R$, then the double interpolated right uncertainty is obtained by:

$$\underline{B}_*^R = \gamma_R \cdot \underline{B}_E^R = \left(\frac{\underline{A}_*^R}{\underline{A}_E^R} \right) \cdot \underline{B}_E^R \tag{19}$$

But, if $\overline{A}_*^L \geq \overline{A}_E^L$, , then the double interpolated right uncertainty is obtained by:

$$\underline{B}_*^R = \gamma_R \cdot \underline{B}_E^R + (1 - \gamma_R) \cdot \underline{B}_U^R \tag{20}$$

where

$$\gamma_R = \left(\frac{\underline{A}_U^R - \underline{A}_*^R}{\underline{A}_U^R - \underline{A}_E^R} \right) \tag{21}$$

V. Empirical comparisons

In this section, we compare the double-linear fuzzy interpolation method (DoLFIn for short) with some well-known methods: [16] (DP), [10] (BTKY), and [7] (BMR). We consider several scenarios:

- all the shapes are similar
- specific observation
- specific premises or specific conclusions
- extremely unspecific observation

Moreover, to extend our comparison to other state of the art approaches, we use some examples shared by the following papers [3] (HS), [4] (CK), [17] (CCL).

Figures Fig. 5 to Fig. 12 should be read as follows. The first row shows the two premises A_1 and A_2 and the observation A_*. The other four rows present the conclusions by each of the four studied approaches. For the comparison the two known conclusions B_1 and B_2 are always the same, and only the obtained result B_* changes between the rows.

A. All the shapes are similar

A first interesting scenario arises when the observation is similar to the premise. In this case, the observation can be considered as a translation of one of the premises. It is a typical case of interpolative reasoning.

In Fig. 5 and Fig. 6, it can be seen that the three approaches BTKY, BMR, and DolFIn propose the exact same conclusion. The unique method which offers a different solution is the DP, which is very unspecific and uncertain.

B. Precise observation

A second interesting case arises when the observation is precise. It is often the case in real-world applications for decision-making process when no fuzzification of the input data is processed.

In Fig. 7 and Fig. 8, it can be seen that the results are different for the four methods. As previously seen, again DP

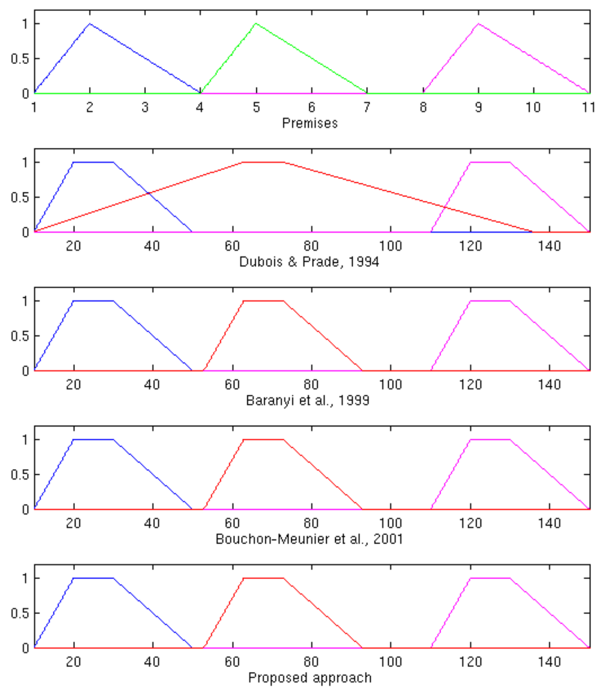

Fig. 5. All the shapes are similar (1)

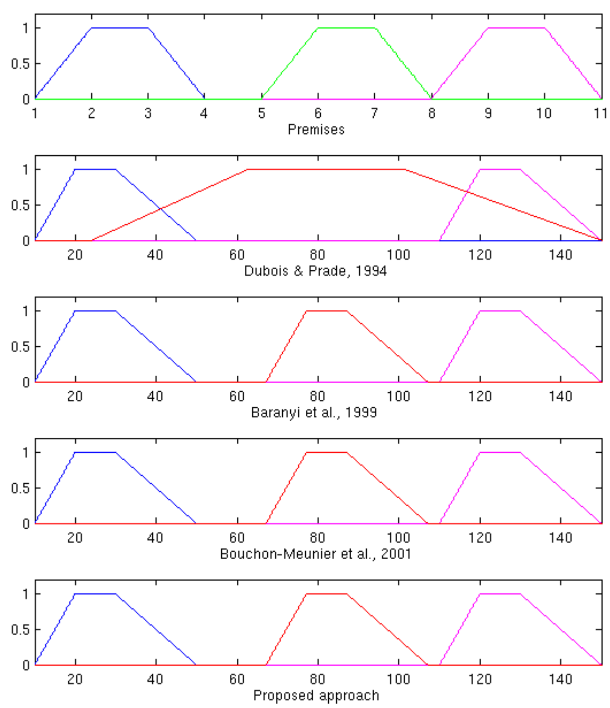

Fig. 6. All the shapes are similar (2)

