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The effect of thermal boundary conditions on dynamos driven by internal heating

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Abstract

The early dynamos of Mars and Earth probably operated without an inner core being present. They were thus exclusively driven by secular cooling and radiogenic heating which can both be modeled by homogeneously distributed heat sources. Some previous dynamo simulations that explored this driving mode found dipole dominated magnetic fields, while other reported multipolar configurations. Since these models differed both in the employed outer thermal boundary conditions and in the size of the inner core, which was still retained for practical reasons, the cause for the variation in field geometry remained unclear. Here we investigate this issue and find that strong dipole dominated fields are preferred for fixed heat flux conditions whereas weaker multipolar fields are typical for fixed temperature conditions. The size of the inner core, on the other hand, proved to be of minor influence. The stronger dipolar fields for fixed heat flux conditions promote larger convective structures. Since the mantle of the terrestrial planets controls the heat flux rather than the temperature at the core-mantle boundary, our re-

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sults suggest that the early dynamos of Mars and Earth would have produced dipole dominated magnetic fields.

Key words: MHD dynamo, internal heating, thermal boundary condition, inner core, paleo-magnetic field

1 **1. Introduction**

2 According to thermal evolution models, Earth had no solid inner core
 3 until about 1-2 billion years ago (Labrosse, 2003; Nimmo, 2007). When an
 4 inner core is absent, a dynamo must be driven by volumetric secular cooling
 5 and possibly radiogenic internal heating, whereas the present geodynamo is
 6 thought to be largely driven by a buoyancy flux from below, arising from the
 7 release of latent heat and compositional enrichment associated with inner
 8 core freezing (e.g. Stevenson et al., 1983; Labrosse, 2003; Nimmo, 2007).
 9 The early Martian dynamo probably also operated without an inner core
 10 and was driven by secular cooling (e.g. Stevenson, 2001). If a growing inner
 11 core had been present, it is difficult to understand why the Martian dynamo
 12 stopped to operate approximately 4.1 billion years ago (Lillis et al., 2008).
 13 In the absence of an inner core the dynamo could have stopped because the
 14 declining heat-flow has led to a subadiabatic temperature gradient in the
 15 fluid core.

16 The presence or absence of an inner core affects the dynamo in various
 17 ways - through its electrical conductivity, through its influence on the ge-
 18 ometry of the flow in the outer core and by its role as a buoyancy source.
 19 Inner core conductivity has been proposed as being essential for stabilizing
 20 the dipole field against too frequent reversals (e.g. Hollerbach & Jones, 1993).

21 Numerical simulations comparing cases with a conducting and an insulating
 22 inner core (Wicht, 2002) and with or without an inner core (Sakuraba &
 23 Kono, 1999) suggest that the differences for the observable field outside the
 24 core are small.

25 The geometrical effect arises because the inner core represents an obstacle
 26 to the preferred pattern of convection, which consists of nearly-geostrophic
 27 convection columns aligned with the rotation axis (e.g. Busse, 2002). This
 28 effect occurs also for non-magnetic convection. Dormy et al. (2004) investi-
 29 gated the onset of thermal convection with homogeneous heat sources in the
 30 inner and outer core. They find that the structure of convection at onset
 31 hardly depends on the inner core radius, provided it is less than approxi-
 32 mately 45 % of the core radius (so-called thick shell regime), and that the
 33 convection is similar to that in a full sphere.

34 Perhaps the most profound difference between dynamos with and without
 35 an inner core comes from the different distribution of buoyancy sources. In
 36 the absence of an inner core, the lack of the buoyancy flux associated with its
 37 growth implies that convection is weaker and Ohmic dissipation lower than
 38 for the present Earth's core. Furthermore, the different distribution of sources
 39 and sinks of buoyancy flux may lead to different morphologies of the magnetic
 40 field. Here we separate the question of the existence of an inner core from that
 41 of the mode of driving convection in the fluid core. We focus on dynamos
 42 in a thick shell where the buoyancy sources are volumetrically distributed
 43 and where the outer boundary represents the sink for the buoyancy flux.
 44 This represents convection that is driven by internal heat sources or secular
 45 cooling or a combination of both. Hereafter we refer to this scenario as

46 internal heating and we use the term basal heating when the buoyancy source
47 is located at the inner core boundary.

48 Previous dynamo simulations proposed that dynamos driven by signifi-
49 cant degrees of internal heating produce different field properties, compared
50 to those with no internal heating. Busse and co-workers (e.g. Grote et al.,
51 2000; Busse, 2002) studied the field morphology for dynamos driven by a
52 combination of internal heating and basal heating, mostly with stress-free
53 mechanical boundary conditions. Depending on the various control param-
54 eters, they found a diversity of field morphologies, comprising dipolar and
55 non-dipolar solutions with various field geometries. Dynamos with dipole-
56 dominated fields are more commonly found with pure basal heating and no-
57 slip mechanical condition (e.g. Christensen et al., 1999). Directly comparing
58 cases of internal heating, of basal heating and of compositional convection
59 (where buoyancy sources at the inner boundary are balanced by volumetric
60 sinks) for the case of no-slip conditions, Kutzner & Christensen (2000, 2002)
61 found that internal heating favours solutions with non-dipolar magnetic fields
62 at the same control parameter values where dipolar dynamos prevail for the
63 other modes of driving convection.

64 For dynamos with an imposed temperature contrast Christensen & Aubert
65 (2006) found that a local Rossby number, which is a measure for the ratio be-
66 tween inertial and Coriolis forces, controls the field structure. At low values
67 the solution is dipolar and at high values (approximately > 0.12) the field
68 at the outer boundary of the dynamo is dominated by higher multipoles.
69 Olson & Christensen (2006) showed that the dipolar-multipolar transitions
70 as a function of the local Rossby number is less sharp for internally heated

dynamos than it is for models with fixed temperature contrast and that non-dipolar solutions persist to lower values of the local Rossby number. The dipole moment was found to be generally weaker at the same value of the buoyancy flux in the internally heated case.

