Accepted Manuscript

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PII: S0167-7152(10)00175-6
DOI: [10.1016/j.spl.2010.06.014](https://doi.org/10.1016/j.spl.2010.06.014)
Reference: STAPRO 5736

To appear in: *Statistics and Probability Letters*

Received date: 5 April 2010

Revised date: 16 June 2010

Accepted date: 25 June 2010

Please cite this article as: Kozubowski, T.J., Podgórski, K., Random self-decomposability and autoregressive processes. *Statistics and Probability Letters* (2010), doi:10.1016/j.spl.2010.06.014

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Random self-decomposability and autoregressive processes

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Abstract

We introduce the notion of random self-decomposability and discuss its relation to the concepts of self-decomposability and geometric infinite divisibility. We present its connection with time series autoregressive schemes with regression coefficient that randomly turns on and off. In particular, we provide a characterization of random self-decomposability as well as that of marginal distributions of stationary time series that follow this scheme. Our results settle an open question related to the existence of such processes.

AMS 2000 subject classification: primary 60E05; secondary 60E07; 62M10

Keywords: Autoregressive process; class L; geometric infinite divisibility; geometric stable law; infinite divisibility; Laplace distribution; Linnik distribution; Mittag-Leffler distribution; non-Gaussian time series; self-decomposable law; stable distribution

1. Introduction

There is a growing interest in non-Gaussian autoregressive (AR) models discussed in Vervaat (1979),

$$X_n = \delta_n X_{n-1} + \varepsilon_n, \quad (1.1)$$

where ε_n and δ_n are independent sequences of independent, identically distributed (IID) random variables (see, e.g., Jayakumar and Mathew, 2002, 2005; Mathew and Jayakumar, 2003, 2005). In particular, the AR structure

$$X_n = \begin{cases} \varepsilon_n & \text{with probability } p \\ \varepsilon_n + cX_{n-1} & \text{with probability } 1 - p, \end{cases} \quad (1.2)$$

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where $p, c \in [0, 1]$, with either exponential or (symmetric and skew) Laplace marginal distributions for $\{X_n\}$, appeared in Dewald and Lewis (1985), Lawrance (1981), Lawrance and Lewis (1981), and Jayakumar and Kuttykrishnan (2008).

Motivated by a question raised by Lawrance (1981), who asked whether similar models can be derived with either gamma or Gaussian stationary distributions, we introduce the class of randomly self-decomposable (RSD) distributions. These provide the only stationary solutions in the AR scheme (1.2) for each $p, c \in [0, 1]$, and bridge the notions of self-decomposability (SD) and geometric infinite divisibility (GID). This class is introduced in Section 2, where we provide its characterization through SD and GID. In Section 3 we discuss connections with AR schemes, followed by examples collected in Section 4, where we also include brief remarks on generalizations to higher-order autoregressive schemes.

2. Random self-decomposability

When we assume stationarity, it is clear that the process (1.2) is well defined whenever the relevant characteristic functions (ChF) satisfy the equation

$$\psi(t) = \psi_{c,p}(t)\{p + (1-p)\psi(ct)\}, \quad (2.1)$$

where ψ and $\psi_{c,p}$ are the ChFs of X_n and ε_n , respectively. This motivates the following definition

Definition 2.1. A distribution with the ChF ψ is said to be randomly self-decomposable (RSD) if for each $p, c \in [0, 1]$ there exists a probability distribution with the ChF $\psi_{c,p}$ satisfying (2.1).

Remark 2.2. In terms random variables, the relation (2.1) takes the form

$$X \stackrel{d}{=} cIX + X_{c,p}, \quad (2.2)$$

where X and $X_{c,p}$ are random variables with the ChFs ψ and $\psi_{c,p}$, respectively, I is a Bernoulli variable with $\mathbb{P}(I = 1) = 1 - p$, and all the variables on the right-hand-side of (2.2) are mutually independent. Note that for $c = 0$ or $p = 1$ the relations (2.1)-(2.2) hold trivially with $\psi_{0,p} = \psi_{c,1} = \psi$ and $X_{0,p} = X_{c,1} = X$.

