Abstract—This paper addresses the problem of controlling wind energy conversion systems (WECS) which involve permanent magnet synchronous generator (PMSG) fed by IGBT-based buck-to-buck rectifier-inverter. The goal of control is to maximize wind energy extraction and this needs letting the wind turbine rotor operate in variable-speed mode. Interestingly, the present study features the achievement of the above energetic goal without resorting to sensors for wind velocity. The control strategy involves: (i) a non-linear regulator designed by the backstepping technique; (ii) a sensorless online reference-speed optimizer designed using the turbine power characteristic to achieve the maximum power point tracking (MPPT) requirement. It is formally shown that the proposed controller actually meets its control objectives. This theoretical result is confirmed by several simulations.

I. INTRODUCTION

Most wind turbines currently in operation are constrained to work at constant speeds (with a tolerable variation of 1-2%) despite wind speed variations. In such operation conditions, the aerodynamic performance is optimal (in the sense of maximum wind energy extraction) only for one wind speed value depending on the considered value of turbine speed. Achieving maximum wind energy extraction in presence of varying wind speed conditions needs a varying turbine speed operation mode. Specifically, the turbine rotor velocity must be controlled so that its power-speed working point is constantly maintained near the optimal position (Fig. 4). This control objective is commonly referred to ‘maximum power point tracking (MPPT)’ and its achievement guarantees optimal aerodynamic efficiency.

In this paper, the focus is made on the wind energy conversion system Fig I. This includes a permanent magnet synchronous generator (PMSG) that converts wind turbine power to output voltage whose amplitude and frequency vary with wind speed. PMSG generators offer several benefits, in wind power applications due to their high power density, high efficiency (as the copper losses in the rotor disappear), and absence of gearbox and reduced active weight. These features make it possible to achieve with PMSG’s high varying speed control performance and highly reliable operation conditions (reduced need for maintenance). In the light of the above discussion, the global system including the wind turbine, the PMSG and the AC/DC/AC power converter has to be controlled to achieve accurate reference-speed tracking. The rotor speed reference ($\omega_{\text{ref}}$) must be updated online, tacking into account the current wind velocity ($v_{\text{win}}$), so that the MPPT requirement is realized. Existing MPPT methods can be divided in two classes. The first one includes those methods using explicitly the turbine power characteristic, e.g. [1]. The second class includes methods based on extremum-seeking techniques not necessitating the knowledge of turbine characteristics, e.g. [2] [5] [3]. In fact, the required wind speed measurement is a sort of average value of wind speed along the turbine blade. Besides this is not easy to measure, the use of wind sensors entails additional cost and inevitably reduce reliability.

Designing rotor speed reference update laws that are able to meet the MPPT requirement without necessitating wind velocity sensors is a crucial issue that has yet to be solved. A solution is presently developed based on the wind turbine aerofoil characteristic curve. The obtained online speed-reference optimizer represents a key feature of our control strategy which, besides, involves turbine rotor speed regulation, DC link voltage regulation and adaptive state observation. Speed regulation is performed using a nonlinear regulator which, in addition, regulates the d-component of the stator current to zero optimizing thus the delivered stator current. The voltage at the DC link between the AC/DC rectifier and inverter is regulated to a constant reference value (Fig. 2). In fact, the DC link voltage reference value should equal the nominal value of the PMS Generator. This regulation loop controls the reactive power control delivered to the grid. All previous regulation loops are designed using the backstepping technique [8]. The nonlinear multiloop (speed, current, voltage) controller thus obtained (Fig. 3) is formally shown, using Lyapunov stability, to meet its control objectives.

II. MODELING THE ASSOCIATION SYNCHRONOUS GENERATOR-AC/DC/AC CONVERTER

A. Modeling of the combination ‘PMSG- DC/AC converter’

The controlled system is illustrated by Fig 1. It includes a combination ‘synchronous Generator-rectifier, on one hand, and a tri-phase DC/AC inverter, the other hand. The rectifier is a AC/DC converter operating, like the DC/AC rectifier, according to the known Pulse Wide Modulation (PWM) principle.
Fig. 1. The AC/DC/AC Converter power circuit in WECS