In contrast to these earlier results, Aubert et al. (2009) found in recent dynamo simulations relatively small differences, at a given value of the convective power, in terms of the magnetic field strength and of the relative dipole contribution to the field at the outer boundary between dynamos with internal heating, basal heating or compositional convection. Their models for internally heated dynamo differed in two respects from earlier ones. Previous models employed a condition of fixed temperature on the outer boundary, whereas Aubert et al. (2009) impose a fixed homogeneous heat flux, which is a more natural condition for dynamo in terrestrial planets. The difference in the thermal boundary condition can have a significant influence on the pattern of convection and the properties of the magnetic field (Sakuraba & Roberts, 2009). Furthermore, most earlier studies used the present radius of the Earth's inner core, 35 % of the core radius, whereas Aubert et al. (2009) reduced the inner core size to 1 - 5 %. Most internally-heated models retained a passive inner core, because the current spectral dynamo codes usually only allow to simulate a spherical shell but not a full sphere. The differences between the results of Aubert et al. (2009) and those of Kutzner & Christensen (2000, 2002) and Olson & Christensen (2006) could be caused by the thermal boundary condition or by the difference in inner core size.

Roberts & Glatzmaier (2001) explored three models related to the past, present and future geodynamo with different sizes of the inner core, using

96 heat flux conditions on the boundaries. In their model with a small inner core
 97 the dipole was found to be more dominant than in the other cases. However,
 98 even their model with a small inner core was mainly driven by basal heating,
 99 because the smaller surface area of the inner core was balanced by a much
 100 faster growth in radius than at present. For this reason there is a strong
 101 difference between the situation without an inner core and that with even a
 102 small inner core. The influence of the inner core size on the dynamo onset
 103 in the case of bottom heated convection was also studied by Heimpel et al.
 104 (2005).

105 In most MHD dynamo simulations fixed temperature conditions have
 106 been the standard (e.g. Kageyama & Sato, 1995; Olson et al., 1999; Takahashi
 107 et al., 2008a). Others used a heat flux boundary condition, but in many cases
 108 the emphasis was on exploring the influence of various pattern of heteroge-
 109 neous heat flux distribution at the core-mantle boundary (e.g. Glatzmaier et
 110 al., 1999; Olson & Christensen, 2002; Christensen & Olson, 2003; Takahashi
 111 et al., 2008b). Stanley et al. (2008) proposed that a strong hemispherical
 112 dichotomy of the heat flux out of the early Martian core has led to a dynamo
 113 operating only in one hemisphere. This could explain the observed uneven
 114 distribution in the magnetization on the Martian crust observed by Mars
 115 Global Surveyor. The question whether the nature of the thermal boundary
 116 condition, fixed flux or fixed temperature, makes a fundamental differences
 117 has been addressed in a few studies only. For non-magnetic rotating con-
 118 vection the heat flux boundary condition favours larger scales of convection
 119 than the temperature condition near the onset of convection (Takehiro et
 120 al., 2002). Comparing dynamo models with different thermal boundary con-

121 ditions, Busse & Simitev (2006) reported no major qualitative differences.
 122 Recently, Sakuraba & Roberts (2009) compared the effect of the boundary
 123 condition for a rapidly-rotating dynamo model (low Ekman number). They
 124 found that the heat flux condition promotes stronger magnetic fields and
 125 larger scales in the velocity and magnetic field.

126 The purpose of this paper is to clarify the influence of thermal boundary
 127 conditions and the size of the inner core on dynamos driven by internal heat-
 128 ing. The inner core is kept only for technical reasons and is made passive in
 129 the sense that it is not a source of buoyancy nor is it electrically conducting.
 130 By varying its size we want to determine if its kinematic influence on the
 131 dynamo is significant. We compare dynamos with uniform temperature and
 132 uniform heat flux, respectively, for otherwise identical sets of control param-
 133 eters. The model setup and the diagnostic parameters that we use to compare
 134 the results are described in Section 2. In Section 3 we demonstrate that the
 135 thermal boundary condition rather than the inner core size has a major role
 136 on the field morphology and in Section 4 we discuss the implications for the
 137 early geodynamo and the Martian dynamo.

138 2. Formulation

139 We model a rotating spherical shell with inner core radius r_i and outer
 140 radius r_o that is filled with an electrically conducting fluid. Convection is
 141 driven by homogeneously distributed volumetric heat sources. We solve the
 142 following dimensionless equations in the Bousinesq approximation: the heat
 143 transport equation (1), the Navier-Stokes equation (2), the induction equa-
 144 tion (3), and the conditions for incompressible fluid and solenoidal field (4):

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{Pr} \nabla^2 T + 1, \quad (1)$$

$$Ek \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) + 2\hat{\mathbf{e}}_z \times \mathbf{u} = -\nabla P + Ra' T \frac{\mathbf{r}}{r_o} + Ek \nabla^2 \mathbf{u} + \frac{1}{Pm} (\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Pm} \nabla^2 \mathbf{B}, \quad (3)$$

$$\nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{B} = 0, \quad (4)$$

where T , $\mathbf{u}$, P and $\mathbf{B}$ are temperature, velocity, non-hydrostatic pressure and magnetic field, respectively. The fundamental scales for length, time, temperature and magnetic field are, respectively, the thickness $D = r_o - r_i$ of the spherical shell, the viscous diffusion time D^2/ν (with ν the kinematic viscosity), $\epsilon D^2/\nu$ (with ϵ the internal heating rate) and $(\rho\mu\lambda\Omega)^{1/2}$ (with ρ the density, μ the magnetic permeability, λ the magnetic diffusivity and Ω the rotation rate). The four non-dimensional control parameters are the Ekman number, Prandtl number, magnetic Prandtl number and modified Rayleigh number:

$$Ek = \frac{\nu}{\Omega D^2}, \quad Pr = \frac{\nu}{\kappa}, \quad Pm = \frac{\nu}{\lambda}, \quad Ra' = \frac{\alpha g_o \epsilon D^3}{\nu^2 \Omega}, \quad (5)$$

where κ , α and g_o are the thermal diffusivity, thermal expansivity and gravity at the outer boundary, respectively.

We impose a zero heat flux at the inner core boundary. On the outer boundary, we assume either a fixed temperature condition, $T(r_o) = 0$, or a uniform heat flux, $\partial T / \partial r(r_o) = -q_o$. We refer to the former as the 'FT' condition and to the latter as the 'FF' condition. In the FF cases, the value of the heat flux at the outer boundary is in balance with the internal heating rate

$$4\pi r_o^2 q_o = \frac{4}{3}\pi (r_o^3 - r_i^3). \quad (6)$$

162 Note that the dimensionless heating rate density is one because of the tem-
 163 perature scale chosen here. When the internal heating rate is zero for an
 164 unchanged heat flux at the outer boundary (secular cooling), the solution
 165 would be identical except for a linear drift of the temperature. The velocity
 166 is forced to vanish on both boundaries (no-slip condition) and the mantle
 167 and inner core are electrically insulating and co-rotating.