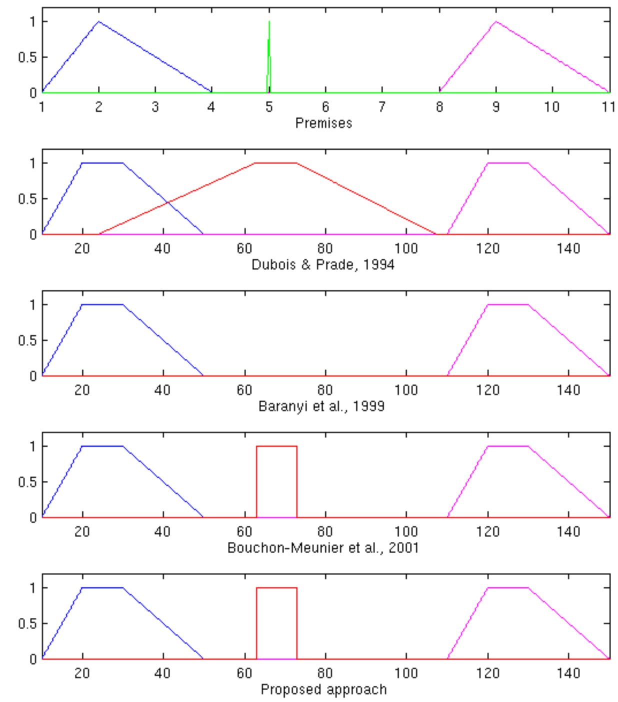

Fig. 7. Precise observation (1)

offers a very specific solution that can hardly be linked to the preciseness of the observation. BTKY proposes a non viable solution. Concerning BMR and DoLFIn, the results are different depending on the form of the premises and the conclusions. In a case, Fig. 7, BMR and DoLFIn methods propose a similar result. On the other case, Fig. 8, the results are very different: BMR proposes a fuzzy solution where DoLFIn constructs a precise solution.

The difference between the situations of Fig. 7 and Fig. 8 is that the forms of the premises are smaller than the forms of the conclusions on the first one, and greater on the second one. In this case, BMR method does not always provide a specific conclusion in presence of a specific observation.

In general, for a precise observation we obtain either a vague solution or no solution at all. While DoLFIn guarantees a specific conclusion in this case.

C. *Precise premises*

When premises are precise and the observation is imprecise and uncertain (see Fig. 9) all the methods except DoLFIn provide a solutions very imprecise and uncertain, sometimes going beyond the scope of the conclusions.

DoLFIn method proposes an interesting solution with a reasonable support size. Moreover, as for the observation and the premises, the solution respects the requirements imposed by the order $\preceq$ defined in Section II-B.

D. *Extremely unspecific observations*

When the observation is in the limit case of an extremely unspecific but precise interval that covers the whole space between the premises, see Fig. 10, DoLFIn is the only approach

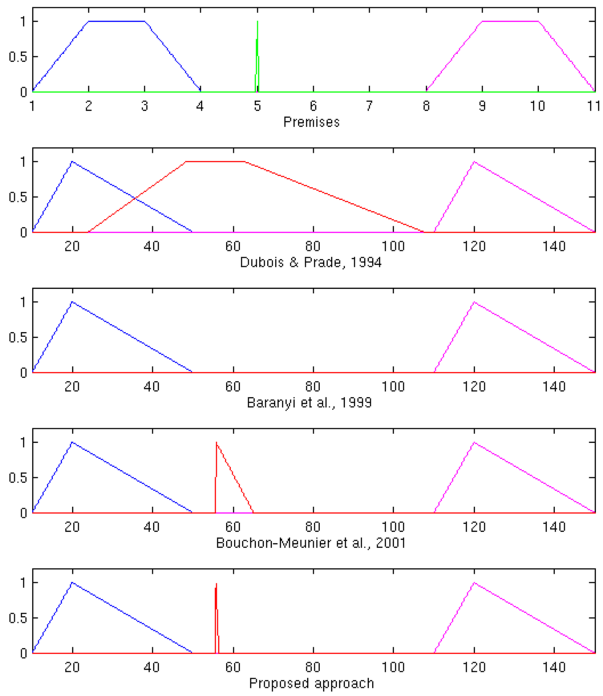

Fig. 8. Precise observation (2)