When $p = 0$, the relations (2.1)-(2.2) reduce to

$$\psi(t) = \psi(ct)\psi_c(t), \quad c \in [0, 1], \quad (2.3)$$

and

$$X \stackrel{d}{=} cX + X_c, \quad (2.4)$$

respectively, where we used the notation $\psi_c = \psi_{c,0}$ and $X_c = X_{c,0}$. This shows that every RSD ChF is also self-decomposable, in other words, $\mathcal{C}_{RSD} \subset \mathcal{C}_{SD}$, where $\mathcal{C}_{RSD}$ and $\mathcal{C}_{SD}$ are the classes of RSD and SD laws, respectively.

On the other hand, when $c = 1$ then (2.1) becomes

$$\psi(t) = \psi_p(t)\{p + (1-p)\psi(t)\}, \quad p \in [0, 1], \quad (2.5)$$

while (2.2) reduces to

$$X \stackrel{d}{=} IX + X_p, \quad (2.6)$$

where we used the notation $\psi_p = \psi_{1,p}$ and $X_p = X_{1,p}$. Solving (2.5) for ψ produces

$$\psi(t) = \frac{p\psi_p(t)}{1 - (1-p)\psi_p(t)}, \quad (2.7)$$

showing that for each $p \in [0, 1]$, the variable X with the ChF ψ can be decomposed into a random sum

$$X \stackrel{d}{=} X_p^{(1)} + X_p^{(2)} + \dots + X_p^{(N_p)}, \quad (2.8)$$

where N_p is a geometric random variable with the distribution

$$\mathbb{P}(N_p = x) = p(1-p)^{x-1}, \quad x = 1, 2, \dots, \quad (2.9)$$

and the $X_p^{(j)}$ are IID copies of X_p with the ChF ψ_p , independent of N_p . In other words, the distribution of X is geometrically infinitely divisible (GID) as defined in Klebanov et al. (1984). Thus, $\mathcal{C}_{RSD} \subset \mathcal{C}_{GID}$, where $\mathcal{C}_{GID}$ is the classes of GID distributions.

The above shows that $\mathcal{C}_{RSD} \subset \mathcal{C}_{GID} \cap \mathcal{C}_{SD}$. The following shows that we actually have an equality here.

Proposition 2.3. *We have $\mathcal{C}_{RSD} = \mathcal{C}_{GID} \cap \mathcal{C}_{SD}$, where $\mathcal{C}_{RSD}$, $\mathcal{C}_{GID}$, and $\mathcal{C}_{SD}$ denote the classes of RSD, GID, and SD laws, respectively. Moreover, whenever $\psi \in \mathcal{C}_{RSD}$, then the ChF $\psi_{c,p}$ in (2.1) can be written as*

$$\psi_{c,p}(t) = \psi_p(ct) \cdot \psi_c(t), \quad (2.10)$$

where ψ_c and ψ_p are given by

$$\psi_c(t) = \psi(t)/\psi(ct) \quad (2.11)$$

and

$$\psi_p(t) = \frac{\psi(t)}{p + (1-p)\psi(t)}, \quad (2.12)$$

respectively.

Proof. Since we have already seen that $\mathcal{C}_{RSD} \subset \mathcal{C}_{GID}$ as well as $\mathcal{C}_{RSD} \subset \mathcal{C}_{SD}$, it follows that $\mathcal{C}_{RSD} \subset \mathcal{C}_{GID} \cap \mathcal{C}_{SD}$. It remains to show that $\mathcal{C}_{GID} \cap \mathcal{C}_{SD} \subset \mathcal{C}_{RSD}$. To see this, note that if the ChF ψ is SD, then for each $c \in [0, 1]$ the quantity (2.11) is a genuine ChF. Similarly, if ψ is GID then (2.12) is another ChF for each $p \in [0, 1]$. Consequently, (2.10) is a well-defined ChF as well, and straightforward algebra shows that (2.1) is valid. This concludes the proof.

3. Relation with autoregressive schemes

In view of Definition 2.1, we have the following obvious result.

Proposition 3.1. *A stationary stochastic process (1.2) is well defined for every $p, c \in [0, 1]$ if and only if the distribution of X_n is randomly self-decomposable.*

It follows that an AR process (1.2) with distribution of X_n given by a ChF ψ and distribution of ε_n given by (2.10) can be obtained if and only if X_n is a RSD random variable. Thus, processes of this form with either (general) gamma or Gaussian distributions of X_n cannot be constructed, as neither of these distributions is GID (although both are SD). This settles the question raised in Lawrance (1981).

