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Abstract
This paper studies the optimal environmental policy in a mixed market when pollution accumulates over time. Specifically, we assume quantity competition between several private firms and one partially privatized firm. The optimal emission tax is shown to be independent of the weight the privatized firm puts on social welfare. The optimal tax rule, the accumulated stock of pollution, firms' production paths and profit streams are identical irrespective of the public firm's ownership status.

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1 Introduction

Most analyses of the regulation of polluting firms have assumed private firms. This assumption ignores an important feature of a number of regulatory settings: the active role of public and (partially) privatized firms as providers of goods and services. As a result of the process of market liberalization in Western Europe and of the transition process in the countries comprising the former Soviet Union, Eastern Europe and Asia, mixed market structures are becoming increasingly common. Actually, public firms compete with private firms in many highly polluting sectors such as energy supply, transportation, iron and steel, chemicals and petrochemicals. They are responsible for releasing large amounts of toxic compounds that accumulates in the environment causing present as well as long-term environmental damages. Thus, the issue of the environmental regulation of mixed markets deserves important consideration.

The purpose of this paper is to analyse efficiency-inducing taxation when the market is served by private and public (or partially privatized) providers. In this connection, we would like to address two questions. First, we want to understand how the mixed market structure affects the design of the optimal corrective tax. Second, we are interested in the welfare effect of privatization when optimal taxation is used before and after privatization. With these purposes in mind, we introduce a partially privatized firm in the model studied by Benchekroun and Van Long (1998).

In line with the literature on mixed oligopolies, we assume that firms with different ownership structures differ in their aims. Since the privatized firm is partly privately owned and partly state-owned, its objective reflects the different interests of its public and private shareholders. Following Bös (1991) and Matsumura (1998) we describe the payoff function of the privatized firm as a weighted average of social welfare and its own profit.

We obtain an irrelevance result. Namely, we find that the optimal linear markovian tax rule which decentralizes the social optimum as an open-loop Nash equilibrium of the differential oligopoly game is independent of the degree of privatization of the public firm. Thus, the optimal environmental policy tells us that technologically identical firms must be taxed the same whatever their ownership status. Furthermore, this result is robust to changes in the information structure of the differential oligopoly game considered. Indeed, the optimal tax rule remains independent of the extent of privatization if we assume that oligopolists use closed-loop strategies. Turning to the welfare effects of privatization, we prove that welfare is unchanged by privatization when the optimal tax rule is used. This result stems directly from the fact that the social optimum is independent of the degree of privatization and thus unique.


Our irrelevance result suggests that mixed oligopolies and private oligopolies should not differ substantially in terms of economic and environmental performance if pollution charge programs are correctly designed. This result seems consistent with experience and empirical evidence which indicate that the economic and environmental consequences of privatization reforms are mixed and vary substantially across sectors and countries\(^1\). Privatization conveys promises of increased productive efficiency and more efficient use of resources, improved access to capital markets and greater investments in cleaner technologies, better management practices and easier access to markets for environmentally friendly goods and services. However, it also involves costs. For example, the decrease in supply as a result of the exercise

\(^1\)For a recent survey of empirical studies of privatization, see Megginson and Netter (2001). Environmental implications of privatisation are extensively reviewed in Lovei and Gentry (2002).
of increased market power may result in a larger economic deadweight loss. In most cases
analysed to date, the quality of environmental regulations and commercial pressure have been
playing a preeminent role in the successes and failures of privatization reforms. Environmental
tax exemptions or lax environmental regulations have resulted in poor environmental perfor-
ance, whatever the ownership structure of the industry (e.g., Lovei and Gentry 2002).

The remainder of this paper is organized as follows. Section 2 describes the basic model.
Section 3 characterizes the social optimum. Optimal corrective tax rules are derived for open-
loop and closed-loop mixed markets in sections 4 and 5, respectively. Section 6 concludes the
paper.

