Multiplicative spectrum of ultrametric Banach algebras of continuous functions

Alain Escassut, Nicolas Mainetti

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by Alain Escassut and Nicolas Maïnetti

Abstract Let K be an ultrametric complete field and let E be an ultrametric space. Let A be the Banach K-algebra of bounded continuous functions from E to K and let B be the Banach K-algebra of bounded uniformly continuous functions from E to K. Maximal ideals and continuous multiplicative semi-norms on A (resp. on B) are studied by defining relations of stickness and contiguousness on ultrafilters that are equivalence relations. So, the maximal spectrum of A (resp. of B) is in bijection with the set of equivalence classes with respect to stickness (resp. to contiguousness). Every prime ideal of A or B is included in a unique maximal ideal and every prime closed ideal of A (resp. of B) is a maximal ideal, hence every continuous multiplicative semi-norms on A (resp. on B) has a kernel that is a maximal ideal. If K is locally compact, every maximal ideal of A, (resp. of B) is of codimension 1. Every maximal ideal of A or B is the kernel of a unique continuous multiplicative semi-norm and every continuous multiplicative semi-norm is defined as the limit along an ultrafilter on E. Consequently, on A as on B the set of continuous multiplicative semi-norms defined by points of E is dense in the whole set of all continuous multiplicative semi-norms. Ultrafilters show bijections between the set of continuous multiplicative semi-norms of A, $Max(A)$ and the Banaschewski compactification of E which is homeomorphic to the topological space of continuous multiplicative semi-norms. The Shilov boundary of A (resp. B) is equal to the whole set of continuous multiplicative semi-norms.

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Introduction and preliminaries:

Definitions and notation:

Let K be a field complete with respect to an ultrametric absolute value $| \, . \, |$. It is well known that the set of maximal ideals is not sufficient to describe spectral properties of an ultrametric Banach algebra: we have to consider the set of continuous multiplicative semi-norms [6], [7], [9], [10], [11],]. Many studies were made on continuous multiplicative semi-norms on algebras of analytic functions, analytic elements and their applications to holomorphic functional calculus [3], [5], [6]. Here we mean to study continuous multiplicative semi-norms on Banach algebras of continuous functions. We will consider two main cases: Banach algebras of bounded continuous functions and Banach algebras of bounded uniformly continuous functions (with an application to Banach algebras of bounded functions).

Definitions and notation: Let E denote a metric space whose distance δ is ultrametric, let A be the Banach K-algebra of bounded continuous fonctions from E to K and let B be the Banach K-algebra of bounded uniformly continuous fonctions from E to K.

We will call *clopen* any closed open subset of E. Let H be a subset of E different from E and $\emptyset$. We will call *codiameter of H* the number $\delta(H, E \setminus H)$ and we will denote it by $\text{codiam}(H)$. The subset H will be said to be *uniformly open* if $\text{codiam}(H) > 0$. Given $a \in E$ and $r > 0$ we will denote by $d(a, r^-)$ the ball $\{x \in E \mid \delta(a, x) < r\}$. Let $\mathcal{F}$ be a filter on E. Given a function f from E to K admitting a limit along $\mathcal{F}$, we will denote by $\lim\limits_{\mathcal{F}} f(x)$ that limit.

Given a set F, we shall denote by $U(F)$ the set of ultrafilters on F. Now, let X be a topological space. Given $\mathcal{F} \in U(X)$, we will denote by $\overline{\mathcal{F}}$ the filter generated by the closures of elements of $\mathcal{F}$. Two ultrafilters $\mathcal{F}$, $\mathcal{G}$ on F will be said to be *sticked* if $\overline{\mathcal{F}}$ and $\overline{\mathcal{G}}$ are secant. We will denote by $(\mathcal{S})$ the relation defined on $U(X)$ as $\mathcal{U}(\mathcal{S})\mathcal{V}$ if $\mathcal{U}$ and $\mathcal{V}$ are sticked.

Next, two filters $\mathcal{F}$, $\mathcal{G}$ on E will be said to be *contiguous* if for every $H \in \mathcal{F},\ L \in \mathcal{G}$, we have $\delta(H, L) = 0$. We shall denote by $(\mathcal{T})$ the relation defined on $U(E)$ as $\mathcal{U}(\mathcal{T})\mathcal{V}$ if $\mathcal{U}$ and $\mathcal{V}$ are contiguous.

An ultrafilter $\mathcal{U}$ on the set E is said to be *principal* if there exists $a \in E$ such that $\mathcal{U} = \{H \subset E \mid a \in H\}$.

A closed open set of E is called *a clopen.*

Remark 1: Let $H \subset E$ be different from $\emptyset$ and from E. Then $\text{codiam}(H) = \text{codiam}(E \setminus H)$.

Remark 2: Two sticked filters on E are contiguous.

Remark 3: A uniformly open subset of E is open and closed.

Remark 4: Let $\mathcal{U}, \mathcal{V}$ be contiguous ultrafilters on E and assume $\mathcal{U}$ is convergent. Then $\mathcal{V}$ is convergent and has the same limit as $\mathcal{U}$. Moreover $\mathcal{U}$ and $\mathcal{V}$ are sticked.

Remark 5: We can construct contiguous ultrafilters that are not sticked. Let $(a_n)_{n\in\mathbb{N}},\ (b_n)_{n\in\mathbb{N}}$ be sequences in K such that

(1) $|a_n| < |a_{n+1}|$,
(2) $\lim\limits_{n\to+\infty} a_n - b_n = 0$,
(3) $|a_n - b_n| \geq \dfrac{1}{n}$.

Let $\mathcal{U}$ be a filter thinner than the sequence (a_n). We will define a filter $\widetilde{\mathcal{U}}$ thinner than the sequence (b_n) completing the example. By definition, $\mathcal{U}$ admits a basis made of images of subsequences $(a_{\sigma(n)}))_{n\in\mathbb{N}}$ of the sequence (a_n). Let $\mathcal{Z}$ be the family of images of such sequences, making a basis of $\mathcal{U}$.

Given such a subsequence $(a_{\sigma(m)})\ m \in \mathbb{N}$, set $Q = \{a_{\sigma(m)} \mid m \in \mathbb{N}\}$ and set $\widetilde{Q} = \{b_\sigma(m) \mid m \in \mathbb{N}\}$. Let $\widetilde{\mathcal{U}}$ be the filter admitting for basis the family $\{\widetilde{Q}, Q \in \mathcal{Z}\}$.

Then we can check that $\mathcal{U}$ is an ultrafilter if and only if so is $\widetilde{\mathcal{U}}$. Indeed, suppose $\mathcal{U}$ is an ultrafilter and suppose $\widetilde{\mathcal{U}}$ is not. Let $\mathcal{V}$ be an ultrafilter strictly thinner than $\widetilde{\mathcal{U}}$.

Let $X \in \mathcal{V} \setminus \widetilde{\mathcal{U}}$. Since $\mathcal{V}$ is thinner than $\widetilde{\mathcal{U}}$, we may assume that X is the image of a subsequence $(b_{\tau(m)})_{m\in\mathbb{N}}$ of the sequence $(b_n)_{n\in\mathbb{N}}$ where this image $\{b_{\tau(m)} \mid m \in \mathbb{N}\}$ is strictly included in the image $\{b_{\sigma(m)} \mid m \in \mathbb{N}\}$ of a subsequence $(b_{\sigma(m)})_{m\in\mathbb{N}}$. But then, the set $\{a_{\tau(m)} \mid m \in \mathbb{N}\}$ is strictly included in $\{a_{\sigma(m)} \mid m \in \mathbb{N}\}$ which belongs to $\mathcal{U}$. But $\{a_{\tau(m)} \mid m \in \mathbb{N}\}$ doesn't belong to $\mathcal{U}$ because if it belonged to $\mathcal{U}$, then $\{b_{\tau(m)} \mid m \in \mathbb{N}\}$ would belong to $\widetilde{\mathcal{U}}$, which is excluded by hypothesis. But then, the filter generated by $\mathcal{U}$ and the set $\{a_{\tau(m)} \mid m \in \mathbb{N}\}$ is strictly thinner than $\mathcal{U}$, a contradiction since $\mathcal{U}$ is an ultrafilter.

Thus we have proved that if $\mathcal{U}$ is an ultrafilter so is $\widetilde{\mathcal{U}}$. The converse is obvious.

Now, by (2), $\mathcal{U}$ and $\widetilde{\mathcal{U}}$ are contiguous. Next, by (1) and (3), both Q, $\widetilde{Q}$ are closed sets such that $Q \cap \widetilde{Q} = \emptyset$ which shows that $\mathcal{U}$ and $\widetilde{\mathcal{U}}$ are not sticked.

Remark 6: Relation $(\mathcal{S})$ is not the equality between ultrafilters, even when the ultrafilters are not convergent. In [13], Labib Haddad introduced the following equivalence relation $(\mathcal{L})$ on ultrafilters. Given two ultrafilters $\mathcal{U}$, $\mathcal{V}$ we write $\mathcal{U}(\mathcal{L})\mathcal{V}$ if there exists an ultrafilter $\mathcal{W}$ such that every closed set H lying in $\mathcal{W}$ also lies in $\mathcal{U}$ and similarly, every closed set H lying in $\mathcal{W}$ also lies in $\mathcal{V}$. So, Relation $(\mathcal{L})$ is clearly thinner than Relation $(\mathcal{S})$. However, it is shown that two ultrafilters $\mathcal{U}$, $\mathcal{V}$ satisfying $\mathcal{U}(\mathcal{L})\mathcal{V}$ may be distinct without converging.

