3D Analytical Model for a Tubular Linear Induction Generator in a Stirling Cogeneration System
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Abstract—This article sets forth a 3D analytical model of a tubular linear induction generator. In the intended application, the slot and edge effects as well as induced current penetration phenomena within the solid mover cannot be overlooked. Moreover, generator optimization within the present context of cogeneration has necessitated a systemic strategy. Reliance upon an analytical modeling approach that incorporates the array of typically-neglected phenomena has proven essential to offering greater computational and analytical flexibility. This article will describe the electromagnetic model of the generator and draw comparisons with a finite element model, in addition to identifying the elements of equivalent electrical diagram and displaying results from the multi-objective optimization study performed using a genetic algorithm.

I. INTRODUCTION

The production of electrical energy must be integrated into an optimal strategy with respect to sustainable development goals. As such, it would appear that a sizable and growing share of all electricity consumption stems, to an increasing extent, from a decentralized cogeneration-based (electricity and heat) or trigeneration-based (electricity, heat and air conditioning) production system [1].

The purpose of the present study lies within this specific scope and proposes exploring new solutions for the autonomous production of electricity in cogeneration mode. While low-power cogenerators do exist, their flexibility in making thermal and electric power adjustments is inadequate and their life cycle generally proves too short with respect to the type of applications involved. The cogenerator studied herein, composed of a Stirling engine coupled directly to a linear generator, is intended to be economical, quiet, ‘clean’, compact and robust. The associated electrical system for performing the electromechanical conversion of energy has been designed as a tubular induction generator with a solid mover. The global optimization of this engine–generator–electronics–load–control cogeneration system requires, for each subsystem, a set of flexible models adapted to a systemic approach. Against this backdrop, an analytical modeling strategy thus seems best suited for our generator. However, given the highly-dynamic operating mode and maximum allowable dimensions, the model must be built to incorporate skin effects along with edge effects.

II. DESCRIPTION OF THE COGENERATOR

As shown in Figure 2, the drive assembly is composed of two Stirling engines working in opposition [2]. The thermal and electric parts are thus very tightly integrated, which serves to constitute a compact electric generating set. The Stirling external-combustion engine is particularly well-adapted to the cogeneration mode, as its pumping chamber may be assimilated with a boiler [3]. Besides this aptitude for cogeneration operations, such a configuration also proves beneficial through: simplifying the kinematics by introducing linear motion; taking advantage of external combustion in the steady state; and facilitating the guidance of mobile parts thanks to the induction system. Such characteristics would suggest much longer life cycles with much less maintenance.

The generator studied is of the tubular linear induction type (see Fig. 3). The mover comprises the drive engine pistons and is made of a solid, conductive non-magnetic material like aluminum. The operating mode (very fast alternating motions), along with the unique structure and its dimensions, make it necessary to develop specific models that incorporate the edge effect as well as dependence of the generator's
electromagnetic parameters on frequency variations (i.e. a dynamic model).

To proceed with this approach, the following hypotheses have been adopted:

- All materials used are isotropic;
- Magnetic saturation never occurs;
- The spatial distribution of primary windings (stator) is presumed to be symmetrical three-phase.

Moreover, the frequency-based approach defined herein has been based on a power supply composed of three-phase sinusoidal currents balanced at a frequency of $\omega_s$. This step will subsequently lead to easily identifying components of the equivalent electrical diagram and thereby extend the design to other types of non-sinusoidal power supplies.

The search for generator performance (in terms of axial force, electromagnetic power, losses, etc.) necessitates solving Maxwell's equations, i.e.:

$$\nabla \times \mathbf{J} = \sigma \nabla \times \mathbf{A}$$

$$\frac{1}{\mu} \nabla \times \mathbf{H} = \mathbf{J}$$

where $\mathbf{J}$ is the current density, $\mathbf{A}$ is the vector potential, $\mathbf{H}$ is the magnetic field, and $\sigma$ and $\mu$ are the electrical conductivity and magnetic permeability of the medium, respectively.