2 The Model

Consider a mixed market consisting of one public firm (indexed by 0) and \( n \) identical private
firms (indexed by 1, 2, ..., \( n \)). Market competition takes place a la Cournot-Nash over the con-
tinuous time period \([0, \infty] \). In each period, firms face a downward sloping inverse demand
function \( p = P(Q) \) where \( Q \equiv \sum_{i=0}^{n} q_i \) with \( q_i \) denoting the quantity produced by firm \( i \).
The total cost function of firm \( i \) is \( C_i(q_i) \) with \( C_i(0) = 0 \), \( C_i'(q_i) > 0 \) and \( C_i''(q_i) \geq 0 \). We assume
that technology is identical across private firms; i.e., \( C_i(q) = C_1(q), \forall q > 0 \) and \( \forall i = 1, 2, \ldots, n \).
However, it may differ between public and private firms; i.e., \( C_0(q) \geq C_1(q) \), \( \forall q > 0 \).
Production of good \( q \) generates emissions of a stock pollutant. Firm \( i \)'s level of pollution emis-
sions is \( e_i = q_i \). The dynamics of the stock of pollution \( S \) is described by

\[
\dot{S}(t) = Q(t) - \delta S(t), \quad S(0) = S_0 \geq 0, \quad (1)
\]

where the coefficient \( \delta > 0 \) reflects the environment’s self-cleaning capacity and \( S_0 \) is the initial size of the pollution stock.

The welfare of society at time \( t \) depends on the current vector of production decisions \( q(t) = (q_0(t), q_1(t), \ldots, q_n(t)) \) and the current stock of pollution \( S(t) \). It is measured by the sum of consumers’ and producers’ surplus less environmental damages. At time \( t \) social welfare is given by

\[
w(t) = \sum_{i=1}^{n} q_i(t) \int_0^{q_0(t)} P(u)du - C_0(q_0(t)) - \sum_{i=1}^{n} C_i(q_i(t)) - D(S(t)), \quad (2)
\]

where the damage function \( D(\cdot) \) measures the economic loss resulting from the current stock of pollution \( S(t) \) and satisfies \( D(0) = 0, D'(0) = 0, \) and \( D'(S) > 0, D''(S) > 0, \forall S > 0 \). Furthermore, we assume that \( P(0) > C_i'(0), \forall i = 0, 1, \ldots, n \).
The social optimum \( q^*(t) = (q^*_0(t), q^*_1(t), \ldots, q^*_n(t)) \) is defined as the solution of the following problem:

\[
\max_{q(t) \geq 0} W = \int_0^{\infty} w(t)e^{-r}dt, \quad (3)
\]

\[
s.t. \dot{S}(t) = Q(t) - \delta S(t), S(0) = S_0 \geq 0, q(t) \geq 0.
\]

Following Benchekroun and Van Long (1998), we suppose that the regulator uses linear Markov
tax rules to regulate pollution. Namely, we assume that each firm is charged a tax \( \tau_i[S(t)] \) per
unit of output, where the unit tax depends only on the current pollution stock \( S(t) \).

The timing of the environmental regulation game is as follows. Prior to market competition,
the regulator announces the markovian tax scheme \( \tau(S) = (\tau_0(S), \tau_1(S), \ldots, \tau_n(S)) \) that will be
applicable to the firms. Then, firms engage in Cournot competition at each subsequent instant of time \( t \in [0, \infty) \) taking as given the tax policy followed by the regulator.

In the remainder of this section, we define firms’ objective functions, specify the information structure of the game and state the problem that the environmental regulator must solve.