Basic results:

First, we will show that Relation $(\mathcal{S})$ may be also defined in terms of clopens.

Theorem 1: *Two ultrafilters $\mathcal{U}, \mathcal{V}$ are sticked if and only if for any two clopens $H \in \mathcal{U}$, $L \in \mathcal{V}$, we have $H \cap L \neq \emptyset$.*

Theorem 2 may be viewed as a particular version of a theorem due to Urysohn, although it is not a direct consequence of a theorem of Urysohn because Urysohn's theorem only concerns functions with values in $[0, 1]$.

Theorem 2: *Let $\mathcal{U}$, $\mathcal{V}$ be two ultrafilters on E that are not sticked. There exist $H \in \mathcal{U}$, $L \in \mathcal{V}$ and $f \in A$ such that $f(x) = 1\ \forall x \in H$, $f(x) = 0\ \forall x \in L$.*

Theorem 3: *Let $\mathcal{U}$, $\mathcal{V}$ be two ultrafilters on E that are not contiguous. There exist $H \in \mathcal{U}$, $L \in \mathcal{V}$ and $f \in B$ such that $f(x) = 1\ \forall x \in H$, $f(x) = 0\ \forall x \in L$.*

Notation: Let T be a K-algebra of bounded functions from E to K.

Given a filter $\mathcal{F}$ on E, we will denote by $\mathcal{I}(\mathcal{F}, T)$ the ideal of the $f \in T$ such that $\lim_{\mathcal{F}} f(x) = 0$. We will denote by $\mathcal{I}^*(\mathcal{F}, T)$ the ideal of the $f \in T$ such that there exists a subset $L \in \mathcal{F}$ such that $f(x) = 0\ \forall x \in L$.

Given $a \in E$ we will denote by $\mathcal{I}(a, T)$ the ideal of the $f \in T$ such that $f(a) = 0$.

We will denote by $Max(T)$ the set of maximal ideals and by $Max_E(T)$ the set of maximal ideals of the form $\mathcal{I}(a,T),\ a \in E$.

Proposition A is easy:

Proposition A: *Let T be a K-algebra of bounded functions from E to K. Given an ultrafilter $\mathcal{U}$ on E, $\mathcal{I}(\mathcal{U},T)$, $\mathcal{I}^*(\mathcal{U},T)$ are prime ideals.*

Theorem 4: *Let $\mathcal{U}$, $\mathcal{V}$ be two ultrafilters on E. Then $\mathcal{I}(\mathcal{U},A) = \mathcal{I}(\mathcal{V},A)$ if and only if $\mathcal{U}$ and $\mathcal{V}$ are sticked. Further, $\mathcal{I}(\mathcal{U},B) = \mathcal{I}(\mathcal{V},B)$ if and only if $\mathcal{U}$ and $\mathcal{V}$ are contiguous.*

Corollary 4.1: *Both Relations $(\mathcal{S})$, $(\mathcal{T})$ are equivalence relations on $U(E)$.*

Remark 7 : Relations $(\mathcal{S})$, $(\mathcal{T})$ are not transitive when applying to the set of all filters on E. However, given a topological space X satisfying the normality axiom, (i.e. any two closed disjoint subsets H, L admit disjoint open neighborhoods), then $(\mathcal{S})$ is transitive for ultrafilters and therefore is an equivalence relation on $U(X)$. Similarly, given a metric space X, then $(\mathcal{T})$ is transitive for ultrafilters and therefore is an equivalence relation on $U(X)$ [13].

Notation: We will denote by $Y_{(\mathcal{S})}(E)$ the set of equivalence classes on $U(E)$ with respect to Relation $(\mathcal{S})$ and by $Y_{(\mathcal{T})}(E)$ the set of equivalence classes on $U(E)$ with respect to Relation $(\mathcal{T})$.

Let $f \in A$ and let ϵ be > 0. We set $D(f,\epsilon) = \{x \in E \mid |f(x)| \leq \epsilon\}$.

We will need the following Proposition that is immediate:

Proposition B: *Let $\mathcal{M}$ be a maximal ideal of A (resp. of B) of the form $\mathcal{I}(\mathcal{U},A)$ (resp. $\mathcal{I}(\mathcal{U},B)$). If $\mathcal{U}$ converges in E then $\mathcal{M}$ is of codimension 1.*

Main Theorems:

Theorem 5 looks like certain Bezout-Corona statements [10], [14]:

Theorem 5: *Let $f_1,...,f_q \in A$ (resp. $f_1,...,f_q \in B$) satisfy*

$$\inf_{x\in E}\big(\max_{1\leq j\leq q} |f_j(x)|\big) > 0.$$

Then there exists $g_1,...,g_q \in A$ (resp. $g_1,...,g_q \in B$) such that

$$\sum_{j=1}^{q} f_j(x)g_j(x) = 1 \ \forall x \in E.$$

Corollary 5.1: *Let I be an ideal of A (resp. of B) different from A (resp. from B). The family $D(f, \epsilon)$, $f \in I$, $\epsilon > 0$, generates a filter $\mathcal{F}_{I,A}$ (resp. $\mathcal{F}_{I,B}$) on E such that $I \subset \mathcal{I}(\mathcal{F}_{I,A}, A)$ (resp. $I \subset \mathcal{I}(\mathcal{F}_{I,B}, B)$).*

By Proposition B, we now have Corollary 5.2:

Corollary 5.2: *Let $\mathcal{M}$ be a maximal ideal of A. There exists an ultrafilter $\mathcal{U}$ on E such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, A)$. Moreover, if $\mathcal{U}$ converges in E, then $\mathcal{M}$ is of codimension* 1.

Corollary 5.3: *Suppose E is complete. Let $\mathcal{M}$ be a maximal ideal of A and let $\mathcal{U}$ be an ultrafilter on E such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, A)$. If $\mathcal{U}$ is a Cauchy ultrafilter, then $\mathcal{M}$ is of codimension* 1.

Corollary 5.4: *For every maximal ideal $\mathcal{M}$ of A, there exists a unique $\mathcal{H} \in Y_{(\mathcal{S})}(E)$ such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, A)$ $\forall \mathcal{U} \in \mathcal{H}$.*

Moreover, the mapping Φ that associates to each $\mathcal{M} \in Max(A)$ the unique $\mathcal{H} \in Y_{(\mathcal{S})}(E)$ such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, A)$ $\forall \mathcal{U} \in \mathcal{H}$, is a bijection from $Max(A)$ onto $Y_{(\mathcal{S})}(E)$.

In the particular case when we consider the discrete topology on E, we have Corollary 5.5:

Corollary 5.5: *For every maximal ideal $\mathcal{M}$ of the Banach K-algebra T of all bounded functions on E, there exists a unique ultrafilter $\mathcal{U}$ on E such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, T)$.*

Moreover, the mapping Φ that associates to each $\mathcal{M} \in Max(T)$ the unique $\mathcal{U}$ such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, T)$ $\forall \mathcal{U} \in \mathcal{H}$, is a bijection from $Max(T)$ onto $U(E)$.

Theorem 6: *Let $\mathcal{M}$ be a maximal ideal of B. There exists an ultrafilter $\mathcal{U}$ on E such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, B)$. Moreover, if $\mathcal{U}$ is a cauchy ultrafilter, then $\mathcal{M}$ is of codimension* 1 .

Corollary 6.1: *For every maximal ideal $\mathcal{M}$ of B there exists a unique $\mathcal{H} \in Y_{(\mathcal{T})}(E)$ such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, B)$ $\forall \mathcal{U} \in \mathcal{H}$.*

Moreover, the mapping Ψ that associates to each $\mathcal{M} \in Max(B)$ the unique $\mathcal{H} \in Y_{(\mathcal{T})}(E)$ such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, B)$ $\forall \mathcal{U} \in \mathcal{H}$, is a bijection from $Max(B)$ onto $Y_{(\mathcal{T})}(E)$.

Theorem 7: *Let K be a locally compact field. Then every maximal ideal of A (resp. B) is of codimension 1.*

The Banashewski compactification of E is directly linked to $Max(A)$.

Definition and notation: Let $\mathcal{B}(E)$ be the boolean ring of clopens provided with the classical laws Δ (the symmetrical difference taking place of the addition) and $\cap$ (taking place of the multiplication).

We will denote by $\Sigma(E)$ the set of homomorphisms from $\mathcal{B}(E)$ to $\mathbb{F}_2$: $\Sigma(E)$ is also called *the Stone space of $\mathcal{B}(E)$* and is provided with the topology of simple convergence, while $\mathbb{F}_2$ is provided with the discrete topology, so $\Sigma(E)$ is compact in the compact space $\mathbb{F}_2^{\mathcal{B}(E)}$.