Remark 3.2. When $p = 0$ we have the standard AR scheme,

$$X_n = cX_{n-1} + \varepsilon_n. \quad (3.1)$$

Non-Gaussian stationary models of the form (3.1) were studied by Anděl (1983) - Gaussian, rectangular, Laplace, Cauchy, exponential, gamma, and mixed exponential, Dewald and Lewis (1985) - Laplace, Lawrance (1981) - exponential, gamma, Laplace, and mixed exponential, Jayakumar et al. (1995) - Linnik, Jayakumar and Pillai (1993) - Mittag-Leffler, Lawrance and Lewis (1980) - exponential, Gaver and Lewis (1980) - exponential, gamma, and mixed exponential, Sim (1994) - logistic, hyperbolic secant, exponential, and Laplace, Gibson (1986) - Laplace, Lawrance and Lewis (1981) - exponential, Jose et al. (2008) - Gaussian-Laplace, Tomy and Jose (2009) - generalized Gaussian-Laplace, and Jayakumar et al. (2009) - skew Laplace. It is important to note that one can not construct autoregressive schemes of the form (3.1) with arbitrary stationary distributions of the $\{X_n\}$ and valid for all $p \in [0, 1]$. Indeed, as observed by Gaver and Lewis (1980) and others, the ChFs ψ and ψ_c of the X_n and ε_n , respectively, must satisfy the relation (2.3), so that the marginal distribution of X_n must belong to the class of SD laws.

Remark 3.3. The case $c = 1$ corresponds to Bernoulli distributed δ_n in (1.1),

$$X_n = \begin{cases} \varepsilon_n & \text{with probability } p \\ \varepsilon_n + X_{n-1} & \text{with probability } 1 - p. \end{cases} \quad (3.2)$$

Stationary models of this form include those where the marginal distribution of the $\{X_n\}$ is skew Laplace (see Jayakumar and Kuttykrishnan, 2007), exponential (see Lawrance and Lewis, 1980, 1981), Linnik (see Anderson and Arnold, 1993, Jayakumar et al., 1995), Mittag-Leffler (see Jayakumar and Pillai, 1993), geometric Mittag-Leffler (see Seetha Lekshmi and Jose, 2004b), geometric Laplace/ α -Laplace and their tailed versions (see Seetha Lekshmi and Jose, 2004a; Seetha Lekshmi et al., 2003), geometric Mittag-Leffler (see Jayakumar and Ajitha, 2003), and geometric generalized Linnik (see Seetha Lekshmi and Jose, 2006). Again, such process are well defined for each $p \in [0, 1]$ whenever the ChFs ψ and ψ_p of X_n and ε_n , respectively, satisfy the relation (2.5). Thus, only GID distributions can appear as the marginal distribution of X_n given by (1.2) with $c = 1$ (see, e.g., Jose and Pillai, 1995; Kozubowski and Podgórski, 2008).

4. Examples

Here is a brief account of examples of autoregressive processes of the form (1.2), which either have already appeared in the literature or can be constructed following the theory above. According to Propositions 3.1 and 2.3, the marginal distribution of X_n must be both GID and SD. Recall that every GID ChF ψ is of the form

$$\psi(t) = \frac{1}{1 - \log \phi(t)}, \quad (4.1)$$

where ϕ is an infinitely divisible (ID) ChF (see, e.g., Klebanov et al., 1984). However, in general, ψ may not be SD. One class that provides a variety of RSD distributions are GID distributions (4.1) with stable ChF ϕ . These are geometric stable (GS) distributions, given by the ChF

$$\psi(t) = (1 + \sigma^\alpha |t|^\alpha \omega_{\alpha, \beta}(t) - i\mu t)^{-1}, \quad (4.2)$$

where $\alpha \in (0, 2]$ is the tail index, $\beta \in [-1, 1]$ is a skewness parameter,

$$\omega_{\alpha, \beta}(x) = \begin{cases} 1 - i\beta \text{sign}(x) \tan(\pi\alpha/2), & \text{if } \alpha \neq 1, \\ 1 + i\beta \frac{2}{\pi} \text{sign}(x) \log |x|, & \text{if } \alpha = 1, \end{cases} \quad (4.3)$$

and $\mu \in R$ and $\sigma \geq 0$ control location and scale, respectively (see, e.g., Klebanov et al., 2006). We shall consider two RSD sub-classes of this family - strictly GS distributions and (skew) Laplace laws.