Such modeling is generally performed in the d-q rotating reference frame because the resulting components \( i_{sd} \) and \( i_{sq} \) turn out to be DC currents. It is shown in [7] that the association PMSG-PWM DC/AC converter model, expressed in the d-q coordinates, can be given by the following state space form:

\[
\begin{align*}
\frac{dx_1}{dt} &= -\frac{F}{J} x_1 - \frac{K_M}{J} x_2 + \frac{T_g}{J} \\
\frac{dx_2}{dt} &= -\frac{R}{L_s} x_2 - p x_1 x_3 + \frac{K_M}{L_s} x_1 - \frac{1}{L_s} u_1 v_{dc} \\
\frac{dx_3}{dt} &= -\frac{R}{L_s} x_3 + p x_1 x_2 - \frac{1}{L_s} u_2 v_{dc}
\end{align*}
\]

(3a) (3b) (3c)

where \( R \) and \( L_s \) are the stator resistor and inductance, \( J, F, p \) are the rotor inertia, viscous coefficient and number of poles pairs. \( T_g \) is the load torque of the generator, \( K_M \) is a flux generator constant. The state variables \( x_1 = \bar{\omega} \) is the rotor speed, \( (x_2 = i_{sq}, x_3 = i_{sd}) \) denote the averaged stator current in dq-coordinate. The inputs control \( u_1 = \bar{i}_{sq}, u_2 = \bar{i}_{sd} \) represent the average d- and q-axis of the three-phase duty ratio system \((s_1, s_2, s_3)\). with \( \tau \) denote the average value on the modulation (MLI) period of \( z \).

B. DC/AC inverter modeling

The inverter circuit (DC/AC) is presented in Fig.1. The power supply net is connected to a converter which consists of a three phase converter that has 6 semiconductors (IGBTs with anti-parallel diodes for bidirectional current flow mode) displayed in three legs 1, 2 and 3. The 6 semiconductors are considered as ideal switches. Only one switch on the same leg can be conducting at the same time.

Applying Kirchhoff’s laws, this subsystem is described by the following set of differential equations:

\[
\begin{align*}
L_0 \frac{d}{dt} [e_{n123}] &= v_{dc} [k_{123}] - [e_{n123}] \\
2C \frac{dv_{dc}}{dt} &= i_r - i_i \\
i_i &= [k_{123}]^T [e_{n123}]
\end{align*}
\]

(4a) (4b) (4c)

where \([e_{n123}] = [e_{n1}, e_{n2}, e_{n3}]^T\) is the input currents in the electric grid, \([e_{123}] = [e_1, e_2, e_3]^T\) is the sinusoidal three-phase net voltages (with known constant frequency \(\omega_n\)), \(v_{dc}\) denotes the voltage in capacitor 2\(C\), \(i_i\) designates the input current inverter, and \(k_j\) is the switch position function taking values in the discrete set \(\{0, 1\}\).

To simplify the three-phase representation (4a) for the synthesis of control laws, the Park transformation is invoked again.

\[
\begin{align*}
\frac{di_{ad}}{dt} &= -\frac{1}{L_0} E_{ad} + \omega_n i_{aq} + \frac{1}{L_0} u_{ad} v_{dc} \\
\frac{di_{aq}}{dt} &= -\frac{1}{L_0} E_{aq} - \omega_n i_{ad} + \frac{1}{L_0} u_{aq} v_{dc} \\
2C \frac{dv_{dc}}{dt} &= i_R - i_i
\end{align*}
\]

(6a) (6b) (6c)

where \([E_{ad}, E_{aq}], (i_{ad}, i_{aq})\) and \((u_{ad}, u_{aq})\) denote the averaged network voltage and current and input control of the inverter in dq-coordinate (Park’s transformation).

The power absorbed by the DC/AC converter is given by the well known expression \(P_{Load} = i_r v_{dc}\). On the other hand, the power released by the network is given by \(P_{OUT} = [e_{n123}]^T [i_{n123}] = E_{nq} u_{nd} + E_{nd} u_{nq}\). Using the power conservation principle, one has \(P_{Load} = P_{OUT}\) or, equivalently \(i_r v_{dc} = E_{nq} u_{nd} + E_{nd} u_{nq}\). Also, from (6a–b) one gets that:

\[
\begin{align*}
2v_{dc} \frac{dv_{dc}}{dt} &= -\frac{1}{C} E_{nd} i_{ad} - \frac{1}{C} (E_{aq} i_{aq} - v_{dc} i_R) \\
\frac{di_{ad}}{dt} &= -\frac{1}{L_0} E_{ad} + \omega_n i_{aq} + \frac{1}{L_0} u_{ad} v_{dc} \\
\frac{di_{aq}}{dt} &= -\frac{1}{L_0} E_{aq} - \omega_n i_{ad} + \frac{1}{L_0} u_{aq} v_{dc}
\end{align*}
\]