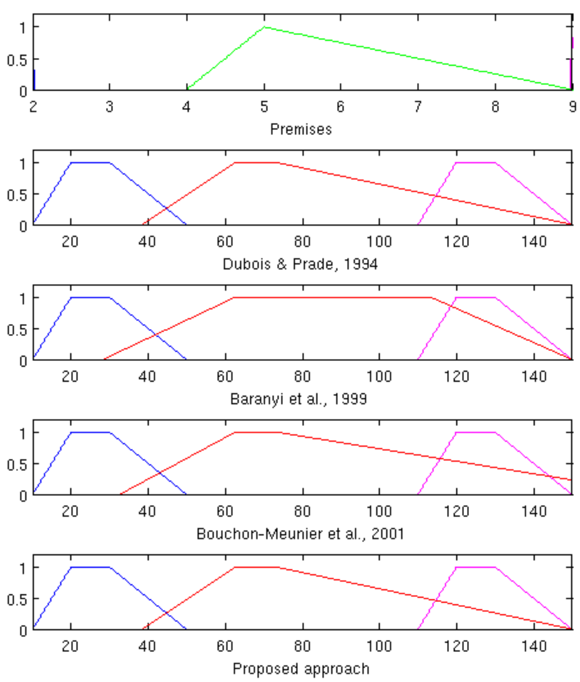

Fig. 9. Precise premises

that leads to an unspecific and precise conclusion. In this case, BTKY does not propose a viable solution, and BMR and DP construct imprecise solutions.

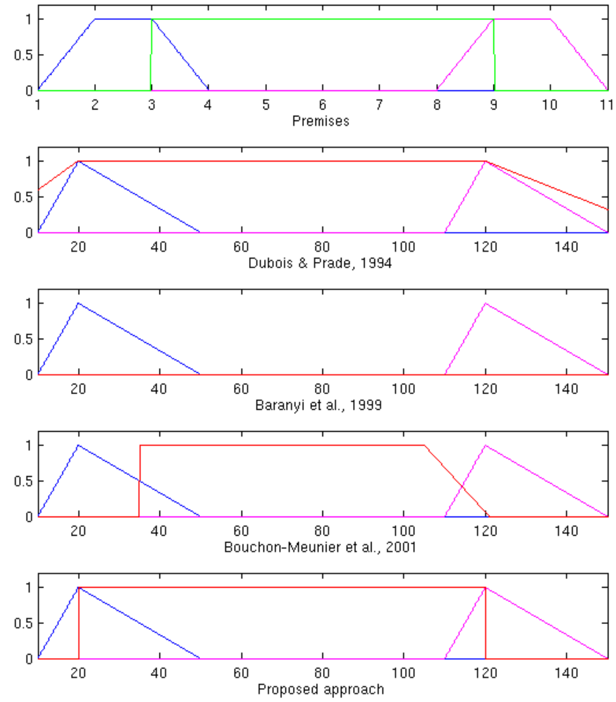

Fig. 10. Extremely unspecific observations

E. Remarkable examples

In figures Fig. 11 to Fig. 12, we extend our comparison by using some remarkable examples pointed out and shared by the following papers [3] (HS), [4] (CK), [17] (CCL). We invite the reader to refer to those papers for further details.

In Fig. 11, it can been seen that DP produces a very unspecific solution and BTKY constructs a non viable solution. The remaining approaches provide similar results with slight differences. BMR proposes a solution with a very steep right slope due to the fact that there is a strong transformation from the right slope of the premises to the observation. The result by DoLFIn is more fuzzy and with a different location than the solution by HS and CK. DoLFIn and CCL produce an almost identical conclusion.

A second remarkable example is presented in Fig. 12. Here again, it can been seen that DP produces a very unspecific conclusion. BTKY constructs a non viable solution. This time all the remaining approaches[1] (BMR, DoLFIn, CCL, and CK) produce a very similar result.

In general DoLFIn is able to provide a reasonable solution to all the remarquable examples identified by the above mentioned authors.

