Towards a new concept for high sensitivity Compton scatter emission imaging
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NEW CONCEPT FOR HIGH SENSITIVITY SCATTERED GAMMA-RAY IMAGING

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ABSTRACT

A new efficient scheme for imaging gamma-ray emitting objects is advocated in this work. A few years back we have put forward the idea of collecting data on Compton scattered rays to reconstruct an object in three dimensions and shown how it works with a collimated detector. To increase drastically the sensitivity of this modality as well as its field of view and resolution we propose now that data acquisition should be performed without collimation. We discuss image formation by scattered radiation in this context by computing and comparing the related Point Spread Functions (PSF) and comment on their properties. We also present numerical simulations to support the attractiveness of this modality.

Key words: Gamma-ray imaging, sensitivity, photon Compton scattering, image formation, object reconstruction.

1. INTRODUCTION

Emission imaging with gamma rays is widely used in numerous fields such as medical imaging, non-destructive testing, gamma astronomy and environmental survey. In conventional nuclear imaging, a collimated gamma camera rotates in space to collect primary radiation emitted by an object under investigation. In this case Compton scatter radiation behaves generally as noise hindering image quality and consequently correction to scatter should be applied.

However recently an interesting new imaging concept, which precisely uses, as imaging agent, scattered radiation by the object medium (instead of primary radiation), has been proposed. A spatially fixed collimated gamma camera records now images labeled by the energy of scattered radiation (or equivalently its scattering angle). It is then shown that the reconstruction of a three-dimensional object is feasible using this data [1, 2, 3]. However in this situation, the image sensitivity is considerably affected due to the presence of the collimator. Only about one out of $10^4$ scattered photons reaches the detector. Therefore in order to record a much larger amount of scattered radiation, we propose to extend the working principle of this gamma-ray imaging to a functioning modality without collimator, as depicted by Fig. 1. However, as can be seen, this procedure differs from other proposals on scattered radiation imaging processes [4, 5, 6, 7], in particular:

- Compton tomography [8], which reconstructs the electron density of the object (instead of its activity density), and uses a moving point-like detector collecting scattered radiation from an external radiation source,
- Compton camera [9], which reconstructs the activity density of an object from scattered radiation using coincidence measurements between a site on a scatter layer-detector and another site on an absorption-detector.

As the true three-dimensional problem implies involved space geometry of scattered rays, we shall first study its two-dimensional counterpart to test the viability of this idea. To this end, we give a careful analysis of the image formation. The corresponding PSF is derived and shall be compared with the previous case where a collimator is present. Next we perform numerical reconstruction of the object radiation activity from simulated scattered radiation (recorded by a camera without collimator) to illustrate the working of this concept.

Fig. 1. Two modalities in emission scattered radiation imaging.
2. IMAGE FORMATION

To understand image formation by scattered radiation, we follow radiation propagation in a two-dimensional (2D) slice (or thin section) of a scattering medium with an electronic density \( n_e \) assumed to be approximately constant. This can be in principle implemented with a slit placed above a detector without collimator, see Fig. 2. Moreover, to concentrate chiefly on scattering effects radiation, attenuation will be left out as working hypothesis.

\[ f(S) \cdot dS \]

\[ \frac{d\phi(D, \omega | S)}{2\pi} = \frac{f(S)|dS|}{2\pi} \cdot 2 \arctan \left( \frac{\sigma}{2|SM|} \right) \cdot \frac{1}{\sigma^2} \]

\[ \times n_e(M) \cdot dS_M \cdot \sigma^2(S) \cdot 2 \arctan \left( \frac{\sigma'}{2|MD|} \right) \cdot \frac{1}{\sigma'^2} \cdot \cos \theta, \]

where \( \theta \) is the angle between the outgoing photon unit vector with the detector normal unit vector, \( |MD| \) the distance from scattering site \( M \) to detection site \( D \). In fact, for a given point source \( S \), there will be two scattering sites \( M_1 \) and \( M_2 \) located on two arcs of circle subtending a scattering angle \( (\pi - \omega) \), as shown in Fig. 4.

The total photon flux density at a site \( D \) is \( g(D, \omega) \), the integral over all source sites and all scattering sites such that the scattering angle is \( \omega \). This last constraint is expressed by a \( \delta \)-function in the integration as

\[ g(D, \omega) = \int \int d\phi(D, \omega | S) \cdot \delta(SMD - (\pi - \omega)) \]

\[ \text{Fig. 3. Geometry of Compton scattering: source site } S, \text{ scattering site } M \text{ and detection site } D \text{ of width } \sigma'. \]

\[ \text{Fig. 2. Two dimensional working with a slit.} \]
As shown before for a camera without collimator, the scattering sites due to a single point source $S$ are located on two circular arcs subtending an angle $(\pi - \omega)$ (Eq. (4)). The photon flux density received at site $D$ is then given by an integration over these two circular arcs (see Fig. 4).

For computation ease, we shall assume a uniform electron density $n_e$ and use polar coordinates $(r, \gamma)$, such that $S = (d, \alpha)$ with $DS = d$ and $M = (r, \gamma)$, with $DM = r$ and $\overrightarrow{DS} \cdot \overrightarrow{DM} = r\, d\cos\gamma$.