Given $a \in E$, we denote by ζ_a the ring homomorphism from $\mathcal{B}(E)$ to $\mathbb{F}_2$ defined as $\zeta_a(O) = 1\ \forall O \in \mathcal{B}(E)$ such that $a \in O$ and $\zeta_a(O) = 0\ \forall O \in \mathcal{B}(E)$ such that $a \notin O$.

We will denote by $\Sigma_E(E)$ the set of the $\zeta_a,\ a \in E$.

We know the following proposition [15]:

Proposition C : *There is a natural bijection between $\Sigma(E)$ and $Max(A)$. Moreover, $\Sigma(E)$ is compact and $\Sigma_E(E)$ is dense in $\Sigma(E)$.*

Here we can describe more precisely this bijection thanks to the following theorem:

Theorem 8 : *Let $\mathcal{U}, \mathcal{V}$ be sticked ultrafilters on E. Then a clopen belongs to $\mathcal{U}$ if and only if it belongs to $\mathcal{V}$.*

Corollary 8.1 *For every maximal ideal $\mathcal{M}$ of A and $\mathcal{U}, \mathcal{V} \in \Phi(\mathcal{M})$, then a clopen belongs to $\mathcal{U}$ if and only if it belongs to $\mathcal{V}$.*

Notation: Given $\mathcal{M} \in Max(A)$ we will denote by $\Xi(\mathcal{M})$ the mapping from $\mathcal{B}(E)$ to $\mathbb{F}_2$ defined as $\Xi(\mathcal{M})(O) = 1$ whenever any $\mathcal{U} \in \Phi(\mathcal{M})$ is secant with O and $\Xi(\mathcal{M})(O) = 0$ whenever any $\mathcal{U} \in \Phi(\mathcal{M})$ is not secant with O.

Theorem 9: *Given $\mathcal{M} \in Max(A)$, $\Xi(\mathcal{M})$ is a ring homomorphism from $\mathcal{B}(E)$ onto $\mathbb{F}_2$.*

Theorem 10: *Ξ is a bijection from $Max(A)$ onto $\Sigma(E)$. Given $a \in E$ then $\Xi(\mathcal{I}(a, A))$ is ζ_a defined above. The restriction of Ξ to $Max_E(A)$ is a bijection from $Max_E(A)$ onto $\Sigma_E(E)$ and $\Sigma_E(E)$ is dense in $\Sigma(E)$.*

Definition: $\Sigma(E)$ is called *the Banaschewski compactification of E.*

We will now examine prime closed ideals of A and B.

Theorem 11: *Let $\mathcal{U}$ be an ultrafilter on E and let $\mathcal{P}$ be a prime ideal included in $\mathcal{I}(\mathcal{U}, A)$ (resp. $\mathcal{I}(\mathcal{U}, B)$). Let $L \in \mathcal{U}$ be a clopen (resp. let $L \in \mathcal{U}$ be uniformly open) and let $H = E \setminus L$). Let u be the function defined on E by $u(x) = 1\ \forall x \in H,\ u(x) = 0\ \forall x \in L$. Then u belongs to $\mathcal{P}$.*

Corollary 11.1: *Let $\mathcal{U}$ be an ultrafilter on E and let $\mathcal{I}^{**}(\mathcal{U}, A)$ the ideal of the $f \in A$ such that there exists a clopen $H \in \mathcal{U}$ such that $f(x) = 0\ \forall x \in H$. Then $\mathcal{I}^{**}(\mathcal{U}, A)$ is included in every prime ideal $\mathcal{P} \subset \mathcal{I}(\mathcal{U}, A)$.*

Corollary 11.2: *Let $\mathcal{U}$ be an ultrafilter on E and let T be the Banach K-algebra of all bounded functions on E. Then $\mathcal{I}^*(\mathcal{U}, T)$ is the smallest prime ideal among all prime ideals $\mathcal{P} \subset \mathcal{I}(\mathcal{U}, T)$.*

Corollary 11.3: *Let $\mathcal{U}$ be an ultrafilter on E and let $\mathcal{I}^{***}(\mathcal{U}, B)$ the ideal of the $f \in B$ such that there exists a uniformly open subset $H \in \mathcal{U}$ such that $f(x) = 0\ \forall x \in H$. Then $\mathcal{I}^{***}(\mathcal{U}, B)$ is included in every prime ideal $\mathcal{P} \subset \mathcal{I}(\mathcal{U}, B)$.*

Theorem 12: *The closure of a prime ideal of A (resp. of B) is a maximal ideal.*

Corollary 12.1: *Let $\mathcal{P}$ be a prime ideal of A (resp. of B). There exists a unique maximal ideal $\mathcal{M}$ of A (resp. of B) containing $\mathcal{P}$.*

Corollary 12.2: *Every prime closed ideal of A (resp. of B) is a maximal ideal.*

Now, since the kernel of a continuous multiplicative semi-norm is a closed prime ideal, we will show Corollary 12.3:

Notation and definition: Let T be a normed commutative K-agebra with unity. We denote by $Mult(T, \| \ . \ \|)$ the set of multiplicative semi-norms of T provided with the topology of simple convergence. Given $\phi \in Mult(T, \| \ . \ \|)$, the set of the $x \in T$ such that $\phi(x) = 0$ is a closed prime ideal and is called *the kernel of ϕ*. It is denoted by $Ker(\phi)$.

We denote by $Mult_m(T, \| \ . \ \|)$ the set of multiplicative semi-norms of T whose kernel is a maximal ideal and by $Mult_1(T, \| \ . \ \|)$ the set of multiplicative semi-norms of T whose kernel is a maximal ideal of codimension 1.

Suppose now T is a K-algebra of bounded functions from E to K normed by the norm of uniform convrergence on E. Let $a \in E$. The mapping φ_a from T to $\mathbb{R}$ defined by $\varphi_a(f) = |f(a)|$ belongs to $Mult(T, \| \ . \ \|)$.

Let $\mathcal{U}$ be an ultrafilter on E. By Urysohn's Theorem, given $f \in T$, the mapping from E to $\mathbb{R}$ that sends x to $|f(x)|$ admits a limit along $\mathcal{U}$. We set $\varphi_{\mathcal{U}}(f) = \lim_{\mathcal{U}} |(x)|$.

Propositions D, E below are immediate and well known:

Proposition D: *Let $T = A$ or B and let $a \in E$. Then $\mathcal{I}(a, T)$ is a maximal ideal of T of codimension 1 and φ_a belongs to $Mult_1(T, \| \ . \ \|)$.*

Notation: Let $T = A$ or B. We denote by $Mult_E(T, \| \ . \ \|)$ the set of multiplicative semi-norms of T of the form $\varphi_a, \ a \in E$. Consequently, by definition, $Mult_E(T, \| \ . \ \|)$ is a subset of $Mult_1(T, \| \ . \ \|)$.

Proposition E is immediate:

Proposition E : *Let $T = A$ or B and let $a \in E$ and let $\mathcal{U}$ be an ultrafilter on E. Then $\varphi_{\mathcal{U}}$ belongs to the closure of $Mult_E(T, \| \ . \ \|)$.*

Now, Corollaries 12.3 is an immediate consequence of Theorem 12 and Propositions D, E:

Corollary 12.3 : *$Mult(A, \| \ . \ \|) = Mult_m(A, \| \ . \ \|)$, $Mult(B, \| \ . \ \|) = Mult_m(B, \| \ . \ \|)$. Further, if K is locally compact then $Mult(A, \| \ . \ \|) = Mult_1(A, \| \ . \ \|)$, $Mult(B, \| \ . \ \|) = Mult_1(B, \| \ . \ \|)$.*

Remark 8: Suppose K is locally compact and E is a disk in an algebraically closed complete ultrametric field. There do exist ultrafilters that do not converge. Let $\mathcal{U}$ be such an ultrafilter. Then $\varphi_{\mathcal{U}}$ belongs to $Mult_1(B, \| \, . \, \|)$ but does not belong to $Mult_E(B, \| \, . \, \|)$.

Remark 9: In $\mathcal{H} \in Y_{(\mathcal{S})}(E)$, the various ultrafilters $\mathcal{U} \in \mathcal{H} \in Y_{(\mathcal{S})}(E)$ define various prime ideals of A. It is not clear whether these ideals are minimal among the set of prime ideals of A. Similarly, in $\mathcal{H} \in Y_{(\mathcal{T})}(E)$ the various ultrafilters $\mathcal{U} \in \mathcal{H} \in Y_{(\mathcal{T})}(E)$ define various prime ideals of B and it is not clear whether these idals are minimal among the set of prime ideals of B.

Remark 10: The ideal $\mathcal{I}^{**}(\mathcal{U}, A)$ is not a prime ideal of A, as the following example shows.

Suppose E is the disk $d(0,1)$ in the field K and let (a_n) be a sequence of limit 0 such that $|a_n| < |a_{n+1}|$. Let $\mathcal{U}$ be an ultrafilter of limit 0. Let $r_n = |a_n|$, $n \in \mathbb{N}$, let $H = \bigcup_{n=0}^{\infty} d(a_n, r_n^-)$ and let $H' = E \setminus H$. Let $f(x) = x \; \forall \, x \in H$, $f(x) = 0 \; \forall \, x \in H'$ and let $g(x) = x - f(x)$. Then $f(x)g(x) = 0 \; \forall x \in E$. However, neither f nor g is identically zero on any clopen belonging to $\mathcal{U}$ because such an clopen must contain the origin that is on the boundary of both H and H'.