168 Aside from the thermal boundary condition and the Rayleigh number, we
 169 vary the size of the inner core, assuming values of either 0.35 or 0.10 for the
 170 ratio

$$\eta = r_i/r_o. \quad (7)$$

171 We vary the modified Rayleigh number Ra' in the range of 350 – 2100 for
 172 each of the four combinations of thermal boundary conditions and inner core
 173 sizes. For cases with the large aspect ratio we also study higher values of
 174 the Rayleigh number. The other parameters are fixed to $Ek = 10^{-4}$, $Pr = 1$,
 175 and $Pm = 3$. The critical Rayleigh numbers Ra'_{crit} for the non-magnetic
 176 convection are 108 (201) and 109 (202) for FT and FF cases, respectively,
 177 at $\eta = 0.35$ (0.10). They have been calculated by monitoring the growth or
 178 decay of weak thermal perturbations of the conductive state. The critical
 179 values are practically identical for the two thermal boundary conditions at
 180 these values of Ek and Pr , in agreement with results by Busse & Simitev
 181 (2006) and Gibbons et al. (2007), while reducing the inner core size from
 182 $\eta = 0.35$ to 0.10 roughly doubles the critical Rayleigh number. In terms of
 183 the supercriticality, our simulations cover the range of 3.2 – 77 (2.2 – 10) at
 184 $\eta = 0.35$ (0.10).

185 We describe the characteristic properties of the solution in terms of time-
 186 averaged non-dimensional measures most of which have been defined in pre-
 187 vious papers (Kutzner & Christensen, 2002; Christensen & Aubert, 2006).
 188 The Nusselt number measures the efficiency of heat transfer and we here use
 189 the definition

$$Nu^* = \frac{\Delta T_s}{\Delta \bar{T}} = \frac{T_s(r_i) - T_s(r_o)}{\bar{T}(r_i) - \bar{T}(r_o)}. \quad (8)$$

190 The overbar indicates the average over a spherical surface and the suffix S
 191 stands for the conductive solution.

192 The amplitude of the rms-velocity u_{rms} is given in terms of the magnetic
 193 Reynolds number

$$Rm = \frac{u_{\text{rms}} D}{\lambda}. \quad (9)$$

194 A measure for the ratio between inertial and Coriolis forces is the local Rossby
 195 number

$$Ro_\ell = Ro \frac{l_u}{\pi} = \frac{u_{\text{rms}} l_u}{\Omega D \pi}, \quad (10)$$

196 where Ro is the conventional Rossby number and l_u is the mean spherical
 197 harmonic degree l in the kinetic energy spectrum

$$l_u = \frac{\sum_l l E_{\text{kin}}^{(l)}}{E_{\text{kin}}}. \quad (11)$$

198 Christensen & Aubert (2006) and Olson & Christensen (2006) found that
 199 for the bottom heated dynamos with FT condition the local Rossby number
 200 controls whether the magnetic field is dominantly dipolar or multipolar.

201 The strength of the magnetic field in the volume is measured by the
 202 Elsasser number

$$\Lambda = \frac{B_{\text{rms}}^2}{\rho \mu \lambda \Omega}, \quad (12)$$

203 which equals the square of the non-dimensional field strength in our scaling.
 204 We characterise the morphology of the magnetic field on the outer boundary
 205 of the dynamo in terms of its dipolarity, defined as the time average power of
 206 the axial dipole field relative to the total power in spherical harmonic degrees
 207 up to twelve:

$$f_{\text{dip}}^{\text{ax}} = \left(\frac{E_{\text{mag}}^{(l=1,m=0)}(r=r_o)}{\sum_{l=1}^{12} \sum_{m=0}^l E_{\text{mag}}^{(l,m)}(r=r_o)} \right)^{1/2}. \quad (13)$$

208 In previous works, the dipolarity f_{dip} was defined in terms of the total dipole
 209 component. Since only the axial part of the dipole has special significance,
 210 we here define the dipolarity for this component alone.

211 Some of our dynamos are not dominated by the dipole but exhibit var-
 212 ious complex field structures as described below. We classify the various
 213 morphologies in terms of the spatial patterns that we observe on the outer
 214 boundary at several snapshots in each run. This classification is supported
 215 by the dominant degree l_B^p and order m_B^p which we obtain from the peak in
 216 the time- and radially- averaged magnetic energy spectrum. To further char-
 217 acterize the magnetic field, we also record time-averaged values of the relative
 218 fractions of the axisymmetric poloidal (P) and toroidal (T) magnetic ener-
 219 gies in the spherical shell volume. The corresponding ratios for the kinetic
 220 energy indicate the relative contributions of zonal wind (D) and meridional
 221 circulation (M), respectively, to the flow. We describe the strength of each
 222 axisymmetric component in terms of the relative fraction of the total energy,
 223 because we are mostly interested in the structural differences in the flow and
 224 the magnetic field. In absolute terms, the total kinetic energy is obtained as
 225 $0.5(Rm/Pm)^2$ and the total magnetic energy is $\Lambda/(2PmEk)$.

226 We use the code MagIC (e.g. Wicht, 2002), which is a modified version of

a code originally developed by Glatzmaier (e.g. Glatzmaier, 1984) and solves the equations with a pseudo-spectral method. The horizontal resolution in terms of the maximum spherical harmonic degree and order is $L = M = 96$, corresponding to $N_\phi = 288$ and $N_\theta = 144$ grid points in azimuthal and latitudinal directions, respectively. In radial direction we use $N_r = 49$ Chebyshev polynomials for cases with $\eta = 0.35$ and $N_r = 71$ for cases with $\eta = 0.10$. The two cases at the highest Rayleigh numbers and $\eta = 0.35$ with FF boundary conditions required $L = M = 128$ and $N_r = 61$.

A dynamo solution with a strong dipolar field was used as initial condition in all our runs. For several cases we confirmed the uniqueness by also starting from multipolar solutions. All runs were integrated for at least two magnetic diffusion times.