The circular arcs have polar equations:

$$r = d \frac{\sin(\omega - \gamma)}{\sin \omega} \quad \text{and} \quad r = d \frac{\sin(\omega + \gamma)}{\sin \omega}, \quad (6)$$

where $\gamma$ is the angle between $\overrightarrow{DS}$ and $\overrightarrow{DM}$. The distance $|SM|$ can be extracted from a simple identity in the triangle $DSM$:

$$|SM| = d \frac{\sin \gamma}{\sin \omega}. \quad (7)$$

And the integration area $dS_M$ is now reduced to the arc element:

$$\sqrt{dr^2 + r^2 d\gamma^2} = \frac{d}{\sin \omega} d\gamma. \quad (8)$$

Hence the PSF is given by the sum of the two integrations on $\gamma$:

$$PSF(D, \omega|S) = \frac{K(\omega)}{\sigma \sigma'} \frac{d}{\sin \omega} \sum_{2\text{Arcs}} \int_0^{\gamma(\omega)} d\gamma \cos \theta(\gamma) \quad (9)$$

where $K(\omega) = 4n_e \sigma^2 C_S(\omega) f_0 / 2\pi$, $\sigma^2 C_S(\omega)$ the differential Compton cross section at scattering angle $\omega$, $f_0$ the intensity of the single point source and $\cos \theta = \sin(\alpha - \gamma)$ if the detector lies along the $0x$ axis and $l$ is the distance between the line detector and the linear lower boundary of the medium $L$. The integration is carried out over the points inside the scattering medium. Therefore when the medium is of finite extent, the limit of the integration $\gamma_{l}(\omega)$, which corresponds to the intersection of the arcs of circle with the scattering medium, should be calculated beforehand, see Fig. 4.

Now if the collimator is mounted on the detector, then only one scattering site $M$, located on the perpendicular to the detector at site $D$, will contribute to detection site $D$, (see Fig. 5). Thus the integration on $\gamma$ is restricted by a delta function which picks out only the corresponding value of $\gamma$, i.e.:

$$\gamma_{col} = \frac{\pi}{2} - \alpha. \quad (10)$$

The resulting PSF expression for a collimated detector is:

$$PSF_{col}(D, \omega|S) = \frac{K(\omega)}{\sigma \sigma'} \frac{d}{\sin \omega} \arctan \left( \frac{\sigma \sin \omega}{2d \sin \frac{\gamma_{col}}{2}} \right) \arctan \left( \frac{\sigma' \sin \omega}{2d \cos(\alpha - \omega)} \right). \quad (11)$$

Now at a fixed scattering angle $\omega$, the PSF curve as function of the detector position with collimator has a Mexican hat shape (see Fig. 6).
while the PSF curve without collimator has a wide Lorentzian shape (see Fig 7).

For example at a scattering angle of 30 degrees, Fig. 8 shows that the PSF without collimator is about 10 times stronger than the PSF with collimator.

4. NUMERICAL RECONSTRUCTION RESULTS

As an illustration of this new imaging concept, we carried out numerical reconstructions of a two-dimensional Shepp-Logan medical phantom from data computed with our model.

The 2D original object (see Fig. 9) is placed at the center of the scattering medium and a unit distance above the detector. A line detector of 55 pixels of 1 unit length, placed on the axis $y = 0$, is simulated. We consider that the scattering medium has the same properties as water. It consists of a discretized square of $55 \times 55$ elements of unit area.

A series of 55 images of the object corresponding to 55 different scattering angles ($12^\circ < \omega < 132^\circ$) have been simulated. We construct the $3025 \times 3025$ weight matrix by computing, for each mesh point source, the PSF at the different scattering angles for each site on the detector. The reconstruction is carried out by inverting this weight matrix using the Singular Value Decomposition method, which is less time
consuming, compared to other reconstruction methods.

Fig. 10. Shepp-Logan phantom reconstruction with collimator

Fig. 11. Shepp-Logan phantom reconstruction without collimator

Fig. 10 and Fig. 11 show the reconstruction results with and without collimator. One observes a better agreement with the original object when the collimator is removed. In Fig. 10, we can see that the part of the object near the detector is better reconstructed than the upper part of the object. The three small structures are invisible. The reconstructive relative error is about 9.13 %. But without collimator (see Fig. 11), the whole object is correctly reconstructed. All structures are visible and the relative error is about $1.17 \times 10^{-4}$ %.

These results have been subsequently validated by Monte-carlo simulations [10].

5. CONCLUSION

The feasibility of image reconstruction using Compton scattered rays detected by a gamma-camera without collimator, operating in a fixed position, is demonstrated in this study. This is the essence of a new concept of high sensitivity imaging, which takes advantage of scattering rays instead of rejecting them as done usually. The main point in this imaging process by Compton scattered radiation is the fact that data acquisition is performed without the usual motion of the detector. This is a major advantage compared to existing imaging systems which require a heavy, bulky and costly mechanical rotation mechanism to move the detector around in space. Work towards an extension to three-dimensional imaging is in progress. The modeling and simulations of multiple scattered radiation in this context will be also subjects of future investigations. These promising results may open the way to new high sensitivity imaging devices which will have applications in nuclear medicine, non-destructive industrial control, high energy astrophysics, environmental survey, etc.

6. REFERENCES