According to our initial hypotheses, the vector potential can be written as:

$$\mathbf{A}(z, r, t) = \varphi(r) e^{j(kz-\omega_t)} \mathbf{u}_\theta$$

A comprehensive understanding of the vector potential at any point within the computation field thus requires determining its radial distribution $\varphi(r)$.

Based on Maxwell's equations and the relation in (5) above, we can derive a quite remarkable differential equation whose general solution consists of a sum of modified Bessel functions of both the 1st and 2nd kind. The vector potential distribution function in each region (index $n$) of the computation space can then be expressed in the following general form, equation (8):

$$\mathbf{A}^{(n)}(z, r, t) = \sum_{m=1}^{\infty} \left[ X^{(n)} I_1 \left( \gamma^{(n)} r \right) + Y^{(n)} K_1 \left( \gamma^{(n)} r \right) \right] e^{j(kz-\omega_t)}$$

The coefficients $X^{(n)}$ and $Y^{(n)}$ are constants determined from the region boundary conditions. $g^{(n)}$, $\mu^{(n)}$, and $\sigma^{(n)}$ are the slip, magnetic permeability and electrical conductivity of region $[n]$, respectively.

Based on this solution, it becomes straightforward to deduce, for each region $[n]$, the expressions for both the axial component of the magnetic field and the azimuthal component of the electric field, equations (9a) et (9b):

$$H_z^{(n)}(z, r, t) = \frac{1}{\mu} \left[ \frac{\partial \mathbf{A}^{(n)}}{\partial t} + \frac{\partial \mathbf{A}^{(n)}}{\partial z} \right] = \frac{1}{\mu} \left[ X^{(n)} I_1 \left( \gamma^{(n)} r \right) - Y^{(n)} K_1 \left( \gamma^{(n)} r \right) \right] e^{j(kz-\omega_t)}$$

$$E_{\theta}^{(n)}(z, r, t) = j \omega A_n = j \omega \left[ X^{(n)} I_1 \left( \gamma^{(n)} r \right) + Y^{(n)} K_1 \left( \gamma^{(n)} r \right) \right] e^{j(kz-\omega_t)}$$

The power exchanged between the mover and the fixed parts (expressed in complex values) corresponds to the difference in input and output power through the mover’s exterior and interior surfaces $S[n]$:

$$P_{tr} = P(s_2) - P(s_1)$$

These power values correspond to the flux of the Poynting vector through surface $S^{[n]}$:

$$P^{(n)} = \frac{1}{2} E_0 \left( \gamma^{(n)}, r^{(n)} \right) H_z^{*} \left( \gamma^{(n)}, r^{(n)} \right) S^{[n]}$$

Figure 3: Generator design geometry

Figure 4: Slotless, infinite-length model
Inserting Equations (9a) and (9b) into (10) yields a power function independent of the time of value:

\[ P(z^{m_1}) = \frac{1}{2} j \omega \sum_{\gamma=0}^{7} \left[ X^{m_1} (l_1 + K_s \gamma) - Y^{m_1} (l_1 - K_s \gamma) \right] S^{m_1} \]

The real part of \( P_n \) corresponds to the transmitted electromagnetic power. The axial electromagnetic force \( F_z \) therefore simply equals:

\[ F_z = \Re \left[ \rho_{t_1} - \rho_{t_2} \right] \]

(11)

Figure 5 presents, for a given generator geometry, the evolution in axial force vs. rotor current frequency output by the analytical model and the finite element model. These results reveal the very strong correlation between these two models in the case where both edge and slot effects have been neglected.

Figure 6: Slotted, infinite-length model

The expressions for the axial component of the magnetic field and the azimuthal component of the electric field, under these conditions, are, equations (14a) and (14b):

\[ H^{(1)}_z(z, r, t) = \sum_{m=1,3,...} j \chi \omega \gamma \left( Y^{m_1} I_1 (r^{m_1,1} \gamma) - Y^{m_1} I_1 (r^{m_1,1} \gamma) \right) e^{i(m \eta + a \lambda z)} \]

\[ E^{(1)}_z(z, r, t) = \sum_{m=1,3,...} j \chi \omega \gamma \left( X^{m_1} I_1 (r^{m_1,1} \gamma) + Y^{m_1} I_1 (r^{m_1,1} \gamma) \right) e^{i(m \eta + a \lambda z)} \]

A comparison between the analytical model and finite element model is displayed in Figure 7 below, with results obtained for the same geometry as before. Here once again, a very strong level of correlation between the two models has been demonstrated.