Similarly, $\mathcal{I}^{***}(\mathcal{U}, B)$ is not a prime ideal of B.

Theorem 13: *$\Phi \circ \Xi^{-1}$ is a homeomorphism from $\Sigma(E)$ onto $Mult(A, \| \, . \, \|)$.*

Corollary 13.1: *The topology of $Mult(A, \| \, . \, \|)$ and this of the Banaschewski compactification induce the same topology on $Max(A)$.*

Corollary 13.2: *The topology of $Mult(A, \| \, . \, \|)$ does not depend on the field K.*

Remark 11: Let F be a set compact for two topologies, admitting a subset E dense for both topologies. In general, we may not conclude that the two topologies are identical, as shows the following example.

Let $F = [0,1]$ be provided with the topology $\mathcal{N}$ induced by this of $\mathbb{R}$ and let $E =]0,1[$. Now, let $\mathcal{Q}$ be the topology on F defined as follows:
For a$\in E$, a neighborhood of a is a subset of F containing an open interval included in E.
A neighborhood of 0 is a subset of F containing a subset of the form $\{0\} \cup]1-\epsilon, 1[$.
A neighborhood of 1 is a subset of F containing a subset of the form $\{1\} \cup]0, \epsilon[$.

So we have defined $\mathcal{Q}$ a topology on F. Of course, $\mathcal{Q}$ is different from $\mathcal{N}$. Then E is obviously dense in F for $\mathcal{Q}$. Next, we can check that F is compact for $\mathcal{Q}$.

Theorem 14: *Let $T = A$ or B. The topology induced on E by this of $Mult_E(T, \| \, . \, \|)$ is equivalent to the metric topology defined by δ.*

Theorem 15 was proved in [15] for the algebra B. We can find it again for A and B in a different way.

Theorem 15: *Let $T = A$ or B and let $\mathcal{M}$ be a maximal ideal of T. Let T' be the field $\frac{T}{M}$. Let θ be the canonical surjection from T onto T'. Given any ultrafilter $\mathcal{U}$ such that $\mathcal{I}(\mathcal{U}, T) = \mathcal{M}$, the quotient norm $\| \ . \ \|'$ on T' is defined by $\|\theta(f)\|' = \lim_{\mathcal{U}} |f(s)|$ and hence is multiplicative.*

Definition: Recall that given a commutative Banach K-algebra with unity T, every maximal ideal of T is the kernel of at least one continuous multiplicative semi-norm [4]. T is said to be *multbijective* if every maximal ideal is the kernel of only one continuous multiplicative semi-norm.

Remark 12: There exist ultrametric Banach K-algebras that are not multbijective [2], [4], [5].

Theorem 16: *A, B are multbijective.*

Corollary 16.1: *The K-algebra of all bounded functions from a set X to K is multbijective.*

By Corollaries 5.2, 5.5 and Theorem 16, we have Corollary 16.2:

Corollary 16.2: *For every $\phi \in Mult(A, \| \ . \ \|)$, there exists an ultrafilter $\mathcal{U}$ on E such that $\phi(f) = \lim_{\mathcal{U}} |f(x)| \ \forall f \in A$.*

Moreover, the mapping $\widetilde{\Phi}$ that associates to each $\phi \in Mult(A, \| \ . \ \|)$ the unique $\mathcal{H} \in Y_{(\mathcal{S})}(E)$ such that $\phi(f) = \lim_{\mathcal{U}} |f(x)| \ \forall f \in A, \ \forall \mathcal{U} \in \mathcal{H}$, is a bijection from $Mult(A, \| \ . \ \|)$ onto $Y_{(\mathcal{S})}(E)$.

Corollary 16.3: *Let T be the Banach algebra of all bounded functions from E to K. For every $\phi \in Mult(T, \| \ . \ \|)$ there exists a unique ultrafilter $\mathcal{U}$ on F such that $\phi(f) = \lim_{\mathcal{U}} |f(x)| \ \forall f \in T$. The mapping $\widetilde{\Phi}$ that associates to each $\phi \in Mult(T, \| \ . \ \|)$ the unique ultrafilter $\mathcal{U}$ such that $\phi(f) = \lim_{\mathcal{U}} |f(x)| \ \forall f \in T$, is a bijection from $Mult(T, \| \ . \ \|)$ onto the set of ultrafilters on F.*

Corollary 16.4: *For every $\phi \in Mult(B, \| \ . \ \|)$ there exists a unique $\mathcal{H} \in Y_{(\mathcal{T})}(E)$ such that $\phi(f) = \lim_{\mathcal{U}} |f(x)| \ \forall f \in B, \ \forall \mathcal{U} \in \mathcal{H}$.*

Moreover, the mapping $\widetilde{\Psi}$ that associates to each $\phi \in Mult(B, \| \ . \ \|)$ the unique $\mathcal{H} \in Y_{(\mathcal{T})}(E)$ such that $\phi(f) = \lim_{\mathcal{U}} |f(x)| \ \forall f \in B, \ \forall \mathcal{U} \in \mathcal{H}$, is a bijection from $Mult(B, \| \ . \ \|)$ onto $Y_{(\mathcal{T})}(E)$.

Remark 13: Consider two ultrafilters $\mathcal{U}, \mathcal{V}$ which are contiguous but not sticked. They define the same maximal ideal and the same multiplicative semi-norm on B, but not on A. This means that for every bounded uniformly continuous fonction f from E to K, we have

$\lim\limits_{\mathcal{U}} |f(x)| = \lim\limits_{\mathcal{V}} |f(x)|$. But there exist bounded continuous functions g from E to K such that $\lim\limits_{\mathcal{U}} |g(x)| \neq \lim\limits_{\mathcal{V}} |g(x)|$. Actually, by Theorem 1, we can find a bounded continuous fonction u such that $\lim\limits_{\mathcal{U}} |u(x)| = 1,\ \lim\limits_{\mathcal{V}} |u(x) = 0$.

Now, by Propositions D and E, we have Corollary 16.5:

Corollary 16.5: *$Mult_E(A, \| \ . \ \|)$ is dense in $Mult(A, \| \ . \ \|)$, $Mult_E(B, \| \ . \ \|)$ is dense in $Mult(B, \| \ . \ \|)$.*

Remark 14: In [8], it is showed that in the algebra of bounded analytic functions in the open unit disk of a complete ultrametric algebraically closed field, any maximal ideal which is not defined by a point of the open unit disk is of infinite codimension. Here, we may ask whether the same holds. In the general case no answer is obvious. We can only answer a particular case:

Theorem 17: *Suppose K is algebraically closed. Let $\mathcal{U}$ be an ultrafilter on K and let $P \in K[x],\ P \neq 0$ satisfy $\lim\limits_{\mathcal{U}} P(x) = 0$. Then $\mathcal{U}$ is a principal ultrafilter.*

As a consequence, we have Theorem 18:

Theorem 18: *Suppose K is algebraically closed. Let F be a closed bounded subset of K with infinitely many points and let $\mathcal{M}$ be a maximal ideal of A (resp. B) which is not principal. Then $\mathcal{M}$ is of infinite codimension.*

Remark 15: Suppose K is algebraically closed and let $E = K$. Then the algebras $A,\ B$ contain no polynomial. In such a case, it is not clear whether maximal ideals not defined by points of K are of infinite codimension.

Remark 15: Concerning uniformly continuous functions, it has been shown that two ultrafilters that are not contiguous define two distinct continuous multiplicative semi-norms.

Now, concerning bounded analytic functions inside the disk $F = \{x \in K\ |x| < 1\}$, in [9], it was shown that the same property holds for a large set of ultrafilters on F. However, the question remains whether it holds for all ultrafilters on F.

Let us recall some results on the Shilov boundary of an ultrametric normed algebra:

Proposition F [5], [6]: *Let T be a normed K-algebra whose norm is $\| \ . \ \|$. For each $x \in T$, let $\|x\|_{si} = \lim\limits_{n\to\infty} \|x^n\|^{\frac{1}{n}}$. Then $\| \ . \ \|_{si}$ is power multiplicative semi-norm on T.*

Definitions: Let T be a normed K-algebra whose norm is $\| \ . \ \|$. We call *spectral semi-norm of T* the semi-norm defined by Proposition F.

We call *Shilov boundary of* T a closed subset S of $Mult(T, \| \, . \, \|)$ that is minimum with respect to inclusion, such that, for every $x \in T$, there exists $\phi \in S$ such that $\phi(x) = \|x\|_{si}$.

Proposition G [5], [7] : *Every normed K-algebra admits a Shilov boundary.*

Theorem 19: *The Shilov boundary S of A (resp. B) is equal to $Mult(A, \| \, . \, \|)$ (resp. $Mult(B, \| \, . \, \|)$).*

The Proofs:

Lemma 1 is classical due to the ultrametric distance of E:

Lemma 1: *For every $r > 0$, E admits a partition of the form $(d(a_i, r^-))_{i \in I}$.*

Definition and notation: A function f from E to K will be said to be *uniformly locally constant* if there exists $r > 0$ such that for every $a \in E$, $f(x)$ is constant in $d(a, r^-)$.