4.1. Strictly GS laws and processes

Strictly GS distributions correspond to strictly stable ChF ϕ in (4.1), so that in (4.2) - (4.3) we either have $\alpha \neq 1$ and $\mu = 0$ or $\alpha = 1$ and $\beta = 0$, with the exponential distribution ($\sigma = 0$) being another special case. Moreover, for a strictly GS random variable X we have

$$X \stackrel{d}{=} p^{1/\alpha} \left(X^{(1)} + X^{(2)} + \dots + X^{(N_p)} \right), \quad p \in (0, 1), \quad (4.4)$$

where the $\{X^{(j)}\}$ are IID copies of X (see, e.g., Klebanov et.al., 1984). Comparing this with (2.8) we see that here $X_p^{(j)} \stackrel{d}{=} p^{1/\alpha} X$, so that in (2.5) we have $\psi_p(t) = \psi(p^{1/\alpha}t)$. By setting $c = p^{1/\alpha}$, the relation (2.5) also shows that X must be SD, with

$$\psi_c(t) = c^\alpha + (1 - c^\alpha)\psi(t) \quad (4.5)$$

in the relation (2.3). Note that in the standard autoregressive scheme (3.1) with a strictly GS $X_n \stackrel{d}{=} X$, the quantity (4.5) will be the ChF of the ε_n . Since here we have an atom at the origin, such processes will suffer from the so called zero defect, which is a well-known phenomenon, where we have successive values of the $\{X_n\}$ follow geometric progression. This is no longer the case in the more general setting (1.2) with a strictly GS distributed X_n . Indeed, in view of Proposition 2.3, the ChF of the error term ε_n is of the form

$$\psi_{c,p}(t) = \psi_p(ct) \cdot \psi_c(t) = \psi(p^{1/\alpha}ct)(c^\alpha + (1 - c^\alpha)\psi(t)), \quad (4.6)$$

so that

$$\varepsilon_n \stackrel{d}{=} p^{1/\alpha} c X'_n + I X''_n, \quad (4.7)$$

where X'_n and X''_n are independent copies of X_n , I is a Bernoulli variable with $\mathbb{P}(I = 1) = 1 - c^\alpha$, and all the variables on the right-hand-side of (4.7) are mutually independent. It is worth noting several special cases.

4.1.1. Exponential laws and processes

When $\sigma = 0$ in (4.2), we obtain exponential distribution with mean μ , which is strictly GS with $\alpha = 1$ in (4.4). This exponential autoregressive process (1.2) is the NEAR(1) process studied in Lawrance (1981) and Lawrance and Lewis (1981).

4.1.2. Mittag-Leffler laws and processes

An important heavy-tail generalization of exponential distribution is the Mittag-Leffler distribution (see, e.g., Klebanov et al., 2006), which is strictly GS with $\alpha < 1$ and $\beta = 1$, most conveniently described via its Laplace transform (LT)

$$\psi(t) = \frac{1}{1 + \lambda t^\alpha}, \quad t > 0. \quad (4.8)$$

A Mittag-Leffler autoregressive process (1.2) appears to be new, and generalized previous models (3.1) and (3.2) with this marginal distribution, mentioned in introduction.

4.1.3. Laplace and Linnik laws and processes

When $\alpha = 2$ and $\mu = 0$ in (4.2), we obtain the classical Laplace distribution, with the ChF

$$\psi(t) = \frac{1}{1 + \sigma^2 |t|^\alpha}, \quad t \in \mathbb{R}, \quad (4.9)$$

with $\alpha = 2$ and $\sigma > 0$. Since this distribution is both GID and SD, we can define a Laplace autoregressive process (1.2) - the NLAR(1) model mentioned in Dewald and Lewis (1985). A further generalization is obtained by considering $\alpha \in (0, 2)$ in (4.9), leading to the (symmetric) Linnik distribution (see, e.g., Klebanov et al., 2006). A Linnik autoregressive process (1.2) appears to be new, and generalized previous models (3.1) and (3.2) with this marginal distribution, mentioned in introduction.