3. Results

The numerical experiments and results are listed in Table 1 in terms of the control parameters, boundary conditions, time-averaged properties of the solutions and the classification of the field morphology. Fig. 1 and Fig. 2 summarize how the field geometry depends on the parameters. Fig. 1 shows the dependence of the dipolarity on the local Rossby number. For low Ro_ℓ values (typically $Ro_\ell < 0.1$), the field at the outer boundary is highly dipolar for the FF boundary condition ($f_{\text{dip}}^{\text{ax}} > 0.45$) but multipolar for the FT condition ($f_{\text{dip}}^{\text{ax}} < 0.15$). The aspect ratio η has a much smaller effect on the dipolarity than the thermal boundary condition. The discrepancy between the findings by Kutzner & Christensen (2000) and Aubert et al. (2009) therefore likely results from the different thermal boundary condition.

251 The strong influence of the thermal boundary condition is also obvious in Fig.
 252 2, which shows a phase diagram for the dependence of the field morphology at
 253 the outer boundary on the aspect ratio η and on the degree of supercriticality
 254 of the Rayleigh number Ra'/Ra'_{crit} . The fields for the FF condition are all
 255 dominated by the axial dipole. For the FT condition the axial dipole is much
 256 weaker and the field shows different multipolar geometries, depending on η
 257 and Ra'/Ra'_{crit} .

258 In the following subsections, we describe typical features of FT (Section
 259 3.1) and FF (Section 3.2) simulations. We focus on the low Ro_ℓ regime,
 260 where the differences are apparent, and compare four cases at $Ra' = 700$,
 261 where Ro_ℓ is between 0.012 (FF, $\eta = 0.10$) and 0.042 (FT, $\eta = 0.35$). For
 262 $Ro_\ell \gtrsim 0.1$, all solutions are expected to be multipolar.

263 3.1. Fixed temperature boundary conditions

264 The fixed temperature boundary conditions promote weaker magnetic
 265 fields, with Elsasser numbers between 0.1 and 4.0, and complex non-dipolar
 266 field geometries, with dominant spherical degrees larger than one (Table 1).
 267 Non-axisymmetric field contributions dominate at the outer boundary as well
 268 as throughout the volume in all FT cases, with the exception of the model at
 269 $Ra' = 700$ and $\eta = 0.10$ where the axisymmetric quadrupole ($l = 2, m = 0$)
 270 is most prominent at the outer boundary. Figure 3 (thin lines) shows the
 271 volume averaged energy spectra at $Ra' = 700$ for both aspect ratios. Har-
 272 monic orders $m = 1$ dominate in the two cases. Axisymmetric contributions
 273 are small and account for not more than 7% of the total magnetic energy
 274 ($P + T$). This increases to 20% for the quadrupolar case at $\eta = 0.10$ and
 275 $Ra' = 700$.

276 A comparison of the spectra for the two aspect ratios reveals that the
 277 thinner shell at $\eta = 0.35$ promotes smaller magnetic length scales: the max-
 278 imum at low degree is less pronounced than at $\eta = 0.10$ and more energy is
 279 concentrated at degrees beyond $l = 15$, resulting in a rather flat spectrum.
 280 This becomes even more pronounced in the cases with large Rayleigh num-
 281 bers $Ra' = 2100$ and 4200 at $\eta = 0.35$, where the spectra show a rather broad
 282 maximum between $l = 7$ and $l = 20$ (not shown). Table 1 illustrates that
 283 the relative contribution of axial dipole $f_{\text{dip}}^{\text{ax}}$ slightly increases with the local
 284 Rossby number Ro_ℓ for either aspect ratio, though it stays small and renders
 285 all FT cases as non-dipolar.

286 Figure 4a and b show typical configuration of the radial field at the outer
 287 boundary for the two FT cases at $Ra' = 700$. Fig. 4a illustrates the hemi-
 288 spherical geometry of the field found at $\eta = 0.35$, where the strong field
 289 patches predominantly concentrate in one hemisphere. The hemispherical
 290 field can be described as a superposition of fields with dipolar and quadrupo-
 291 lar symmetries, each of which does not require an equatorial asymmetry of
 292 the flow (which we do not observe). Fig. 4b shows the equatorially symmet-
 293 ric field we find at $\eta = 0.10$. The axisymmetric magnetic fields depicted in
 294 Figure 5 confirm that either field characteristic, hemispherical or equatorially
 295 symmetric, prevails throughout the shell. A configuration dominated by an
 296 equatorially symmetric octopole ($l = 3, m = 1$) is observed at an aspect ratio
 297 of $\eta = 0.35$ and a low Rayleigh number of $Ra' = 350$ (not shown here).

298 Grote et al. (2000) find hemispherical dynamos at parameters close to the
 299 ones employed here, at $Pm \sim 3$, $Pr = 1$ and $Ek \sim 10^{-4}$, with the stress-free
 300 boundary conditions. They also report that dipolar dynamos are recovered

at larger magnetic Prandtl numbers, which we have not investigated so far. Further we observe oscillatory behaviours of the field, as reported in Busse & Simitev (2006). For example, in some models the activity switches between hemispheres. The quadrupolar field shows a periodic oscillation in which structures in the axisymmetric poloidal and toroidal fields propagate poleward in each hemisphere.

The fluid motion is dominated by non-axisymmetric contributions. The energies in the axisymmetric flows ($D+M$) amount to only 5% and 8% of the total kinetic energy in the cases of $\eta = 0.35$ and $\eta = 0.10$, respectively. Figure 6 illustrates contours of the z -component of the vorticity in the equatorial plane. The convection cells concentrate in the outer regions of the shell, where the conductive temperature gradient is steeper than deep inside of the shell. This concentration of convection cells on the outer regions is obvious in the case with a small inner core, in which the Rayleigh number is closer to the critical value than that in the case with a large inner core. The helicity, $\overline{\mathbf{u} \cdot (\nabla \times \mathbf{u})}$, is therefore concentrated near the outer boundary (Figure 5c and f). The kinetic energy spectra show broad peaks at $8 \lesssim m \lesssim 11$ and $5 \lesssim m \lesssim 9$ for $\eta = 0.35$ and 0.10 (thin solid lines in Figure 7a and b), respectively, and little dependence on the Elsasser number. We cannot observe the effect that Lorentz forces promote larger flow scales, which some magnetoconvection simulations found for $\Lambda > \mathcal{O}(Ek^{1/3})$ or others for $\Lambda > \mathcal{O}(1)$ (Jones (2007) and Wicht et al. (2009) for recent overviews).