Figure 7: Flux density calculated using finite elements for \( v = v_1 \), and the force developed for a slotted, infinite-length machine vs. the rotor current frequency (b)

B. Finite length analytical model

In order to take account of the edge effects produced by the finite length of the stator, we have undertaken the same approach as before, yet this time in introducing a new amplitude (±1) modulation function \( \Delta_m \) of the current density \( \lambda_x \) (see Fig. 8). The modulation length \( L \), chosen arbitrarily, is such that: \( L >> L \).

\[ \Delta_m(z) = \sum_{m=1,3,...} \frac{4}{\pi \xi \sigma} \sin \left( \frac{\pi m z}{L} \right) \sin \left( \frac{2 \pi r \xi}{L} \right) \]

(15)

The expression of the modulated primary current then becomes (equation (16)):

\[ \lambda_x(z, r, t) = \Delta \left( \frac{12}{\pi \xi} \sum_{m=1,3,...} \frac{1}{m \sigma} \sin \left( \frac{m \pi z}{L} \right) \sin \left( \frac{2 \pi r \xi}{L} \right) \right) \]

with: \( k_{x_1} = \frac{\pi}{5} \left( m + \frac{1}{2} \pi \frac{2r \xi}{L} \right) \), \( k_{x_2} = \frac{\pi}{5} \left( m - \frac{1}{2} \pi \frac{2r \xi}{L} \right) \)
Using the same equations as above, the expression derived for vector potential is, equation (17) :

\[
A^{[n]}(z,x,y) = \sum_{\xi=L, \eta=L} \left[ \frac{1}{\xi_d} \left( \sum_{i=1}^{m} \left( \chi_{n}^{[n]} Y_{i}^{[n]} A_{i}^{[n]} K_{i}^{[n]} \right) \right) \right] e^{i \xi_d y} + \sum_{\xi=L, \eta=L} \left[ \frac{1}{\xi_d} \left( \sum_{i=1}^{m} \left( \chi_{n}^{[n]} Y_{i}^{[n]} A_{i}^{[n]} K_{i}^{[n]} \right) \right) \right] e^{i \xi_d y}
\]

where:

\[
y^{[n]} = \sqrt{k_{n}^{[n]} + j \gamma \nu_{n}^{[n]} (\mu \sigma)^{[n]}},
\]

\[
y^{[n]} = \sqrt{\nu_{n}^{[n]} + j \gamma \nu_{n}^{[n]} (\mu \sigma)^{[n]}},
\]

\[
g^{[n]} = 1 + \chi (g - 1) \left( m + \frac{2 \pi}{L} \right),
\]

\[
g^{[n]} = 1 + \chi (g - 1) \left( m + \frac{2 \pi}{L} \right)
\]

\[\Delta s = \frac{\partial A^{[n]}}{\partial r} \]

Remark:
For \( \frac{L}{\tau} \neq 1 \), we return to the case of an infinite-length machine.

The finite stator length also necessitates modulating the airgap permeance, which translates the increase in airgap at the generator edges. By applying the same modulation function as the one described above over the magnetic field, we obtain:

where \( A^{[n]} \) is given in equation (17).

**Figure 8: Slotted, finite-length model**

**Figure 9: Force developed for a slotted, finite-length machine**

Furthermore, the expression for electric field \( E_{d} \) may be deduced directly from the vector potential expression: \( E_{d} = j \omega A \).

As shown in Figure 9 results, obtained using the same generator dimensions as before, the discrepancy between the two models remains relatively small.

An exhaustive study on the validity of the analytical model for various geometric parameters (with special emphasis on the ratio \( \frac{L}{\tau} \)) has been done. We show figure 10 the end effect reduction factor.

**Figure 10: Strength reduction versus (generator length / pole pitch)**

C. Schéma électrique equivalent

Afin d’établir les équations électriques représentatives du générateur asynchrone et son schéma équivalent, il est nécessaire de calculer les différentes inductances et résistances primaire et secondaire.