(7a) (7b) (7c)

Let us introduce the state variables \(x_4 = \bar{v}_{dc}, \quad x_5 = \bar{i}_{ad}, \quad x_6 = \bar{i}_{aq}\) and \(u_3 = \bar{u}_{ad}, u_4 = \bar{u}_{aq}\) represent the average d- and q-axis components of the three-phase duty ratio system \(k_i\).

The state space equations obtained up to now are put together to get a state-space model of the whole system including the AC/DC/AC converters combined with the synchronous generator. For convenience, the whole model is rewritten here for future reference:

\[
\begin{align*}
\frac{dx_1}{dt} &= -\frac{F}{J} x_1 - \frac{K_M}{J} x_2 + \frac{T_g}{J} \\
\frac{dx_2}{dt} &= -\frac{R}{L_s} x_2 - p x_1 x_3 + \frac{K_M}{L_s} x_1 - \frac{1}{L_s} u_1 v_{dc} \\
\frac{dx_3}{dt} &= -\frac{R}{L_s} x_3 + p x_1 x_2 - \frac{1}{L_s} u_2 v_{dc} \\
\frac{dx_4}{dt} &= -\frac{1}{C} E_{ad} x_5 - \frac{1}{C} (E_{aq} x_6 - v_{dc} i_R) \\
\frac{dx_5}{dt} &= -\frac{1}{L_0} E_{ad} + \omega_n x_6 + \frac{1}{L_0} u_3 v_{cd} \\
\frac{dx_6}{dt} &= -\frac{1}{L_0} E_{aq} - \omega_n x_5 + \frac{1}{L_0} u_4 v_{cd}
\end{align*}
\]

(9a) (9b) (9c) (9d) (9e) (9f)
III. CONTROL OBJECTIVES

There are five operational control objectives:

CO1. Optimization of wind energy extraction: one must determine the optimal reference speed that makes the extracted energy be maximal.

CO2. Speed regulation: the machine speed \( \omega \) must track a given reference signal \( \omega_{\text{ref}} \). This reference has been obtained from the MPPT strategy (e.g. [1]).

CO3. The inverter output currents \( (i_{n1}, i_{n2}, i_{n3}) \) must be sinusoidal with the same frequency as the supplied power grid and the reactive power in the AC grid must be well regulated.

CO4. Controlling the continuous voltage \( v_{dc} \) making it track a given reference signal \( v_{dc\text{ref}} \). This generally is set to a constant value equal to the nominal voltage entering the converter and machine.

CO5. Regulate the current \( i_{sd} \) to a reference value \( i_{d\text{ref}} \), preferably equal to zero in order to optimize the stator current (see e.g. [12]).

IV. STATE FEEDBACK CONTROLLER DESIGN

A. Wind sensorless rotor speed reference optimization

The optimization strategy is to search a reference speed \( \omega_{\text{opt}} \), in order to achieve optimal speed ratio working conditions of the wind turbine to capture the maximum energy from the wind. The wind power acting on the swept area of the blade \( A \) is a function of the air density \( \rho \) (\( \text{kg/m}^3 \)) and the wind speed \( v_{\text{win}} \) (\( \text{m/s} \)). The transmitted power \( P(W) \) is generally deduced from the wind power using the power coefficient \( C_p \), as:

\[
P = \frac{1}{2} C_p \rho A v_{\text{win}}^3
\]

The power coefficient \( C_p \) is a non-linear function of the tip speed-ratio \( \lambda = R \omega / v_{\text{win}} \) (where \( R \) is the turbine radius), which depends on the wind velocity and the rotation speed of the generator rotor \( \omega \) (\( \text{rad/s} \)). Fig. 4 represents the transmitted power according to the rotor PMSG speed for various values of the wind speed.