Figure 8 shows snapshots of the axisymmetric temperature distribution, zonal flow and axisymmetric meridional circulation. Plume-like convection is shown to set in inside the tangent cylinder (hereafter referred as TC) at

Rayleigh numbers which are a few times the critical Rayleigh number. Once it has started, it can transport heat effectively inside the TC as well. This effect is responsible for the latitudinal dependence of the zonally averaged temperature in the case with $\eta = 0.35$ and $Ra' = 700$ (Fig. 8a). The plumes drive a strong meridional circulation (Fig. 8c) and are also responsible for the strong thermal winds arising inside the TC (Fig. 8b). For $\eta = 0.10$, the TC occupies only a small fraction of the shell volume. The effect of the plumes on the overall flow is therefore reduced (Fig. 8d and f). The latitudinal temperature contrast between inside and outside the TC and thus the thermal winds around the TC (Fig. 8e) are weaker than those for $\eta = 0.35$. The small inner core promotes simple structures of axisymmetric flow. The zonal flow outside the TC is strongly geostrophic.

3.2. Fixed heat-flux boundary conditions

The FF conditions at the outer boundary promote dipole-dominated stronger fields with Elsasser numbers between 3 and 11 as long as the local Rossby number remains below roughly 0.1. Axial dipole contributions clearly dominate the field at the outer boundary as well as throughout the shell. Consequently, $l = 1$ and $m = 0$ harmonics dominate the magnetic energy spectra at $Ra' = 700$, shown in Fig. 3 (thick lines). Axisymmetric field contributions now carry a sizable fraction of the total magnetic energy: between 10% and 30% for $\eta = 0.35$ and 30% and 44% for $\eta = 0.10$. All this changes in the large Rayleigh number cases $Ra' \geq 6300$ with $Ro_\ell \geq 0.120$ (see Table 1), where the dipole has lost its dominance and reverses more or less continuously.

Fig. 9 and 10 illustrate the spatial pattern of the field for $Ra' = 700$. The

351 radial field at the outer boundary shown in Fig. 9 demonstrates that the
 352 dipole dominates for both aspect ratios, but also that the azimuthal length-
 353 scales are larger for $\eta = 0.10$ than for $\eta = 0.35$. This may in part be due to the
 354 lower degree of supercriticality in the case with a small inner core. However,
 355 comparisons for the mean wavenumbers of the flow and the magnetic field,
 356 l_u and l_B , between cases with $\eta = 0.35, Ra' = 700$ and $\eta = 0.10, Ra' = 1400$,
 357 which have a similar degree of supercriticality, suggests that even under these
 358 conditions a larger inner core fosters smaller scales: l_B (l_u) is 10.1 (13.7) for
 359 $\eta = 0.35, Ra' = 700$ and 8.3 (10.6) for $\eta = 0.10, Ra' = 1400$. Fig. 10b shows
 360 that the axisymmetric poloidal field lines are distorted inside the TC by the
 361 plume flow for $\eta = 0.35$. This effect seems negligible for $\eta = 0.10$ where
 362 the lines close to the rotation axis are nearly parallel to it (Fig. 10e). The
 363 toroidal fields shown in Fig. 10a and d also differ between the two cases.
 364 Two flux patches can be found in each hemisphere, one closer to the outer
 365 boundary near the equator and the other deeper inside the shell. The deep
 366 patches seem to be caused by an ω -effect near the TC boundary, but they
 367 probably contribute little to the poloidal field production through an α -effect
 368 because of the weak helicity in this region (see Fig. 10c and f). These patches
 369 lie closer to the rotational axis for $\eta = 0.10$ than for $\eta = 0.35$.

370 Fig. 11 compares the axisymmetric temperature, zonal flow, and merid-
 371 ional circulation for the two FF cases at $Ra' = 700$. The two solutions are
 372 similar but differ significantly from the FT cases shown in Fig. 8. In the FF
 373 case, the zonal flow shows strong variations along the z -direction (Fig. 11b
 374 and e). While Reynolds stress rules the zonal flow outside the TC in the FT
 375 solutions, thermal wind dominates in the FF cases. The pattern is similar to

that of typical bottom heated dynamo simulations with strong dipolar magnetic fields. According to Aubert (2005), the associated Lorentz forces help to balance the Coriolis force and thereby partly release the Taylor-Proudman constraint and therefore allow for a large degree of z -dependence. Heat transport is more efficient in the equatorial region than in the regions closer to the poles. This can be quantified in terms of a co-latitude dependent Nusselt number:

$$Nu^*(\theta) = \frac{\Delta T_s}{\Delta \bar{T}^\phi(\theta)} = \frac{T_s(r_i) - T_s(r_o)}{\bar{T}^\phi(r_i, \theta) - \bar{T}^\phi(r_o, \theta)}, \quad (14)$$

where the overbar with the suffix ϕ indicates the average over longitude. For the snapshots in Fig. 11a and d, $Nu^*(\pi/2) = 4.3$ and $Nu^*(0) = 2.1$ for $\eta = 0.35$, $Nu^*(\pi/2) = 2.7$ and $Nu^*(0) = 1.5$ for $\eta = 0.10$. This latitudinal dependence in the convective heat transport results in a latitudinal temperature gradient in the FF cases. For the models at $Ra' = 700$ depicted in Fig. 11a and d, it amounts to 1.4 for $\eta = 0.35$ and 2.3 for $\eta = 0.10$, in terms of a normalized measure, $[\bar{T}^\phi(r_o, \pi/2) - \bar{T}^\phi(r_o, 0)]/\bar{T}(r_o)$. These latitudinal temperature gradients drive the strong thermal winds below the outer boundary in the equatorial region, shown in Fig. 11b and e. The temperature distributions are also compliant with the large scale meridional circulation (Fig. 11c and f), involving rising hot material around the equatorial plane and colder descending flow at the TC boundary. The meridional circulation contributes less than 1 % to the kinetic energy, but is significantly stronger in the FF case than in the FT case (compare M in Table 1). The effective magnetic Reynolds number for the meridional circulation, $Rm^M = Rm\sqrt{M}$, is estimated to 4 - 45. Since Rm^M is larger than one, the circulation can affect the magnetic field.