### The Polluting Oligopoly

Let us assume that the environmental regulator imposes a tax \( \tau_i(S) \) on each unit of pollution produced by firm \( i \) (\( i = 0, 1, 2, \ldots, n \)). Then, firm \( i \)’s instantaneous profit level is \( \pi_i(t) = P[q_i(t)]q_i(t) - C_i[q_i(t)] - \tau_i(S(t))q_i(t) \) where \( Q_{-i}(t) = -q_i(t) + \sum_{i=0}^{n} q_i(t) \). Following Bös (1991) and Matsumura (1998), private firms are considered to be profit maximizers while the privatized firm’s instantaneous objective is a weighted average of social welfare and its own profit, \( f_0(t) = (1 - \theta)w(t) + \theta \pi_0(t) \), where \( \theta \in [0, 1] \). In this formulation, the weight \( (1 - \theta) \) describes the extent to which the government is able to control the behavior of the public firm through the shares that it has retained. If \( \theta = 1 \) the privatized firm behaves as a private oligopolist whereas if \( \theta = 0 \) the privatized firm behaves as a welfare-maximizing public firm. Here, \( \theta \) is assumed to be exogenously given and readily observable by the firms. Accordingly, each private firm solves

\[
\max_{q_i(t) \geq 0} \Pi_i = \int_0^\infty \pi_i(t)e^{-rt}dt \tag{4}
\]

\[
s.t. \dot{S}(t) = Q(t) - \delta S(t), \quad S(0) = S_0 \geq 0. \tag{5}
\]

whereas the privatized firm solves

\[
\max_{q_0(t) \geq 0} F_0 = \int_0^\infty f_0(t)e^{-rt}dt, \tag{6}
\]

\[
s.t. \dot{S}(t) = Q(t) - \delta S(t), \quad S(0) = S_0 \geq 0. \tag{7}
\]

The specific sets of strategies that are available to the firms depend on the information structure of the game. In this paper we restrict our attention to open-loop (OL) and closed-loop (CL) information structures. Under an OL-information structure, firms are unable to observe the current state of the game. Consequently, they condition their strategies only on time. Namely, each firm \( i \) (\( i = 1, \ldots, n \)) uses an OL-strategy; i.e., a decision rule of the form \( q_i(t) = \phi_i(t) \).

By contrast, under a CL-information structure, firms are able to observe the current state of the game and use this information to revise their strategies at each point of time. Each firm \( i \) (\( i = 1, \ldots, n \)) uses a CL-strategy; i.e., a decision rule of the form \( q_i(t) = \phi_i(S(t)) \). Whatever the information structure considered, the relevant equilibrium concept is the Nash equilibrium. Let us recall that an OL (CL) Nash equilibrium is a profile of OL (CL) strategies that are mutual best responses.

### The Environmental Regulator

At a prior stage the environmental regulator determines the system of linear Markov tax rules \( \tau(S) = (\tau_0(S), \ldots, \tau_n(S)) \) to regulate pollution. Having determined firms’ optimal behaviors in the oligopoly subgame, it selects the tax scheme \( \tau(S) \) so as to maximize aggregate social welfare. Formally, the optimal tax scheme \( \tau^*(S) \) is obtained by solving

\[
\max_{\tau(S)} W^e = \int_0^\infty w^e(t)e^{-rt}dt, \tag{8}
\]

\[
s.t. \dot{S}(t) = Q(t) - \delta S(t), S(0) = S_0 \geq 0, q(t) \geq 0. \tag{9}
\]
where the superscript \( e \in \{ \text{ol}, \text{cl} \} \) indicates whether variables are evaluated at the OL or CL-Nash equilibrium of the underlying dynamic oligopoly game\(^2\).

3 The Social Optimum

The social optimum can be obtained by solving problem (3)\(^3\). First we derive the necessary conditions for optimality\(^4\). Second we characterize the steady state solution. Let \( \lambda_r \) denotes the costate (or adjoint) variable associated with \( \dot{S} \). Assuming interior solutions, the maximum principle implies the following necessary and sufficient optimality conditions

\[
-\lambda_r = P(Q) - C_i'(q_i), \quad \forall i = 0, \ldots, n, \tag{10}
\]

\[
\dot{\lambda}_r = \lambda_r r - \partial H_r / \partial S = \lambda_r (r + \delta) + D'(S), \tag{11}
\]

\[
\lim_{t \to \infty} e^{-rt} \lambda_r(t)S(t) = 0, \tag{12}
\]

along with equation (1). From Equation (10), the costate variable \( \lambda_r \) is negative and represents the shadow cost of the pollution stock. Furthermore, solving these conditions yields

\[
C_i'(q_i) = C_j'(q_j), \quad \forall i = 0, 1, \ldots, n. \tag{13}
\]