L’inductance et la résistance primaires d’une phase sont aisément définies :

\[
R_{1} = p \alpha \frac{\sigma D_{n}}{S},
\]

\[
L_{1} = \mu_{0} n \frac{S_{n}}{2} \left( \frac{D_{n}}{h} + D_{n} \right)
\]

With, \( p \), length pitch pole number, \( n \), spire number, \( b \), coil thickness, \( D \), mean coil diameter, \( D_{a} \), mean airgap diameter, \( h \), coil height, \( e \), airgap thickness, \( S_{a} \), airgap surface, (airgap cut by \( \theta \),r plane)

Au secondaire, nous utilisons le modèle analytique. Il nous permet de connaître les puissances actives et réactives aux frontières des régions, fer, entrefer, mover en intégrant sur ces frontières le vecteur de Poynting (voir paragraphe III.A). De ces puissances, nous pouvons déduire les impédances du secondaire, ainsi que l’impédance magnétisante en fonction de la fréquence des courants induits.

\[
P_{(\omega)} = \frac{1}{2} E_{d}( \gamma_{t} r_{a} ) \cdot \mathbf{H}_{z}^{*} ( \gamma_{t} r_{a} ) \cdot S_{a},
\]
et, \( \bar{Q}_{(in)} = \frac{1}{2} E_0 (\gamma_a \bar{r}_a) \cdot H_z (\gamma_a \bar{r}_a) \cdot S_m \).

La Figure 11 présente les puissances active et réactive qui sont à l’origine des valeurs de R2 et L2.

![Figure 11: Ptot Qtr versus mover versus eddy currents frequency](image)

IV. ÉTUDE GLOBALE DU COGÉNÉRATEUR

Comme il est rappelé dans l’introduction, l’étude du cogénérateur a été partagé selon trois disciplines, électrotechnique, thermique et automatique. La modélisation du générateur présentée dans cette article est complétée d’une modélisation du moteur Stirling en vue d’optimiser globalement le cogénérateur et de le contrôler.

A. Modélisation du moteur stirling

La modélisation du moteur thermique est reprise des travaux de Julien Boucher [12]. Le cycle thermodynamique du moteur Stirling comporte deux isochores et deux isothermes. Par hypothèse, les échanges se réalisent en mode adiabatique. (\( P, V = \) constante). Le moteur est à double effet, à piston libre.

Le point de fonctionnement du moteur se caractérise par la grandeur nous permettant d’assurer le pilotage de l’ensemble.

La modélisation du moteur Stirling en vue d’optimiser le cogénérateur pris dans son ensemble.

Cette etude se devait d’être complétée par une démarche d’optimisation globale du cogénérateur pris dans son ensemble.

Cette optimisation avait pour but de répondre à deux objectifs

- minimiser les pertes du générateur,
- minimiser la taille des composants de l’ondeur.
L’optimisation a été réalisée avec un logiciel utilisant un algorithme génétique NSGA II. L’avantage d’un tel algorithme est la possibilité de prendre en compte un grand nombre de paramètres, associés à des objectifs et des contraintes multiples, ces objectifs et ces contraintes étant calculés avec les modèles électriques et thermiques précédemment décrits.

La figure [14] donne un exemple de résultat d’optimisation. Il concerne la minimisation des pertes dans le générateur et la minimisation de la taille des semi-conducteurs de l’onduleur.

Figure 13: Optimisation process

La figure [14] donne un exemple de résultat d’optimisation. Il concerne la minimisation des pertes dans le générateur et la minimisation de la taille des semi-conducteurs de l’onduleur.

V. CONCLUSION

This article has set forth the steps behind development of a 3D analytical model for the tubular linear induction generator with inclusion of both slot and edge effects. Utilizing a frequency-based approach, this model proves to be relatively accurate with respect to the finite element model. Besides calculating the electromagnetic performance of the generator, our new model makes it possible to identify elements of equivalent dynamic electrical diagram [9]. Also, this article highlights the multi-objective optimization studies conducted within the specific scope of co-generation operations [11] through applying a genetic algorithm [13].

REFERENCES