Fig. 12 illustrates contours of the z -components of the vorticity in the equatorial plane. A comparison with the equivalent illustration for the FT cases in Fig. 6 reveals that the flow is now much more large scale. The kinetic energy spectra with respect to spherical harmonic order m show (thick solid lines in Figure 7) a dominance of low wavenumbers contributions in the FF cases, compared to the FT case. The mean degree l_u of the time-averaged spectra, listed in Table 1, confirms the increase in scale. For example, in the cases with $\eta = 0.10$, we find l_u values between 9 and 12 for FF conditions, whereas it lies between 13 and 15 for FT conditions.

The effect of thermal boundary conditions on the onset of convection and finite-amplitude convection are examined by Takehiro et al. (2002), Busse & Simitev (2006) and Gibbons et al. (2007). All three articles report that larger convective cells appear at the onset when FF conditions are employed at moderately small Ekman numbers. To explore whether the same effect is responsible for the differences found in our dynamo calculations, we also ran non-magnetic convection models with fixed flux at $Ra' = 700$ for the two aspect ratios (listed in the lowest two rows of Table 1). Thick dotted lines in Figure 7a and b show the time-averaged kinetic energy spectra and Figure 13 illustrates the spatial pattern in the non-magnetic FF case at $\eta = 0.10$. For both aspect ratios, non-magnetic convection shows flow structures that are similar to those found in the FT dynamos, for example, a geostrophic zonal wind and small-scale non-axisymmetric flow. We also find that the multipolar FF dynamo case with $Ra' = 8400$ and $\eta = 0.35$, which has a strong magnetic field with $\Lambda > 7$, has similarly structures and length scales as the non-magnetic convection or dynamos with a fixed temperature condition.

425 These observations suggest that a strong dipolar field is essential to maintain
 426 a balance between Lorentz force and Coriolis force that leads to the change
 427 of convective scale and differences in axisymmetric flow structures.

428 An interesting question is whether the differences in the flow between
 429 the non-magnetic FT and FF cases, even though they are weak, are directly
 430 causing the difference in the field morphology between FT and FF dynamos.
 431 If this is the case, the different field structure should appear, kinematically, in
 432 an early stage after a weak seed field is added on the non-magnetic convection.
 433 We tested this possibility for the FF case at $Ra' = 700$ and $\eta = 0.35$. Figure
 434 14 shows the time evolution of the dipolarity, the field strength and the mean
 435 spherical harmonic degree of the flow. The initial seed field grows to a field
 436 of weak strength ($\Lambda \sim 0.3$) and multipolar morphology ($f_{\text{dip}}^{\text{ax}} \sim 0.11$) with a
 437 length scale ($l_u \sim 17$) similar to that for the non-magnetic stage, and stays in
 438 this state for approximately 1.3 magnetic diffusion time. After that, the field
 439 starts to increase in strength and dipolarity and the flow length scale becomes
 440 larger, settling finally at $\Lambda \sim 8.6$, $f_{\text{dip}}^{\text{ax}} \sim 0.65$ and $l_u \sim 14$. This shows that
 441 the difference in flow structure, that is caused directly by the different thermal
 442 boundary conditions, is not sufficient to explain the different magnetic field
 443 morphologies. The slight change in the flow is sufficient to trigger, in the FF
 444 case, the growth of a dipolar field, however, it requires the back-reaction of
 445 this field on the flow structure for the dipole field to fully develop and reach
 446 its field strength.

447 4. Conclusion and Discussion

448 In order to explore the early Martian dynamo and the early geodynamo,
 449 which both probably operated without an inner core being present, we con-
 450 ducted dynamo simulations purely driven by volumetric internal heating.
 451 Previous attempts in this direction have shown conflicting results concerning
 452 the magnetic field geometry. However, the simulations differed in the ther-
 453 mal boundary condition at the outer boundary and the size of an inner core
 454 that was retained for practical reason. Our simulations demonstrate that
 455 the discrepancies in these finding result from the thermal boundary condi-
 456 tions rather than the different inner core sizes. The fixed heat flux (FF)
 457 condition promotes dipole-dominated strong magnetic fields when the local
 458 Rossby number is not too large. As in the case of bottom-heated convec-
 459 tion, the critical values for the local Rossby number are around 0.1. The
 460 condition of fixed temperature (FT) always yields multipolar and somewhat
 461 weaker fields, at least at an Ekman number of 10^{-4} , irrespective of the inner
 462 core size. Numerical limitations prevented us from exploring inner cores with
 463 radii smaller than 10 % of the total core radius, but the results by Aubert et
 464 al. (2009) for sizes down to 1 % are consistent with our results. Note that the
 465 inner core size is more important for bottom-heated cases where its surface
 466 provides the main buoyancy sources (Heimpel et al., 2005).

467 In addition to the differences in the magnetic field, the FF and FT sim-
 468 ulations also show profound differences in the flow structure. FF cases have
 469 thermal wind-dominated zonal flows, larger meridional circulation cells and
 470 a generally larger convective flow scale. This seems to be promoted by the
 471 stronger dipole field, which can more efficiently balance the Coriolis force.

472 The resulting changes in the axisymmetric temperature distribution, the
 473 zonal wind structure and the meridional circulation have already been re-
 474 ported by Aubert (2005). The change towards larger length scales found in
 475 the non-axisymmetric pattern of the convective flow is an additional effect
 476 by the enhanced role of Lorentz forces associated with a dominantly dipolar
 477 field.

478 In the volume-heated cases we discuss here, the convective action and
 479 generation of magnetic field by helical motion is concentrated at larger radii
 480 than in the bottom-heated cases (compare Fig. 4 in Olson et al. (1999)).
 481 Therefore the inner core size has a minor influence while the outer thermal
 482 boundary condition has a profound impact. In the (predominantly) bottom-
 483 heated cases, dipole-dominated fields are the norm for not too vigorous flow.
 484 In the volume-heated cases, the thermal boundary condition controls whether
 485 a dipolar or multipolar configuration is assumed.