Optimality requires that the aggregate output be produced at least cost; i.e., marginal costs of the last unit of output must be equal across firms. Using Equations (10) and (13) to eliminate the social shadow cost from (11), we obtain the following system of equations

\[
P'(Q)\dot{Q} - C''_i(q_i)q_i = (r + \delta)[P(Q) - C'_i(q_i)] - D'(S), \quad \forall i = 0, 1, \ldots, n. \tag{14}
\]

The steady state is then defined by \((\dot{q}^\infty_i + \sum_{i=1}^n \dot{q}^\infty_i) = \hat{Q}^\infty = \delta \hat{S}^\infty\), where \( \hat{S}^\infty \) satisfies

\[
P(\delta \hat{S}^\infty) = C_i'(\hat{q}^\infty_i) + D'(\hat{S}^\infty)/(r + \delta) \tag{15}
\]

and the optimal allocation of production is given by \( C_i'(\hat{q}^\infty_i) = C_j'(\hat{q}^\infty_j), \forall i, j(i \neq j) \in \{0, 1, \ldots, n\} \). Condition (15) states that production should be allocated so that marginal benefits equal marginal production costs plus the present value of marginal external damages. It can be clearly seen from above that firms’ ownership structure is immaterial from the point of view of the social planner. Indeed, social optimality requires exclusively that allocative efficiency and cost efficiency conditions be satisfied.

4 Open-Loop Mixed Oligopoly

We assume that firms are unable to revise their production paths once they have made their choices; i.e., we assume an open-loop information structure. An OL-Nash equilibrium is a

\(^2\)Recall that \( w = CS(Q) + \pi_0 + \sum_{i=1}^n \pi_i - D(S) + \tau_0(S)q_0 + \sum_{i=1}^n \tau_i(S)q_i \) where \( CS(Q) = \int_0^Q P(u)du - pQ \) is the consumer’s surplus. Social welfare rewrites as

\[
w = \int_0^Q P(u)du - P(Q)Q + P(Q)q_0 - C_0(q_0) - \tau_0(S)q_0 - \sum_{i=1}^n (P(Q)q_i - C_i(q_i) - \tau_i(S)q_i) + \sum_{i=0}^n \tau_i(S)q_i - D(S),
\]

\[^3\]Detailed derivations are available in Claude and Tidball (2006)

\[^4\]Because of the convexity properties of the problem, the necessary conditions are also sufficient conditions for optimality.
and (21) with (14), the following conditions obtain:

\[ P'(Q)q_i + P(Q) - C'_i(q_i) - \tau_i(S) + \lambda_i = 0, \quad \forall i \neq 0, \quad (16) \]

\[ P(Q) - C'_0(q_0) + \lambda_0 + \theta \left[ -\tau_0(S) + q_0 P'(Q) \right] = 0, \quad (17) \]

\[ \dot{\lambda}_i = \lambda_i r - \partial H_i / \partial S = \lambda_i (r + \delta) + \tau'_i(S) q_i, \quad \forall i \neq 0, \quad (18) \]

\[ \dot{\lambda}_0 = \lambda_0 r - \partial H_0 / \partial S = \lambda_0 (r + \delta) + \theta \tau'_0(S) q_0 + (1 - \theta) D'(S), \quad (19) \]

together with (1) and the \((n + 1)\) transversality conditions, \( \lim_{t \to \infty} e^{-rt} \lambda_i(t) S(t) = 0, \quad \forall i = 0, 1, \ldots, n \). Using (16) to eliminate \( \lambda_i \) from (18), the following conditions obtain,

\[ P'' \dot{Q} q_i + P' \dot{q}_i + P' \dot{Q} - C''_i \dot{q}_i - \tau'_i \dot{S} = (r + \delta) [P' q_i + P - C'_i - \tau_i] - \tau'_i q_i, \quad \forall i = 1, \ldots, n, \quad (20) \]