![Fig. 4: Turbine Power Characteristics (Pitch angle=0°).](image)

The summits of these curves give the maximum ‘extractable’ power \( P_{\text{opt}} \) and so represent the optimal points. Each one of these points is characterized by the optimal speed \( \omega_{\text{opt}} \). It is readily seen from Fig. 4 that for any wind velocity value, say \( \omega_{\text{win}} \), there is a unique couple \((\omega_{\text{opt}}, P_{\text{opt}})\) that involves the largest extractable power. The set of all such optimal couples \((\omega_{\text{opt}}, P_{\text{opt}})\) is represented by the blue curve in Fig. 4. A number of such couples have been collected from Fig. 4 and interpolated to get a polynomial function \( \omega_{\text{opt}} = F(P_{\text{opt}}) \). Let the obtained polynomial be denoted:

\[
F(P) = h_n P^n + h_{n-1} P^{n-1} + \ldots + h_1 P + h_0
\]

The function \( F(.) \) defines the speed-reference optimizer (Fig. 5). Indeed, suppose that, at some instant, the wind velocity is \( v_{\text{win}}^0 = 14 \text{m/s} \) and the rotor speed is \( \omega_0 \). One can see on Fig. 4 which transmitted wind power \( P_0 \) corresponds to this couple \((v_{\text{win}}^0, \omega_0)\). The point is that \( P_0 \) is easily online computed. Given only the value of \( P_0 \), the speed-reference optimizer gives a new rotor speed reference value \( \omega_2 = F(P_0) \). This corresponds to point A on Fig. 4. Assume that, a speed regulator (that has to be determined) is available that makes the machine rotor rotate at the new speed reference i.e. \( \omega = \omega_2 \). Then, according to Fig. 4, the wind turbine provides a new extractable power value equal to \( P_2 \). Then, the speed-reference optimizer will suggest (to the speed regulator) a new speed reference \( \omega_3 \) (point B). This process will continue until the achievement of the optimal point \((P_{\text{opt}}, \omega_{\text{opt}})\) (point C on Fig. 4).

B. Speed regulator design for synchronous generator

The regulator design is based on equations (9a-b) where the input signal \( u_1 \) stands as the actual input, in order to guarantee speed reference tracking. Following the backstepping technique [8], let \( z_1 \) denotes the speed tracking error:

\[
z_1 = x_1 - \omega_{\text{ref}} = x_1 - x_1^* \quad (12)
\]

In view of (9a), the above error undergoes the following equation:

\[
z_i = ( -F x_i - K_m x_i + T_s ) / J - \dot{x}_i \quad (13)
\]

In (13), the quantity \( \alpha = -(K_m / J) x_i \) stands up as a (virtual) control input for the \( z_i \)-dynamics. Let \( \alpha^* \) denote the stabilizing function (yet to be determined) associated to \( \alpha \). It is easily seen from (13) that if \( \alpha = \alpha^* \) with:

\[
\alpha^* = [c(\dot{z}_i + (F / J)x_i - (T_s / J) + \dot{x}_i)](\text{with } c_1 > 0) \quad (14)
\]

Indeed, if \( \alpha = \alpha^* \) one will have \( \dot{z}_i = -c_1 z_i \) which clearly is asymptotically stable with respect the Lyapunov function:
\[ V_1 = 0.5z_1^2 \] 

In effect, one then has:

\[ \dot{V}_1 = z_1\dot{z}_1 = -c_1z_1^2 < 0 \]  

As \( \alpha = -(K_M / J)x_z \), is just a virtual control input, one cannot set \( \alpha = \alpha^* \). Nevertheless, the above expression of \( \alpha^* \) retained as a first stabilization function and a new error is introduced:

\[ z_2 = \alpha - \alpha^* \]  

Using (14)-(17), it follows from (13) that the \( z_1 \)-dynamics undergoes the following equation:

\[ \dot{z}_1 = -c_1z_1 + z_2 \]  

The next step consists in determining the control input \( u_i \) so that the \( (z_1, z_2) \) error system is asymptotically stable. First, let us obtain the trajectory of the error \( z_2 \). Deriving this with respect to time and using (17) gives:

\[ \dot{z}_2 = -(K_M / J)z_2 - \alpha^* \]

Using (14) and (9a-b) in (19), one gets:

\[ \dot{z}_2 = \chi(x,t) - c_1^2z_1^2 + c_1z_1 + (K_M / J)u_i v_{dc} \]  

where

\[ \chi(x,t) = (R_s x_1 + pL_s x_1 - F \dot{x}_s / F^T_s + F / J - \dot{x}_s) \]  