Lemma 2: *The set of bounded uniformly locally constant functions from E to K is a K-subalgebra of B and is dense in B*

Proof: It is obvious that S is a K-algebra and is included in B. We will check that S is dense in B. Let $f \in B$ and let ϵ be > 0. There exists $r > 0$ such that $|f(x) - f(y)| \leq \epsilon$ for all $x, \ y \in E$ such that $\delta(x, y) \leq \epsilon$. Now, by Lemma 1, E admits a partition of the form $(d(a_i, r^-))_{i \in I}$. Let h be the function defined by $h(x) = f(a_i) \ \forall x \in d(a_i, r^-)$. Clearly, $\|f - h\| \leq \epsilon$.

Remark 16: Lemma 2 suggests that in our general study, we can't find an interesting complete subalgebra of B.

Notation: We will denote by $| \, . \, |_\infty$ the Archimedean absolute value of $\mathbb{R}$.

Lemma 3: *Let $m, \ M \in \mathbb{R}_+^*$ and let $f \in A$. Then the sets $H = \{x \in E \mid \, | \, |f(x)| - m|_\infty \geq M\}, L = \{x \in E \mid \, | \, |f(x)| - m|_\infty \leq M\}$, are clopen. Moreover, if $f \in B$, then H, L are uniformly open.*

Lemma 4: *Let H be a clopen. Then the characteristic function u of H belongs to A. Moreover, if H is uniformly open, then u belongs to B.*

Given a bounded function in the set E, $|f(x)|$ obviously takes values in a compact of $\mathbb{R}$, therefore the following Lemma 5 comes from Urysohn's Theorem [1].

Lemma 5: *Let $\mathcal{U}$ be an ultrafilter on E. Let f be a bounded function from E to K. The function $|f|$ from E to $\mathbb{R}_+$ defined as $|f|(x) = |f(x)|$ admits a limit along $\mathcal{U}$. Moreover, if K is locally compact, then $f(x)$ admits a limit along $\mathcal{U}$.*

Lemma 6: *Let $\mathcal{U}, \mathcal{V}$ be sticked (resp. contiguous) ultrafilters on E and let $f \in A$ (resp. let $f \in B$). Then $\lim\limits_{\mathcal{U}} |f(x)| = \lim\limits_{\mathcal{V}} |f(x)|$.*

Lemma 7 is immediate:

Lemma 7: *Let $f \in B$ and let $\widetilde{E}$ be the completion of E. Then f has continuation to a function $\widetilde{f}$ uniformly continuous on $\widetilde{E}$.*

Proof of Theorem 1: If $\mathcal{U}, \mathcal{V}$ are sticked, then by definition, given a clopen $H \in \mathcal{U}$ and a clopen $L \in \mathcal{V}$ we have $H \cap L \neq \emptyset$.

Now, suppose that two ultrafilters $\mathcal{U}, \mathcal{V}$ are not sticked. We can find closed subsets $F \in \mathcal{U},\ G \in \mathcal{V}$ of E such that $F \cap G = \emptyset$. For each $x \in F$, let $r(x)$ be the distance from x to G and let $H = \bigcup_{x \in F} d(x, \big(\frac{1}{2}r(x)\big)^{-})$. So, H is open. Suppose H is not closed and let $(c_n)_{n \in \mathbb{N}}$ be a sequence of H converging to a point $c \in E \setminus H$. Since the distance is ultrametric, each point c_n belongs to a ball $d(a_n, r(a_n)^{-})$ with $a_n \in F$. Suppose the sequence $(r(a_n))_{n \in \mathbb{N}}$ does not tend to 0. There exists a subsequence $(r(a_{q(m)}))_{m \in \mathbb{N}}$ and $s > 0$ such that $r(a_{q(m)}) \geq s\ \forall m \in \mathbb{N}$ and consequently, c belongs to one of the balls $d(a_{q(m)}, \big(\frac{1}{2}r(a_{q(m)})\big)^{-})$, a contradiction. Thus, the sequence $(r(a_n))_{n \in \mathbb{N}}$ must tend to 0. But since a_n belongs to F and since F is closed, clearly c lies in F, a contradiction. Thus, H is a clopen . By Construction, H belongs to $\mathcal{U}$ and satisfies $H \cap G = \emptyset$, hence H does not belong to $\mathcal{V}$. Now, let $L = E \setminus H$. Then L also is a clopen that does not belong to $\mathcal{U}$. But since $\mathcal{V}$ is an ultrafilter that is not secant with H, it is secant with L and hence L belongs to $\mathcal{V}$, which ends the proof.

Proof of Theorem 2: Since $\mathcal{U}, \mathcal{V}$ are not sticked, by Theorem 1 we can find clopens $H \in \mathcal{U},\ L \in \mathcal{V}$ of E such that $H \cap L = \emptyset$. Then the set $H' = E \setminus H$ also is a clopen. Let u be the characteristic function of H. Since H and H' are open, u is continuous, which ends the proof.

Proof of Theorem 3: Since $\mathcal{U}$ and $\mathcal{V}$ are not contiguous, there exist $H \in \mathcal{U},\ L \in \mathcal{V}$ such that $\delta(H, L) = \mu > 0$. Let $H' = \{x \in E \mid \delta(x, H) \leq \frac{\mu}{2}\}$. Then H' is a clopen containing H and by ultrametricity, we can check that $\delta(H', L) \geq \mu$. Let u be the function defined in E by $u(x) = 1\ \forall x \in H'$ and $f(x) = 0\ \forall x \in E \setminus H'$. Since u is constant in any ball of diameter $\frac{\mu}{2}$, u belongs to B.

Proof of Proposition A: It is obvious and well known that $\mathcal{I}(\mathcal{U}, T)$ is prime. Let us check that so is $\mathcal{I}^*(\mathcal{U}, T)$. Suppose $\mathcal{I}^*(\mathcal{U}, T)$ is not prime. There exists $f,\ g \notin \mathcal{I}^*(\mathcal{U}, T)$ such that $fg \in \mathcal{I}^*(\mathcal{U}, T)$. Thus, there exists $L \in \mathcal{U}$ such that $f(x)g(x) = 0\ \forall x \in L$, but neither f nor g are identically zero on L. Let F be the set of the $x \in L$ such that $f(x) = 0$ and let G be the set of the $x \in L$ such that $g(x) = 0$. Then $F \cup G = L$, hence $\mathcal{U}$ is secant at least with one of the two sets F and G. Suppose it is secant with F. The intersection of $\mathcal{U}$ with F is a filter thinner than $\mathcal{U}$, hence it is $\mathcal{U}$. Thus, f is identically zero on a set $F \in \mathcal{U}$, a contradiction. And similarly if it is secant with G.

Proof of Theorem 4: First, if $\mathcal{U}$ and $\mathcal{V}$ are not sticked, by Theorem 2 we have $\mathcal{I}(\mathcal{F}, A) \neq \mathcal{I}(\mathcal{G}, A)$.

Now, suppose that $\mathcal{U},\ \mathcal{V}$ are sticked and let $f \in \mathcal{I}(\mathcal{U}, A)$. Let ϵ be > 0 and let $H \in \mathcal{U}$ be such that $|f(x)| \leq \epsilon\ \forall x \in H$. Since $|f|$ has a limit l along $\mathcal{V}$, we can find $L \in \mathcal{V}$ such that $|f(x) - l| \leq \epsilon\ \forall x \in L$. Since f is continuous, it satisfies $|f(x)| \leq \epsilon\ \forall x \in \overline{H}$ and $|f(x) - l| \leq \epsilon\ \forall x \in \overline{L}$. But since $\mathcal{U},\ \mathcal{V}$ are sticked, there exists $a \in \overline{H} \cap \overline{L}$, hence $l \leq 2\epsilon$. And since ϵ is arbitrary, then $l = 0$ and hence $f \in \mathcal{I}(\mathcal{V}, A)$. Thus, $\mathcal{I}(\mathcal{U}, A) \subset \mathcal{I}(\mathcal{V}, A)$. And symmetrically, we have $\mathcal{I}(\mathcal{V}, A) \subset \mathcal{I}(\mathcal{U}, A)$, hence the two ideals are equal.

Suppose now that $\mathcal{U},\ \mathcal{V}$ are not contiguous. By Theorem 3, there exist $H \in \mathcal{U},\ L \in \mathcal{V}$ and $f \in B$ such that $f(x) = 1\ \forall x \in H,\ f(x) = 0\ \forall x \in L$. Consequently, f belongs to $\mathcal{I}(\mathcal{U}, B)$ but does not belong to $\mathcal{I}(\mathcal{V}, B)$. Thus, $\mathcal{I}(\mathcal{F}, B) \neq \mathcal{I}(\mathcal{G}, B)$.