where \( P = P(Q) \), \( C'_i = C'_i(q_i) \) and \( \tau_i = \tau_i(S) \). Similarly, from (17) and (19), we get:

\[ C'_0 \dot{q}_0 - P' \dot{Q} + \theta \left[ \tau'_0 \dot{S} - P'' \dot{Q} q_0 - P' \dot{q}_0 \right] = (1 - \theta) D' + \theta \tau'_0 q_0 + (r + \delta) \left[ C'_0 - P + \theta (\tau_0 - P' q_0) \right] \quad (21) \]

We proceed with the stability analysis of the system defined by (20) and (21) together with (1). The OL-Nash equilibrium steady state pollution stock \( \hat{S}^\infty \) must satisfy the following system of \((n + 1)\) equations:

\[ (r + \delta) (C'_i - P) = (P' q_i - \tau_i)(r + \delta) - \tau'_i q_i, \quad \forall i = 1, \ldots, n, \quad (22) \]

\[ (r + \delta) (C'_0 - P) + D' = \theta \left[ (P' q_0 - \tau_0)(r + \delta) - \tau'_0 q_0 + D' \right]. \quad (23) \]

Now we are in a position to study how the environmental regulator can decentralize the social optimum as an OL-Nash equilibrium of the oligopoly game. The regulator designs \( \tau(S) \) so that firms optimality conditions match the socially optimal conditions. By comparison of (20) with (14) and (21) with (14), the following conditions obtain:

\[ \tau'_i \dot{S} - P'' \dot{Q} q_i - P' \dot{q}_i - (r + \delta) (\tau_i - P' q_i) + D' - \tau'_i q_i = 0, \quad (24) \]

\[ \tau'_0 \dot{S} - P'' \dot{Q} q_0 - P' \dot{q}_0 - (r + \delta) (\tau_0 - P' q_0) + D' - \tau'_0 q_0 = 0. \quad (25) \]

From Section 3, we know that \((r + \delta) (C'_i - P(\delta \hat{S}^\infty)) + D'(\hat{S}^\infty) = 0\), so that (22) and (23) become:

\[ P(\delta \hat{S}^\infty) = C'_i + \tau_i(\hat{S}^\infty) + q_i(\hat{S}^\infty) \left[ \tau'_i(\hat{S}^\infty) / (r + \delta) - P'(\delta \hat{S}^\infty) \right], \quad \forall i = 0, \ldots, n \quad (26) \]

The profile of markovian tax rules \( \tau(S) \) must satisfy conditions (24) and (25) and \( \tau(\hat{S}^\infty) \) must satisfy the \((n + 1)\) steady state conditions (26). Clearly, these conditions are independent of \( \theta \). We thus obtain the irrelevance result stated in the following proposition:

**Proposition 4.1.** When the environmental regulator uses efficiency inducing taxation in order to regulate a polluting oligopoly, the optimal linear-Markov taxation scheme, the time-path of pollution accumulation, firms’ time-paths of production and profit streams are identical irrespective of whether i) all \((n + 1)\) firms behave as profit maximizers or ii) a partially privatized firm competes in quantities with \(n\) private firms.
The basic intuition for proposition 1 is simple. To begin with, consider the two limiting cases: the regulation of a private oligopoly and that of a pure mixed oligopoly. The first is obtained by setting $\theta = 1$ in the objective of the public firm. In this case, the public firm is a profit maximizer and the problem boils down to the regulation of a private polluting oligopoly. From Benchekroun and Van Long (1998), we know that there exists an optimal tax rule which induces firms to follow the socially optimal production path. The second is obtained by setting $\theta = 0$. In this case, the privatized firm maximizes aggregated social welfare. Corrective taxation does not affect the output decision of the public firm directly; the tax only affects the behavior of the public firm through its effect on private firms’ output levels. Now, suppose that the regulator uses the tax rule obtained in the private oligopoly case to regulate the pure mixed oligopoly. Then, private firms follow the optimal production path. Since the public firm seeks to maximize social welfare, its best response to the behavior of private firms is also to follow the socially optimal production path.