The error equations (18) and (20) are given in more compact form:

\[ \dot{z}_1 = -c_1z_1 + z_2 \]  

\[ \dot{z}_2 = \chi(x,t) - c_1^2z_1^2 + c_1z_1 + (K_M / J)u_i v_{dc} \]  

To determine a stabilizing control law for (22b), let us consider the quadratic Lyapunov function candidate:

\[ V_2 = V_1 + 0.5z_1^2 = 0.5z_1^2 + 0.5z_2^2 \]  

Using (18), the time derivative of \( V_2 \) can be rewritten as:

\[ \dot{V}_2 = -c_1z_1^2 + z_1\dot{z}_1 + \frac{1}{2}(c_1^2 + 1)z_1 + \chi(x,t) \]  

This shows that, for the \( (z_1, z_2) \)-system to be globally asymptotically stable, it is sufficient to choose the control \( u_i \) so that:

\[ \dot{V}_2 = -c_1z_1^2 - c_2z_2^2 \]  

where \( c_2 > 0 \) is a new design parameter. In view of (24a), equation (24b) is ensured if:

\[ \dot{z}_2 = -c_2z_2 - z_1 \]  

Comparing (25) and (22b) yields the following backstepping control law:

\[ u_i = -JL_s(c_1 + c_2)z_2 - (c_1^2 + 1)z_1 + \chi(x,t) \]  

\[ (3K_M v_{dc}) \]

C. d-axis current regulation

The d-axis current \( x_3 \) undergoes equation (9c) in which the following quantity:

\[ v = p x_2 x_1 - u_2 v_{dc} / \dot{L}_s \]  

As the reference signal \( i_{dc} \text{ref} \) is null, it follows that the tracking error \( z_3 = x_3 - i_{dc} \text{ref} \) undergoes the equation:

\[ \dot{z}_3 = -(R_s / \dot{L}_s)z_3 + v \]  

To get a stabilizing control signal for this first-order system, consider the quadratic Lyapunov function \( V_3 = 0.5z_3^2 \). It is easily checked that, if the virtual control is let to be \( v = -(R_s / \dot{L}_s + c_3)z_3 \) where \( c_3 > 0 \), then:

\[ \dot{V}_3 = -c_3z_3^2 \]  

which is negative definite. Furthermore, substituting (29) in (28), one gets the closed-loop equation:

\[ \dot{z}_3 = -c_3z_3 \]  

Now, it is readily observed that the actual control input is obtained substituting (29) into (27) and solving the resulting equation for \( u_2 \). So doing, one gets:

\[ u_2 = (c_3 z_3 - (R_s / \dot{L}_s)z_3 + p x_2 x_1) / v_{dc} \]  

From (22a), (25) and (31), it is clear that the subsystem is globally exponentially stable.

D. Reactive power and DC voltage controller

In controlling a PFC, the main objective is to obtain a sinusoidal output current and the injection or extraction a desired reactive power in the electric network. The continuous voltage \( v_{dc} \) must track a given reference signal \( v_{dc} \text{ref} \). These objectives lead to two control loops. The first loop ensures the regulation of the DC voltage \( x_4 \) and the second ensures the injection of the desired reactive power.

1) DC voltage loop

Based on equations (9d-e), a first equation involving the control input \( u_3 \) will now be designed, using the backstepping technique [8], so that the squared DC-link voltage \( x_4 = v_{dc}^2 \) tracks well any reference signal \( x_{dc}^\text{def} = v_{dc}^2 \text{ref} > 0 \). As the system (9d-e) is of relative degree 2, the design toward that equation is performed in two steps.

Step 1. Let \( z_4 \) denote the speed tracking error:

\[ z_4 = x_4 - x_4^\star \]  

In view of (9d), the above error undergoes the following equation:

\[ \dot{z}_4 = -E_{out} x_4 / C + \beta(x_{\text{ref}} + z_2) - \dot{x}_4 \]
\[
\beta(z) = \begin{cases} 
-E_{nd} x_5 / C & \text{if } z \leq 0 \\
0 & \text{if } z > 0 
\end{cases}
\]

In (35), the quantity \( \alpha_1 = -E_{nd} x_5 / C \) stands up as a (virtual) control input for the \( z_4 \)-dynamics because the actual control input \( u_4 \) act on \( z_4 \) indirectly through \( \alpha_1 \). Following the backstepping design technique, the Lyapunov function candidate is considered as: \( V_4 = 0.5 z_4^2 \). Deriving \( V_4 \) along the trajectory of (35) yields:

\[
\dot{V}_4 = z_4 \dot{z}_4 = -z_4 \left( E_{nd} x_5 / C - \beta(x, z) + \dot{x}_4 \right)
\]