Finally, suppose that $\mathcal{U},\ \mathcal{V}$ are contiguous. Let $f \in \mathcal{I}(\mathcal{U}, B)$. Let $l = \lim\limits_{\mathcal{V}} |f(x)|$, suppose $l > 0$ and let $L \in \mathcal{V}$ be such that $|\ |f(x)| - l|_\infty \leq \dfrac{l}{3}\ \forall x \in L$, hence $|f(x)| \geq \dfrac{2l}{3}\ \forall x \in L$. Let $H \in \mathcal{U}$ be such that $|f(x)| \leq \dfrac{l}{3}\ \forall x \in H$. Since $f \in B$, there exists $\rho > 0$ such that $\delta(x, y) \leq \rho$ implies $|f(x) - f(y)| \leq \dfrac{l}{4}$. And since f is uniformly continuous, there exist $a \in H,\ b \in L$ such that $\delta(a, b)| \leq \rho$, hence $|f(a) - f(b)| \leq \dfrac{l}{4}$, a contradiction because $|f(a)| \leq \dfrac{l}{3}$ and $|f(b)| \geq \dfrac{2l}{3}$.

Proof of Proposition B: Suppose that $\mathcal{U}$ converges to a point a. Given $f \in A$ (resp. $f \in B$), we have $\lim\limits_{\mathcal{U}} f(x) = f(a)$. So, the mapping θ from A (resp. B) to K defined as $\theta(f) = f(a)$ admits $\mathcal{M}$ for kernel and hence $\mathcal{M}$ is of codimension 1.

Proof of Theorem 5: Let $M = \inf_{x \in E}(\max_{1 \leq j \leq q} |f_j(x)|)$. Let $E_j = \{x \in E \mid |f_j(x)| \geq M\},\ j = 1, ..., q$ and let $F_j = \bigcup\limits_{m=1}^{j} E_m,\ j = 1, ..., q$. Let $g_1(x) = \frac{1}{f_1(x)} \forall x \in E_1$ and $g_1(x) = 0\ \forall x \in E \setminus E_1$. Since $|f_1(x)| \geq M\ \forall x \in E_1$, $|g_1(x)|$ is clearly bounded.

Suppose first $f_1, ..., f_q \in A$. Since E is ultrametric, each E_j is obviously a clopen and so is each F_j. And since f_1 is continuous g_1 is continuous, hence belongs to A.

Now, suppose that $f_1, ..., f_q \in B$. By Lemma 3, E_1 has a strictly positive codiameter ρ and so does $E \setminus E_1$. Then g_1 is obviously uniformly continuous in $E \setminus E_1$. And, since $|f_1(x)| \geq M\ \forall x \in E_1$, g_1 is uniformly continuous in E_1. Hence it is uniformly continuous in E. Thus g_1 belongs to B.

Suppose now we have constructed $g_1, ..., g_k \in A$ (resp. $g_1, ..., g_k \in B$) satisfying $\sum\limits_{j=1}^{k} f_j g_j(x) = 1\ \forall x \in F_k$ and $\sum\limits_{j=1}^{k} f_j g_j(x) = 0\ \forall x \in E \setminus F_k$. Let g_{k+1} be defined on E by $g_{k+1}(x) = \dfrac{1}{f_{k+1}(x)}\ \forall x \in F_{k+1} \setminus F_k$ and $g_{k+1}(x) = 0\ \forall x \in E \setminus (F_{k+1} \setminus F_k)$. Then g_{k+1} is bounded.

Now we can check that $\sum\limits_{j=1}^{k+1} f_j g_j(x) = 1\ \forall x \in F_{k+1}$ and $\sum\limits_{j=1}^{k} f_j g_j(x) = 0\ \forall x \in E \setminus F_{k+1}$.
So, by an immediate recurrence, we can get bounded functions $g_1, ..., g_q$ such that

$\sum_{j=1}^{q} f_j g_j(x) = 1 \ \forall x \in E.$

Now suppose that $f_1, ..., f_q \in A$. Since F_k and F_{k+1} are clopens, so is $E \setminus (F_{k+1} \setminus F_k)$ and consequently, g_{k+1} is continuous. Similarly as for g_1, since $|f_{k+1}(x)| \geq M \ \forall x \in E_{k+1}$, $|g_{k+1}(x)|$ is clearly bounded, hence belongs to A. And similarly, if $g_1, ..., g_k \in B$, g_{k+1} belongs to B for the same reason as g_1 above. So, by induction, we can get $g_1, ..., g_q \in B$ such that $\sum_{j=1}^{q} f_j g_j(x) = 1 \ \forall x \in E$.

Proof of Theorem 6: By Theorem 5, there exists an ultrafilter $\mathcal{U}$ on E such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, B)$. Now, suppose that $\mathcal{M}$ is a Cauchy ultrafilter. Since the functions of B are uniformly continuous, by Lemma 7 they have continuation to the completion $\widetilde{E}$ of E and $\mathcal{U}$ defines an ultrafilter that converges in $\widetilde{E}$ to a point a. Given $f \in B$, let $\widetilde{f}$ be the continuation of f in $\widetilde{E}$: we have $\lim\limits_{\mathcal{U}} f(x) = \widetilde{f}(a)$. So by Proposition B $\mathcal{M}$ is of codimension 1.

Proof of Theorem 7: Let $\mathcal{M}$ be a maximal ideal of A (resp. B). By Corollary 4.2 there exists an ultrafilter $\mathcal{U}$ such that $\mathcal{M} = \mathcal{I}(\mathcal{U}, A)$ (resp. $\mathcal{M} = \mathcal{I}(\mathcal{U}, B)$). Let $f \in A$ (resp. $f \in B$). By Lemma 2, the function f has a limit $\chi(f)$ along $\mathcal{U}$. Thus, the mapping χ from A (resp. B) to K is a K-algebra homomorphism and therefore $\mathcal{M}$ is of codimension 1.

Proof of Theorem 8: Let $\mathcal{U}, \mathcal{V}$ be sticked ultrafilters and let O be a clopen that belongs to $\mathcal{U}$. Suppose it does not belong to $\mathcal{V}$. Then $\mathcal{V}$ is secant with $E \setminus O$. But since $\mathcal{V}$ is an ultrafilter, $E \setminus O$ belongs to $\mathcal{V}$. But $E \setminus O$ is a clopen, hence it has a non-empty intersection with O (because $\mathcal{U}$ and $\mathcal{V}$ are sticked), a contradiction. Thus, O belongs to $\mathcal{V}$, which proves Theorem 8.

Lemma 8 : *Let O be a clopen and let $\mathcal{U}$ be an ultrafilter that is not secant with O. There exists a clopen L that belongs to $\mathcal{U}$ and satisfies $L \cap O = \emptyset$.*

Proof : Let H be a clopen that belongs to $\mathcal{U}$ and let $L = H \setminus O$. Since $\mathcal{U}$ is not secant with O, it is secant with L. But since both H, O are clopen, so is L. And by definition, $L \cap O = \emptyset$.

Proof of Theorem 9: Let $\mathcal{U} \in \Phi(\mathcal{M})$. Let O_1, O_2 be clopen and set $\theta = \Xi(\mathcal{M})$. We first have to check that $\theta(O_1 \Delta O_2) = \theta(O_1) + \theta(O_2)$ in $\mathbb{F}_2$. Let $\mathcal{U} \in \Phi(\mathcal{M})$.

If O_1 belongs to $\mathcal{U}$ and if $O_2 \notin \mathcal{U}$, the conclusion is immediate. Similarly, so is it whenever $O_1 \notin \mathcal{U}$ and $O_2 \notin \mathcal{U}$. Now, consider the case when $O_1 \in \mathcal{U}$ and $O_2 \in \mathcal{U}$.

Then $O_1 \cap O_2$ belongs to $\mathcal{U}$. But since $\mathcal{U}$ is an ultrafilter, it cannot be secant with $(O_1 \cup O_2) \setminus (O_1 \cap O_2)$. Consequently, $\theta(O_1 \Delta O_2) = 0$. So we have checked that $\theta(O_1 \Delta O_2) = \theta(O_1) + \theta(O_2)$.

Concerning $\theta(O_1 \cap O_2)$, clearly, $\theta(O_1 \cap O_2) = 1$ if and only if both O_1, O belong to $\mathcal{U}$ i.e. $\theta(O_1) = \theta(O_2) = 1$, hence $\theta(O_1 \cap O_2) = \theta(O_1)\theta(O_2)$. This finishes proving that θ is a ring homomorpism.

Proof of Theorem 10: Let us check that Ξ is injective. Suppose $\mathcal{M}_1,\ \mathcal{M}_2$ are two distinct maximal ideals such that $\Xi(\mathcal{M}_1)=\Xi(\mathcal{M}_2)$. Let $\mathcal{U}_1\in\Phi(\mathcal{M}_1),\ \mathcal{U}_2\in\Phi(\mathcal{M}_2)$. Since $\mathcal{U}_1,\ \mathcal{U}_2$ are not sticked, by Theorem 1 there exists a clopen $O\in\mathcal{U}_1$ that does not belong to $\mathcal{U}_2$. Consequently, $\Xi(\mathcal{M}_1)(O)=1,\ \Xi(\mathcal{M}_2)(O)=0$, which proves that Ξ is injective.