Now, consider intermediate cases, $\theta \in [0, 1]$, in which the partially privatized firm deviates from strict welfare maximization without being exclusively profit oriented. Suppose that the regulator uses the tax rule obtained in the private oligopoly case to regulate the mixed market. Now, corrective taxation affects the behavior of the public firm directly since it appears in its profits. Public and private owners of the public firm have a common interest in following the socially optimal production path. Indeed, it would be the policy chosen by the public shareholders if they were the unique owners of the privatized firm and the choice of private shareholders if they were the unique owners of the privatized firm.

### 5 Closed-Loop Mixed Oligopoly

We now proceed by considering the broader class of CL-strategies in order to prove that our irrelevance result is not contingent upon assumptions regarding the informational structure. Since optimality conditions for private firms are independent of $\theta$, we may restrict our attention to the behavior of the partially privatized firm. Now, each firm assumes that the strategies used by its competitors are a function of the accumulated stock. Accordingly, firm 0 chooses the output path $q_0^*(t)$ which maximizes its discounted payoff $F_0$ subject to (1). Let $Q_{-0}(S) = \sum_{i=1}^n \phi_i(S)$. Assuming interior solutions, the optimality conditions are

$$\lambda_0 = \left( C_0' - P \right) + \theta \left[ \tau_0 - q_0P \right],$$

$$\dot{\lambda}_0 = (1 - \theta) \left[ \sum_{i=1}^n C_i' \phi_i' + D' - P'Q_{-0}' \right] + \theta q_0 \left[ \tau_0' - P'Q_{-0}' \right] + \lambda_0 \left[ (r + \delta) - Q_{-0}' \right],$$

together with (1) and $\lim_{t \to \infty} e^{-rt} \lambda_0(t)S(t) = 0$. Following the same steps as in Section 4, one obtains the following condition on $\tau_0(S)$:

$$\Lambda + Q_{-0}'(\tau_0 + (C_i' - P)) = 0$$

where $\Lambda = \tau_0 \dot{S} - P' \dot{Q}q_0 - (r + \delta) (\tau_0 - P' q_0) + D' - \tau_0 q_0$ is the bracketed term in (25). Again, we observe that this equation will be satisfied or not regardless of the value of $\theta$. Furthermore, following the same steps as in Section 4, it is straightforward to show that the corresponding steady state condition is independent of $\theta$. The system of linear Markov tax rules must satisfy a system of differential equations that is independent of $\theta$ and thus the following proposition obtains:

**Proposition 5.1.** When the environmental regulator uses efficiency inducing taxation in order to regulate a polluting oligopoly, the optimal linear-Markov taxation scheme, the time-path of
pollution accumulation, firms’ time-paths of production and profit streams are identical irrespective of whether i) all \((n + 1)\) firms behave as profit maximizers or ii) a partially privatized firm competes in quantities with \(n\) private firms.

6 Conclusion

We considered efficiency-inducing taxation for a polluting oligopoly in which a partially privatized firm competes with private firms. The analysis of this paper provided some answers to hitherto neglected questions about the interaction between privatization and environmental taxation. Assuming that the partially privatized firm maximizes a weighted average of social welfare and its own profit, we proved that the optimal corrective tax scheme is independent of the weight the privatized firm puts on its own profit; i.e., the extent of privatization. This result tells us that technologically identical privatized and private firms should be taxed the same even if they have different incentives to produce. It was shown that this conclusion holds regardless of whether firms use OL or CL strategies. Turning to the welfare effect of privatization, we proved that social welfare is unchanged by privatization when the optimal environmental policy is implemented. Indeed, we showed that the optimal environmental policy decentralizes the social optimum. Then, since the social optimum is unique and the optimal tax policy is independent of the extent of privatization, the same level of aggregate welfare is achieved irrespective of the ownership status of the public firm.

References