486 The enlarged length scale in FF cases is reminiscent to the effects re-
 487 ported for magnetoconvection or Cartesian dynamo simulations where a
 488 strong enough imposed-magnetic field around $\Lambda = \mathcal{O}(1)$ promotes large
 489 and more vigorous flow structures (see Wicht et al. (2009) and references
 490 therein). The magnetoconvection simulations show a more drastic regime
 491 change than observed here from a weak-field branch with small-scale flow
 492 structures, whose typical wavenumbers are $\mathcal{O}(Ek^{1/3})$, to a strong-field branch
 493 with flow structures of $\mathcal{O}(1)$. They also demonstrate that the effect becomes
 494 larger for smaller Ekman numbers. Takahashi et al. (2008a) report a decrease
 495 in the mean harmonic degree of the flow, l_u , of approximately 20% when com-
 496 paring a dynamo simulation with a non-magnetic convection simulation at

497 $Ek = 2 \times 10^{-6}$. This is the parameter range where magneto-convection calcu-
 498 lations already see a much more drastic regime change. In our FF cases with
 499 $Ra' = 700$ and $Ek = 10^{-4}$, the mean harmonic degree l_u in the FF dynamo
 500 case is approximately 25% lower than that in the non-magnetic FF convec-
 501 tion case. A change in length scale comparable to that reported by Takahashi
 502 et al. (2008a) thus occurs also at larger Ekman number, $Ek = 10^{-4}$.

503 In our simulations the flow vigor is smaller in the FF cases. For example,
 504 we find $Rm = 204$ in the FT case at $Ra' = 700$ and $\eta = 0.35$ but only
 505 $Rm = 153$ in the respective FF simulation. This is not really in line with the
 506 picture of a weak field or a strong field branch, where a strong magnetic field
 507 not only increases the wavelengths of the flow but also the velocity. However,
 508 it agrees with the observation in many dynamo simulations that periods of
 509 stronger magnetic energy go along with reduced kinetic energy.

510 Sakuraba & Roberts (2009) showed that the fixed heat flux condition
 511 promotes scale change in a simulation at Ekman number $Ek \sim 2.4 \times 10^{-6}$
 512 with a combination of internal and basal heating. They also claimed that the
 513 difference in scale between cases with fixed flux and fixed temperature exists
 514 only at low Ekman numbers. However, we find that the effect can already be
 515 observed at a much larger Ekman number of $Ek = 10^{-4}$. We confirmed the
 516 importance of the FF condition, which leads to a decrease of approximately
 517 30% in mean harmonic degree l_u and a drastic increase of energy carried
 518 by low-order harmonics in the flow (Fig. 7). A difference is that we find
 519 multipolar fields when using a FT condition, whereas Sakuraba & Roberts
 520 (2009) obtain a dipole-dominated field also in the FT case. The difference
 521 may be attributed to the lower Ekman number or the partial basal heating

522 in their model.

523 It remains unclear how the different flow structures created by the thermal
 524 boundary conditions promote the different field morphologies. Results from
 525 kinematic dynamo studies propose that axial dipole parity is not favoured
 526 when the axisymmetric flow components are weak (e.g. Gubbins & Gibbons,
 527 2009). This does not agree with our findings where axial dipolar fields are
 528 compatible with rather small axisymmetric flows (compare our FF experi-
 529 ments, corresponding to $-0.12 \lesssim D \lesssim -0.05$ and $-0.005 < M \lesssim -0.002$,
 530 with Fig. 1 or 11 in Gubbins & Gibbons (2009)). The discrepancy between
 531 preferred parities in kinematic and the present MHD dynamos may suggest
 532 that mode selection is not explained only by linear kinematic process. Our
 533 simulation starting from non-magnetic convection with a small seed field, in
 534 which a strong dipolar field emerges only after a long transient stage with
 535 a weak multipolar field, shows that the back-reaction of the Lorentz forces
 536 modifying the flow plays an important role for selecting the field morphology.
 537 This mode selection seems to depend on a subtle balance, which the thermal
 538 boundary condition can tip one way as the other.

539 The mantle of terrestrial planets imposes a heat flux rather than a tem-
 540 perature on the boundary to the dynamo region. Fixed flux simulations are
 541 more appropriate than simulations with fixed temperature to model dynamos
 542 in terrestrial planets in general, the early Martian dynamo, and early geody-
 543 namo before the inner core developed in particular. Our simulations suggest
 544 that the early dynamos had dipole-dominated fields.

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Table 1: Dynamo cases with $Ek = 1 \times 10^{-4}$, $Pm = 3$, $Pr = 1$. In the second column labeled 'b.c.', FT and FF represent uniform temperature and heat flux boundary conditions, respectively. 'Morp.' indicates field morphologies. The fractions of axisymmetric parts of poloidal/toroidal kinetic (M/D) and magnetic energies (P/T) to total kinetic and magnetic energies, respectively, are represented in %. The four last rows are for non-magnetic convection cases with $Ek = 1 \times 10^{-4}$, $Pr = 1$. Cases highlighted by bold font are illustrated in Figures 4-13.