Now, let us check that Ξ is surjective. Let $\theta\in\Sigma(E)$. The family of clopens O satisfying $\theta(O)=1$ clearly generates a filter $\mathcal{F}$. Let $\mathcal{U}$ be an ultrafilter thinner than $\mathcal{F}$ and let $\mathcal{M}=\mathcal{I}(\mathcal{U},A)$. We will check that $\theta=\Xi(\mathcal{M})$. Let O be clopen that belongs to $\mathcal{U}$. Then $\mathcal{F}$ is secant with O, hence $\theta(O)=1$. Now, let $\mathcal{V}$ be an ultrafilter sticked to $\mathcal{U}$. By Theorem 8, the clopens that belong to $\mathcal{U}$ are the same as those which belong to $\mathcal{V}$. Consequently, $\theta(O)=\Xi(\mathcal{M})(O)$ for every clopen which belongs to any $\mathcal{V}\in\Phi(\mathcal{M})$.

And now, let O be a clopen that does not belong to any $\mathcal{V}\in\Phi(\mathcal{M})$. Let us take again $\mathcal{U}\in\Phi(\mathcal{M})$. By Lemma 8, there exists a clopen $L\in\mathcal{V}$ such that $O\cap L=\emptyset$. Then $O\Delta L=O\cup L$ belongs to $\mathcal{U}$. Hence $\theta(L\cup O)=1=\theta(L)$ and consequently, $\theta(O)=0$, which finishes proving that $\theta=\Xi(\mathcal{M})$. So, Xi is surjective.

$\Sigma(E)$ is compact because it is closed in $\mathbb{F}_2^{\mathcal{B}(E)}$. By definition, we have $\Xi(\mathcal{I}(a,A))=\zeta_a\ \forall a\in E$. Let us check that $\Sigma_E(E)$ is dense in $\Sigma(E)$. Let $\theta=\Xi(\mathcal{M})$ ($\mathcal{M}\in Max(A)$) and let $O_1,...,O_q$ be clopens. We may assume that $\theta(O_j)=1\ \forall j=1,...,k$ and $\theta(O_j)=0\ \forall j=k+1,...,q$. Let $\mathcal{U}\in\Phi(\mathcal{M})$. Then $\mathcal{U}$ is secant with $O_1,...,O_k$ and is not with $O_{k+1},...,O_q$. Let $a\in\bigcap\limits_{j=1}^{k}O_j$, hence clearly $\theta(0_j)=\zeta_a(O_j)\ \forall j=1,...,q$. This finishes the proof of Theorem 10.

Proof of Theorem 11: We know that $u\in A$ (resp. $u\in B$). By construction, $1-u$ does not belong to $\mathcal{I}(\mathcal{U},A)$ (resp. $\mathcal{I}(\mathcal{U},B)$) because $\lim\limits_{\mathcal{U}}u(x)=1$. But $u(1-u)=0$, hence u belongs to $\mathcal{P}$ because $\mathcal{P}$ is prime.

Proof of Theorem 12: Let $\mathcal{P}$ be a prime ideal of A included in a maximal ideal $\mathcal{M}=\mathcal{I}(\mathcal{U},A)$, (resp. $\mathcal{M}=\mathcal{I}(\mathcal{U},B)$). Let f belong to $\mathcal{I}(\mathcal{U},A)$, (resp. $\mathcal{I}(\mathcal{U},B)$). Let us take $\epsilon>0$ and find $h\in\mathcal{P}$ such that $\|f-h\|\le\epsilon$. By Lemma 3 we can find a clopen $L\in\mathcal{U}$, (resp. a uniformly open subset $L\in\mathcal{U}$) such that $|f(x)|\le\epsilon\ \forall x\in L$. Let u be the characteristic function of $E\setminus L$. By Theorem 11 u belongs to $\mathcal{P}$ and hence so does uf. We then check that $\|f-uf\|\le\epsilon$. Thus, $\mathcal{P}$ is dense in $\mathcal{M}$.

Notation: Let $T=A$ or B. Given $f_1,...,f_q\in T,\ \epsilon>0$, on $Mult_E(T,\|\ .\ \|)$ we will denote by $W(\varphi_a,f_1,...,f_q,\epsilon)$ the set of the φ_x such that $|\ |f_j(x)|-|f_j(a)|\ |_\infty<\epsilon\ \forall j=1,...,q$.

Definitions and notation: Let $T=A$ or B. Given $\phi\in Mult(T,\|\ .\ \|)$ and $f_1,...,f_q\in T,\ \epsilon>0$, we will denote by $W(\phi,f_1,...,f_q,\epsilon)$ the set of the ψ such that $|\ |\phi(f_j)|-|\psi(f_j)|\ |_\infty<\epsilon\ \forall j=1,...,q$. Such neighborhoods of ϕ will be called *basic neighborhoods of ϕ.*

By definition of the topology of simple convergence on $Mult(T,\|\ .\ \|)$ we know that the set of basic neighborhoods of ϕ makes a fundamental system of neighborhoods of ϕ.

Similarly, given $\zeta \in \Sigma(E)$ and clopens $O_1, ..., O_q$ we will denote by $Z(\zeta, O_1, ..., O_q)$ the set of the ξ such that $\zeta(O_j) = \xi(O_j)\ \forall j = 1, ..., q$. Such neighborhoods of ζ will be called *basic neighborhoods of* ζ.

Then by definition of the topology of simple convergence on $\Sigma(E)$, we know that the set of basic neighborhoods of ζ makes a fundamental system of neighborhoods of ζ.

Given $\psi \in Mult(A, \|\ .\ \|)$, we set $\overline{\psi} = \Xi \circ \Phi^{-1}(\psi)$.

Lemma 9: quad *Let $\phi \in Mult(A, \|\ .\ \|)$, let $O_1, ..., O_q$ be clopens and let $\epsilon \in]0, 1[$. Let $D = \bigcap_{j=1}^{q} O_j$ and let u be the characteristic function of D. Then, given $\psi \in Mult(A, \|\ .\ \|)$, ψ belongs to $W(\phi, u, \epsilon)$ if and only if $\overline{\psi}$ belongs to $Z(\overline{\phi}, O_1, ..., O_q)$.*

Proof: Let $\mathcal{U}$ be an ultrafilter such that $Ker(\phi) = \mathcal{I}(\mathcal{U}, A)$. Of course $\phi(u)$ is equal to 0 or 1. Since $\epsilon < 1$, we can see that $\psi(u) = 1$ if and only if $\mathcal{U}$ is secant with D and $\psi(u) = 0$ if and only if $\mathcal{U}$ is not secant with D. But this holds if and only if $\overline{\psi}(O_j) = 1\ \forall j = 1, ..., q$, hence $\overline{\psi}$ belongs to $Z(\overline{\phi}, O_1, ..., O_q)$.

Proof of Theorem 13: Let $\phi \in Mult(A, \|\ .\ \|)$ and consider first a basic neighborhood of $\overline{\phi}$: $Z(\overline{\phi}, O_1, ..., O_q)$. Since $\bigcap_{j=1}^{q} O_j$ is a clopen, its characteristic function u belongs to A. Then by Lemma 9, given $\epsilon \in]0, 1[$, a $\psi \in Mult(A, \|\ .\ \|)$ belongs to $W(\phi, u, \epsilon)$ if and only if $\overline{\psi}$ belongs to $Z(\overline{\phi}, O_1, ..., O_q)$. Consequently, any basic neighborhood of $\overline{\phi}$ is the image of a basic neighborhood of ϕ by the bijection $\Xi \circ \Phi^{-1}$. Therefore, the topology of $Mult(A, \|\ .\ \|)$ is at least as thin as this of $\Sigma(E)$.

Now, conversely, consider a basic neighborhood of ϕ: $W(\phi, f_1, ..., f_q, \epsilon)$ with $\epsilon \in]01[$. The set of the $x \in E$ such that $|\ |f_j(x)| - \phi(f_j)|_\infty \leq \epsilon\ \forall j = 1, ..., q$ is a clopen D hence its characteristic function u belongs to A. Now, let $\zeta \in \Sigma(E)$. It is of the form $\Xi(\mathcal{M})$ where $\mathcal{M}$ is a maximal ideal $\mathcal{I}(\mathcal{U}, A)$ with $\mathcal{U}$ an ultrafilter on E. And then, given any clopen O, we have $\Xi(\mathcal{M})(O) = 1$ if and only if $\mathcal{U}$ is secant with O. Thus, ζ belongs to $Z(\overline{\phi}, D)$ if and only if ultrafilters $\mathcal{U}$ such that $\Xi^{-1}(\zeta) = \mathcal{I}(\mathcal{U}, A)$ are secant with D. Now, suppose that ζ belongs to $Z(\overline{\phi}, D)$. Of course, $Ker(\Phi \circ \Xi^{-1}(\zeta))$ is equal to $\mathcal{I}(\mathcal{U}, A)$. Set $\widehat{\zeta} = \Phi \circ \Xi^{-1}(\zeta)$. Since $\mathcal{U}$ is secant with D, the inequality $|\lim_{\mathcal{U}} |f_j(x)| - \phi(f_j)|_\infty \leq \epsilon$ holds for every $j = 1, ..., q$. Therefore, $\widehat{\zeta}$ belongs to $W(\phi, f_1, ..., f_q, \epsilon)$. This proves that $\Xi \circ \Phi^{-1}(W(\phi, f_1, ..., f_q, \epsilon))$ contains a neighborhood of ζ in $\Sigma(E)$ and finishes proving that the two topologies are equivalent.