This suggests for the (virtual control) \( \alpha_1 \) the following control law:

\[
\alpha_1^* = -c_4 z_4 - \beta(x, z) + \dot{x}_4
\]

with \( c_4 > 0 \) a design parameter. Indeed, substituting \( \alpha_1^* \) to \( \alpha_1 = -E_{nd} x_5 / C \) gives \( \dot{V}_4 = -c_4 \dot{z}_4^2 \) which clearly is negative definite in \( z_4 \). As \( \alpha_1 \) is just a virtual control input, one can not set \( \alpha_1 = \alpha_1^* \). Nevertheless, the above expression of \( \alpha_1^* \) is retained and a new error is introduced:

\[
z_5 = \alpha_1 - \alpha_1^* \quad (38)
\]

Using (37), it follows from (35) that the \( z_4 \)-dynamics undergoes the following equation:

\[
\dot{z}_4 = -c_4 z_4 + z_5 \quad (39)
\]

**Step 2.** Now, the aim is to make the couple of errors \( (z_4, z_5) \) vanish asymptotically. The trajectory of the error \( z_5 \) is obtained by time-derivation of (38) i.e.:

\[
\dot{z}_5 = -E_{nd} \dot{x}_5 / C + c_5 \dot{z}_4 + \beta(x, z) - \dot{x}_4 \quad (40)
\]

Using (39) and (9d-e) in (40) yields:

\[
\dot{z}_5 = \beta_1(x_{i=1...6}, z_{i=1...5}) - \frac{E_{nd}}{L_0} u_3 v_{cd} \quad (41)
\]

With \( \beta_1(x, z) = c_2 \dot{z}_4 + \beta(x, z) - \dot{x}_4 + \frac{E_{nd}}{C L_0} - \frac{E_{nd}}{C} \omega_n x_6 \n
To determine a stabilizing control law for (9d-e), let us consider the quadratic Lyapunov function candidate:

\[
V_5 = 0.5 z_4^2 + 0.5 z_5^2 \quad (42a)
\]

Using (39)-(41), one gets from (42a) that:

\[
\dot{V}_5 = -c_5 z_5^2 + z_3 \left( z_4 + \beta_1(x_{i=1...6}, z_{i=1...5}) - \left( E_{nd} / C L_0 \right) u_3 v_{cd} \right) \quad (42b)
\]

This suggests for the control variable \( u_3 \) the following choice:

\[
u_3 = \left( c_5 z_5 + z_4 + \beta_1(x_{i=1...6}, z_{i=1...5}) \right) / E_{nd} v_{cd} \quad (43)
\]

where \( c_5 > 0 \) is a new design parameter. Indeed, substituting (43) in (42b) yields:

\[
\dot{V}_5 = -c_4 z_4^2 - c_5 z_5^2 < 0 \quad (44)
\]

Now, substituting (43) in (41) one obtains the DC voltage closed-loop control system:

\[
z_4 = -c_4 z_4 + z_5; \quad \dot{z}_4 = -c_4 z_4 - z_4 \quad (45)
\]

**2) Reactive Power loop**

Here, the focus is made on the control objective CO3 that involves the reactive power \( Q_n \) which is required to track its reference \( Q_n^* \). The electrical reactive power injected in the grid is given by \( Q_n = E_{ng} x_6 - E_{ng} x_5 \). To harmonize notation throughout this section, the corresponding tracking error is denoted \( z_6 = Q_n - Q_n^* \). It follows from (9e-f) that \( z_6 \) undergoes the differential equation:

\[
\dot{z}_6 = \beta_2(x_5, x_6) + v_{cd} \left( E_{nd} u_4 - E_{ng} u_5 \right) / L_0 \quad (46)
\]

with \( \beta_2(x_5, x_6) = -\omega_n (E_{nd} x_5 + E_{ng} x_6) - Q_n^* \).