η	b.c.	Ra'	Nu^*	Rm	Ro_ℓ	l_u	Λ	$f_{\text{dip}}^{\text{ax}}$	l_B^P	m_B^P	Morp.	$M[\%]$	$D[\%]$	$P[\%]$	$T[\%]$
0.35	FT	350	1.6	136	0.024	16.5	0.06	0.052	3	1	Octo.	0.02	4.0	2.1	1.2
0.35	FT	450	1.9	153	0.031	19.2	0.17	0.081	2-4	2	Hemis.	0.02	4.1	3.1	1.3
0.35	FT	700	2.5	204	0.043	19.6	0.48	0.085	3-5	1	Hemis.	0.04	4.0	2.1	1.3
0.35	FT	2100	4.9	372	0.078	19.8	1.95	0.091	7-15	1-3	Multi.	0.09	4.3	1.8	1.4
0.35	FT	4200	7.1	511	0.110	20.2	4.02	0.075	9-20	1-3	Multi.	0.16	4.4	1.4	1.3
0.10	FT	700	1.8	148	0.021	13.0	0.03	0.0002	2	0-1	Quad.	0.02	7.3	13.1	6.2
0.10	FT	1400	2.8	235	0.036	14.6	0.48	0.12	3-6	1	Hemis.	0.03	5.4	4.3	2.3
0.10	FT	2100	3.6	297	0.046	14.7	0.96	0.13	3-11	1	Hemis.	0.05	5.0	3.6	2.3
0.35	FF	450	2.1	119	0.016	12.1	5.64	0.72	1	0	Dip.	0.19	11.8	21.8	17.2
0.35	FF	700	2.7	153	0.023	13.7	8.24	0.66	1	0	Dip.	0.27	9.6	18.2	13.5
0.35	FF	2100	5.1	303	0.061	18.8	9.71	0.59	1	0	Dip.	0.23	4.9	10.5	5.1
0.35	FF	4200	7.8	450	0.088	18.3	12.99	0.52	1	0	Dip.	0.30	7.0	5.7	4.5
0.35	FF	6300	10.2	608	0.120	18.6	9.14	0.26	9-13	0-1	Multi.	0.36	13.1	3.4	4.0
0.35	FF	8400	11.5	724	0.147	19.1	7.53	0.16	8-17	1	Multi.	0.39	11.9	2.3	2.3
0.10	FF	700	2.0	123	0.012	9.4	3.52	0.83	1	0	Dip.	0.12	9.1	29.6	14.8
0.10	FF	1400	3.1	181	0.021	10.6	8.49	0.62	1	0	Dip.	0.44	10.1	18.6	15.8
0.10	FF	2100	3.9	230	0.028	11.2	11.01	0.59	1	0	Dip.	0.44	9.5	13.8	17.3
0.10	FT	450	1.2	94	0.010	10.4	—	—	—	—	No	0.01	5.9	—	—
0.10	FF	450	1.4	89	0.010	10.9	—	—	—	—	No	0.01	6.5	—	—
0.35	FF	700	2.2	—	0.040	18.2	—	—	—	—	—	0.04	10.6	—	—
0.10	FF	700	1.7	—	0.019	12.5	—	—	—	—	—	0.02	11.4	—	—

Figure 1: Relative axial-dipole strength versus local Rossby number. White and black symbols represent cases with fixed temperature (FT) and heat flux (FF) boundary conditions, respectively. The aspect ratio η is indicated by the shapes of the symbols, 0.35 for squares and 0.10 for circles. The relative axial dipole strength of the present (2000) geomagnetic field is indicated by the broken line.

Figure 2: Phase diagrams for the field morphologies at $r = r_o$ with FT (a) and FF (b) conditions. A filled square indicates axial dipolar field, a filled triangle - quadrupolar field, a circle - octopolar field, an asterisk - hemispherical field, an empty triangle - higher degree-dominated field and a cross - no dynamo.

Figure 3: Magnetic spectra, averaged in time and radius and normalized by the total magnetic energy, for spherical harmonic degree l (solid lines) and order m (dashed lines) at $Ra' = 700$ with aspect ratios $\eta = 0.35$ (a) and 0.10 (b). Thin and thick lines represent cases for FT and FF conditions, respectively.

Figure 4: Radial component of the magnetic field at the outer boundary for FT boundary condition at $Ra' = 700$ with aspect ratios $\eta = 0.35$ (a) and 0.10 (b) at snapshots. Colour represents intensity and direction of the field: yellow to red for outward field and light blue to dark blue for inward. Contour intervals are 0.125 (a) and 0.025 (b).

Figure 5: Axisymmetric parts of the toroidal field (a,d), poloidal field line (b,e) and helicity (c,f) for FT condition for $\eta = 0.35$ (upper row, a-c) and 0.10 (lower row, d-f) with $Ra' = 700$, at the same instant in time as in Figure 4a and b, respectively. Dash-dotted (solid) lines for negative (positive) values. Contour steps are 0.10 (a), 0.025 (d), 20000 (c) and 10000 (f).

Figure 6: Axial(z) component of vorticity in the equatorial plane for FT condition at $\eta = 0.35$ (a) and 0.10 (b) with $Ra' = 700$ at the same instant in time as in Figure 4a and b, respectively. Solid lines for positive values. Contour interval is 600.

Figure 7: Time-averaged kinetic energy spectra as function of harmonic order, normalized by total kinetic energy, for $\eta = 0.35$ (a) and 0.10 (b) with $Ra' = 700$. Thin solid, thick solid and dotted lines represent the dynamo case with FT condition, the dynamo case with FF condition and the non-magnetic convection with FF condition, respectively.

Figure 8: Axisymmetric parts of temperature (a,d), zonal (b,e) and meridional flow (c,f) for FT condition at $\eta = 0.35$ (upper row, a-c) and 0.10 (lower row, d-f) with $Ra' = 700$ at the same instant in time as in Figure 4a and b, respectively. Dashed lines for negative values and clockwise circulation, respectively. Contour steps are 0.02 (a,d) and 10 (b,e).

Figure 9: Radial component of the field at the outer boundary for FF condition at $Ra' = 700$ with aspect ratio $\eta = 0.35$ (a) and 0.10 (b). Contour interval is 0.25 .

Figure 10: Axisymmetric parts of the toroidal field (a,d), poloidal field line (b,e) and helicity (c,f) for FF condition for $\eta = 0.35$ (upper row, a-c) and 0.10 (lower row, d-f) with $Ra' = 700$, at the same time as in Figure 9, respectively. Contour steps are 0.50 (a), 0.25 (d) and 5000 (c,f).

Figure 11: Axisymmetric temperature (a,d), zonal (b,e) and meridional flow (c,f) for FF condition at $\eta = 0.35$ (upper row, a-c) and 0.10 (lower row, d-f) with $Ra' = 700$ at the same time as in Figure 9a and b, respectively. Contour steps are 0.02 (a,d) and 10 (b,e).

Figure 12: Axial(z)-component of vorticity in the equatorial plane for FF condition at $\eta = 0.35$ (a) and 0.10 (b) with $Ra' = 700$, at the same time as in Figure 9a and b, respectively. Contour step is 600.

Figure 13: Non-magnetic convection with FF condition for $\eta = 0.10$ and $Ra' = 700$: z -component of vorticity in the equatorial plane (a), axisymmetric temperature (b), zonal flow (c) and meridional flow (d). Contour steps are 600 (a), 0.02 (b) and 10 (c).

Figure 14: Dipolarity, Elsasser number and mean harmonic degree of flow (from top to bottom) versus time for FF dynamo case at $Ra' = 700$ and $\eta = 0.35$.