Proof of Theorem 14: Let $a \in E$. The filter of neighborhoods of a admits for basis the family of balls $d(a, r^-) = \{x \in E \mid \delta(x, a) \leq r\},\ r > 0$. But we can check that such a ball is induced by a neighborhood of φ_a with respect to both topologies of $Mult_E(A, \|\ .\ \|)$ and $Mult_E(B, \|\ .\ \|)$. Given $\varphi_a \in Mult_E(T, \|\ .\ \|)$, we set $W'(\varphi_a, f_1, ..., f_q, \epsilon) = W(\phi_a, f_1, ..., f_q, \epsilon) \cap Mult_E(T, \|\ .\ \|)$. Let $r \in]0, 1[$. By Lemma 4 there exists $u \in B$ such that $u(x) = 0\ \forall x \in \mathcal{B}(a, r)$ and $u(x) = 1\ \forall x \in E \setminus d(a, r^-)$.

Consequently, $W'(\varphi_a, u, r)$ is the set of the φ_b such that $|b - a| \leq r$. This holds when we consider A as when we consider B and hence the topology of $Mult_E(B, \| \ . \ \|)$ as this of $Mult_E(A, \| \ . \ \|)$ is thinner or equal to the metric topology of E. Now, since each f_j is continuous, the set of the $x \in E$ such that $| \ |f_j(x)| - |f_j(a)| \ |_\infty \leq \epsilon \ \forall j = 1, ..., q$ is open and hence contains a ball $d(a, r^-)$ of E. Consequently, the topology of E is thinner or equal to this of $Mult_E(T, \| \ . \ \|)$, which finishes proving that the topology induced on E by $Mult(T, \| \ . \ \|)$ coincides with the metric topology of E.

Proof of Theorem 15: Let $t \in T'$ and let $f \in T$ be such that $\theta(f) = t$. Let $\mathcal{U}$ be an ultrafilter such that $\mathcal{I}(\mathcal{U}, T) = \mathcal{M}$. So, $\|t\|' \geq \lim_{\mathcal{U}} |f(s)|$. Conversely, let $W \in \mathcal{U}$ be such that $|f(x)| \leq \lim_{\mathcal{U}} |f(s)| + \epsilon \ \forall x \in W$. There exists $f_1, ..., f_q \in \mathcal{M}$ and $\epsilon > 0$ sucht that $\bigcap_{j=1}^{q} D(f_j, \epsilon) \subset W$. Let $X = \bigcap_{j=1}^{q} D(f_j, \epsilon)$.

Suppose $\mathcal{M}$ is a maximal ideal of T. There exists $u \in T$ such that $u(x) = 0 \ \forall x \in X, \ u(x) = 1 \ \forall x \in F \setminus X$. Then $u(1 - u) = 0$. But $1 - u \notin \mathcal{M}$. Hence, u belongs to $\mathcal{M}$. Then $\theta(f - uf) = \theta(f) = t$. But by construction, $(f - uf)(x) = 0 \ \forall x \in F \setminus X$ and $(f - uf)(x) = f(x) \ \forall x \in X$. Consequently, $\|f - uf\| \leq \lim_{\mathcal{U}} |f(s)| + \epsilon$ and therefore $\|t\|' = \|\theta(f - uh)\| \leq \lim_{\mathcal{U}} |f(s)| + \epsilon$. This finishes proving the equality $\|\theta(f)\|' = \lim_{\mathcal{U}} |f(s)|$. Now, such a norm defined as $\|\theta(f)\|' = \lim_{\mathcal{U}} |f(s)|$ is obviously multiplicative. The proof concerning A is exactly similar, the set X being then closed and open by Lemma 3 and the proof concerning B is also similar the set X being uniformly open.

Proof of Theorem 16: Let $\mathcal{M}$ be a maximal ideal of A (resp. of B) and let A' be the field $\frac{A}{\mathcal{M}}$ (resp. let B' be the field $\frac{B}{\mathcal{M}}$). By Theorem 15, the quotient norm of A' (resp. of B') is multiplicative. But then, A' (resp. B') admits only one continuous multiplicative semi-norm: its quotient K-algebra norm. Consequently, A (resp. B) admits only one continuous multiplicative semi-norm whose kernel is $\mathcal{M}$, which proves that A (resp. B) is multbijective.

Proof of Theorem 17: Let $P(x) = \prod_{j=1}^{q} (x - a_j)$. Let $\mathcal{F}$ be the the filter admitting for basis the sets $\Lambda(r) = \bigcup_{j=1}^{q} d(a_j, r)$. Suppose first that $\mathcal{U}$ is not secant with $\mathcal{F}$. There exists $\rho > 0$ and $H \in \mathcal{U}$ such that $\Lambda(\rho) \cap H = \emptyset$. Then $|P(x)| \geq \rho^q \ \forall x \in H$, a contradiction to the hypothesis $\lim_{\mathcal{U}} P(x) = 0$. Consequently, $\mathcal{U}$ is secant with $\mathcal{F}$. Hence it is obviously secant with the filter of neighborhoods of one of the points a_j and therefore, it converges to this point.

Proof of Theorem 18: Indeed, by Theorem 17 the ideal $\mathcal{M}$ contains no polynomials, hence $\frac{A}{\mathcal{M}}$, (resp. $\frac{B}{\mathcal{M}}$) contains a subfield isomorphic to $K(x)$.

Proof of Theorem 19: Given $\psi \in Mult(A, \| \,.\, \|)$, $f_1, ..., f_q \in A$, $\epsilon > 0$, we set

$$V(\psi, f_1, ..., f_q, \epsilon) = \{\phi \in Mult(A, \| \,.\, \|) \mid |\phi(f_j) - \psi(f_j)|_\infty \leq \epsilon \ , j = 1, ..., q\}.$$

By definition of the topology of simple convergence, the filter of neighborhoods of ψ admits for basis the family of sets

$$V(\psi, f_1, ..., f_q, \epsilon), q \in \mathbb{N}^*, \ f_1, ..., f_q \in A, \ \epsilon > 0.$$

Henceforth we take $\epsilon \in]0, \frac{1}{2}[$.

Suppose that the Shilov boundary of A is not equal to $Mult(A, \| \,.\, \|)$ and let $\psi \in Mult(A, \| \,.\, \|) \setminus S$. Since S is a closed subset of $Mult(A, \| \,.\, \|)$, the set $Mult(A, \| \,.\, \|) \setminus S$ is an open subset of $Mult(A, \| \,.\, \|)$ and hence, there exist $f_1, ..., f_q$ such that $V(\psi, f_1, ..., f_q, \epsilon) \subset (Mult(A, \| \,.\, \|) \setminus S)$. Let $L = \{x \in E \mid |\psi(f_j) - |f(_j(x)| \ |_\infty \leq \dfrac{\epsilon}{2}\}$. By Lemma 3 L is a clopen. Consequently, by Lemma 4 the characteristic function u of L belongs to A and obviously satisfies $\psi(u) = 1$. On the other hand, we have $u(x) = 0 \ \forall x \notin L$.

Now, there exists $\theta \in S$ such that $\theta(u) = \|u\| = 1$. Consider the neighborhood $V(\theta, f_1, ..., f_q, u, \frac{\epsilon}{2})$. Since $Mult_1(A, \| \,.\, \|)$ is dense in $Mult(A, \| \,.\, \|)$, particularly, for every $\varphi_a \in V(\theta, f_1, ..., f_q, u, \frac{\epsilon}{2})$ we have $| \ |u(a)| - \theta(u)|_\infty \leq \frac{\epsilon}{2}$, hence $|u(a)| \geq 1 - \frac{\epsilon}{2} > 0$. But since $\theta \in S$, we have $V(\theta, f_1, ..., f_q, \frac{\epsilon}{2}) \cap V(\psi, f_1, ..., f_q, \frac{\epsilon}{2}) = \emptyset$ and so much the more: $V(\theta, f_1, ..., f_q, u, \frac{\epsilon}{2}) \cap V(\psi, f_1, ..., f_q, u, \frac{\epsilon}{2}) = \emptyset$. Let $H = \{a \in E \mid \varphi_a \in V(\theta, f_1, ..., f_q, u, \frac{\epsilon}{2})\}$. Then $H \cap L = \emptyset$. But by definition of u, we have $u(a) = 0 \ \forall a \in H$, a contradiction.

The proof of the statement concerning B is similar. Following the same notation with B in place of A, we only have to remark that by lemma 3 here, L is a uniformly open subset of E. Consequently, by Lemma 4 the characteristic function u of L belongs to B. The end of the proof follows the same way as this for A.

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Alain Escassut
Laboratoire de Mathématiques UMR 6620
Université Blaise Pascal
(Clermont-Ferrand)
Les Cézeaux
63177 AUBIERE CEDEX
FRANCE

Alain.Escassut@math.univ-bpclermont.fr

Nicolas Mainetti
Département GEA Aurillac
IUT GEA Aurillac
100 rue de l'Egalité
15000 Aurillac
FRANCE

Nicolas.Mainetti@iut.u-clermont1.fr