[1]The comparison with HS is not available because this example is not treated in [3].

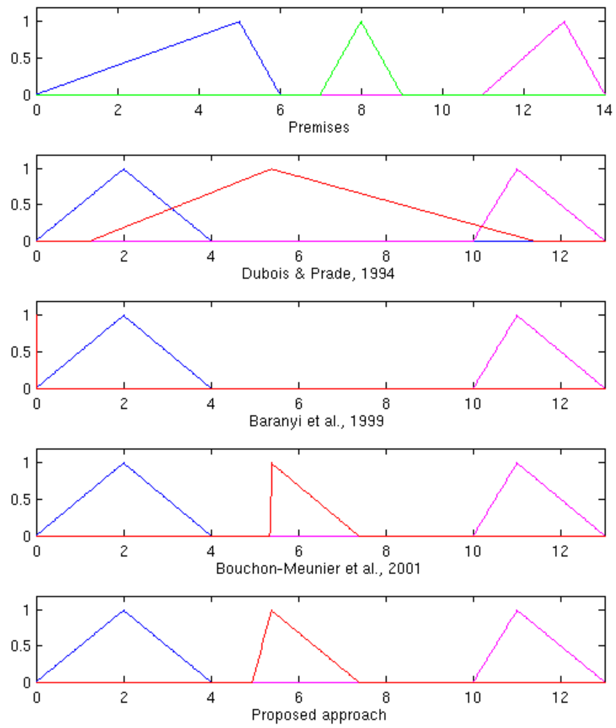

Fig. 11. Remarkable example (1)

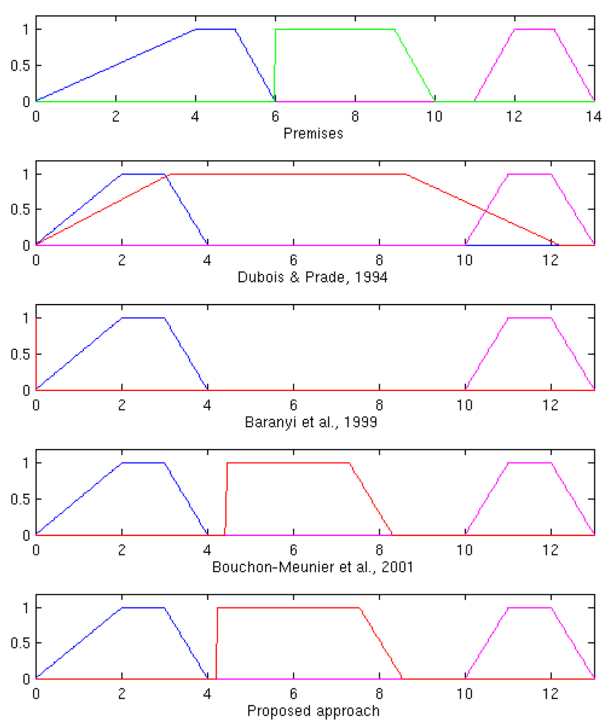

Fig. 12. Remarkable example (2)

F. Discussion

It can be seen in the presented comparison that for each of the scenarios, a group of methods (not always the same ones) provides adequate solutions.

DP proposes always a solution which is very unspecific and imprecise. BTKY generates conclusions often very pertinent but suffers of a problem of viability. BMR is very robust and can handle a large set of particular cases without having a problem of viability thanks to its standardization step [7]. However, the constructed solution can have a fuzzy conclusion with a precise observation, or a conclusion which is out of the scope of the known conclusions.

The proposed DoLFIn method has none of the mentioned drawbacks and proposes a pertinent and viable solution in all the studied cases without the use of a standardization step.

VI. Conclusion

Our method is based on the deviation to the expected linearly interpolated solution and constrains the constructed solution between extreme cases. We discovered that from the fundamental assumptions, mathematically, two scenarios appear: one where the form is expected to diminish but constrained to be positive and a second where the form is expected to grow but is constrained by the two known rules.

The presented study suggests that DoLFIn produces always a pertinent solution for a large set of diverse situations. Its pertinence is reinforced by the fact that each time the solution coincides with at least the result of another method.

An extensive and detailed comparison with a larger set of examples and methods coded in the FRI toolbox [18], is under development. We believe that its conclusion will not reveal any major differences with the conclusions of the presented paper. Future works will focus on the extension of the presented method to general shaped fuzzy sets and to the more complex multi-premise rules interpolation problem.

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