As the equation (46) is a first order, it can be (globally asymptotically) stabilized using a simple proportional control law:

\[
u_{cd} \left( E_{nd} u_4 - E_{ng} u_5 \right) / L_0 = -c_6 \dot{z}_6 - \beta_2(x_5, x_6) \quad (47)
\]

with \( c_6 > 0 \). Then the control law \( u_4 \) is given as:

\[
u_4 = \left( -L_0 (c_6 z_6 + \beta_2(x_5, x_6)) / v_{cd} + E_{ng} u_5 \right) / E_{nd} \quad (48)
\]

It can be easily checked that the dynamic of \( z_6 \) undergoes the following equation:

\[
\dot{z}_6 = -c_6 z_6 \quad (49)
\]

Then, from (45) and (49), it is clear that the closed loop subsystem is globally exponentially stable and the error vector \( (z_4, z_5, z_6) \) converges exponentially fast to zero, whatever the initial conditions.

**V. SIMULATION**

The closed loop system is simulated, within the Matlab/Simulink environment, using the electro-mechanical characteristics of Table 1. The controller, mathematically defined by (26), (32), (43) and (48) is simulated. The used numerical values for the design parameters are chosen as follows:

- \( c_1 = 7; c_2 = 4.10^4; c_3 = 10^3; c_4 = 40; c_5 = 10^3; c_6 = 2.10^4 \)

that proved to be suitable. The controller performances will be evaluated in presence of (time-varying) wind velocity. According to the control design, the remaining closed loop inputs are kept constant: \( i_{ald} = 0 A, V_{ald} = 700 V \) and \( Q_{ald} = 0 \text{ VAR} \).

### Table I: WECS System Characteristics

<table>
<thead>
<tr>
<th>Grid voltage</th>
<th>380V - 220V/50Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC/DC/AC</td>
<td>L_0 = 1.5 mH; C= 22 mF; PMW: 10 KHz</td>
</tr>
<tr>
<td>Synchronous</td>
<td>5 kW; R_e = 0.5 Ohm; L_s = 8.5 mH; ( q_1 = 0.576 \text{ Wb} ); p = 4</td>
</tr>
<tr>
<td>Generator</td>
<td>( J=2.2 \text{Nm/rad/s} ); ( F=0.001417 \text{ Nm/rad/s} )</td>
</tr>
<tr>
<td>Wind Turbine</td>
<td>blade radius = 2 m.</td>
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</table>
A. Illustration of optimization strategy

Fig 8a shows that the wind speed signal reference takes a low, medium and high value (equal to 8.4, 12.6, 14 and step to 11.2 m/s at times 0, 10, 20 and 30s respectively). With these values of wind speeds, the turbine generates the rotor torque shown in Fig 8b. Block optimization of speed, studied in section 3.2. Referring to the turbine power characteristics (Fig 4), these active powers correspond to optimal values of each wind speeds.

Fig. 8. a) Wind speed; b) Optimized Rotor Speed reference

B. Illustration of the controller performances

In this subsection, the controller performances are illustrated by the curves in Fig. 9. Figs 9a and 9b show that the machine speed, $\omega_1 = \omega$ and the d-component of the stator current, $x_3 = i_{sd}$ perfectly converge to their respective references. The tracking quality is quite satisfactory as the response time (after each change in the wind speed) is less than 1 s. Curves (10a) and (10b) respectively show the reactive power injected in the three-phase network $Q_d$ (equals zero) and electric power $P_{dc}$, produced by the machine and transferred to the grid through the tri-phase inverter.

Fig. 11a shows that the DC-link voltage $V_{dc}$ is tightly regulated: it quickly settles down after each change in the wind speed. The wave frame of the line current $i_{al}$ is showed in fig 11b. It is seen that the current amplitude changes whenever the wind velocity varies. The current remains (almost) all time sinusoidal and in phase with the network voltage complying with the PFC requirement.

VI. CONCLUSION

We have addressed the problem of sensor less control of the wind energy conversion system. Maximum wind energy extraction is achieved by running the wind turbine generator in variable-speed mode without using the sensor wind velocity. The controlled system is an association including wind turbine, permanent magnet synchronous aero-generator and AC/DC/AC converter connected with a tri-phase network. The system dynamics have been described by the averaged sixth order nonlinear state-space model (9a-f). The multi loops nonlinear controller defined by the control laws (26), (32), (43) and (48) has been designed. The backstepping design technique is used. The controller has been designed to: (i) satisfactory rotor speed reference tracking for extracting maximum power; (ii) tight regulation of the stator d-axis; (iii) power factor correction; (iv) well regulated DC-link voltage (vdc). These results have been confirmed by a simulation study.

REFERENCES