4.2. Skew Laplace laws and processes

The ChFs of geometric stable distributions with $\alpha = 2$ reduce to

$$\psi(t) = \frac{1}{1 + \sigma^2 t^2 - i\mu t}, \quad t \in \mathbb{R}, \quad (4.10)$$

and define the class $AL(\mu, \sigma)$ of asymmetric Laplace (AL) distributions, studied extensively in Kotz et al. (2001). The densities of AL distributions admit an explicit form,

$$f(x) = \frac{1}{\sigma} \frac{\kappa}{1 + \kappa^2} \begin{cases} \exp(-\frac{\kappa}{\sigma}x) & \text{if } x \geq 0 \\ \exp(\frac{1}{\kappa\sigma}x) & \text{if } x < 0, \end{cases} \quad (4.11)$$

where the alternative parameter $\kappa > 0$ is related to μ via the relations

$$\kappa = \frac{2}{\frac{\mu}{\sigma} + \sqrt{4 + (\frac{\mu}{\sigma})^2}}, \quad \frac{1}{\kappa} - \kappa = \frac{\mu}{\sigma}. \quad (4.12)$$

Since all AL distributions are both, SD and GID, they are also RSD, and can serve as the marginal distributions of X_n in the autoregressive scheme (1.2). Proposition 2.3 and straightforward algebra shows that the ChF of the error term ε_n in this model is of the form

$$\psi_{c,p}(t) = \psi_p(ct) \cdot \psi_c(t) = \frac{1}{1 + p\sigma^2 c^2 t^2 - ip\mu ct} \left\{ \pi_1 + \pi_2 \frac{1}{1 - i\frac{\sigma}{\kappa}t} + \pi_3 \frac{1}{1 + i\sigma\kappa t} \right\}, \quad (4.13)$$

where

$$\pi_1 = c^2, \quad \pi_2 = (1 - c) \frac{1 + c\kappa^2}{1 + \kappa^2}, \quad \pi_3 = (1 - c) \frac{c + \kappa^2}{1 + \kappa^2}. \quad (4.14)$$

Here, the error term admits the stochastic representation

$$\varepsilon_n \stackrel{d}{=} cX + I_1 \{I_2 E_1 + (I_2 - 1)E_2\}, \quad (4.15)$$

where X has the asymmetric Laplace $AL(\mu p, \sigma\sqrt{p})$ distribution, E_1 and E_2 are exponential random variables with parameters κ/σ and $1/\kappa\sigma$, respectively, I_1 and I_2 are Bernoulli variables with $\mathbb{P}(I_1 = 1) = 1 - \pi_1$ and $\mathbb{P}(I_2 = 1) = \pi_2/(1 - \pi_1)$, respectively, with all the variables on the right-hand-side of (4.15) being mutually independent. This autoregressive scheme appeared in Jayakumar and Kuttykrishnan (2008), while its special cases with $c = 1$ and $p = 0$ were studied in Jayakumar and Kuttykrishnan (2007) and Jayakumar et al. (2009), respectively.

4.3. Generalizations to higher order autoregressive schemes

There several non-equivalent ways to generalize autoregressive schemes with random coefficients to higher order autoregressive models. Starting with Lawrance and Lewis (1980), a scheme of generalizing (1.2) through

$$X_n = \begin{cases} \varepsilon_n & \text{with probability } p = 1 - p_1 - \dots - p_r, \\ \varepsilon_n + cX_{n-1} & \text{with probability } p_1, \\ \dots & \dots \\ \varepsilon_n + cX_{n-1} + \dots + cX_{n-r} & \text{with probability } p_r, \end{cases} \quad (4.16)$$

has been discussed on several occasions. Specifically, the case of $c = 1$ appears in Lawrance and Lewis (1982), Dewald and Lewis (1985) (the second order case), Jayakumar and Kuttykrishnan (2007), Jayakumar and Ajitha (2003), Seetha Lekshmi and Jose (2004b), Seetha Lekshmi and Jose (2004a), Seetha Lekshmi and Jose (2006). It is clear that the relation (2.1) when written as

$\psi(t) = \psi_{c,p}(t)\{p + p_1\psi(ct) + \dots + p_r\psi(ct)\}$ is providing with a characterization of distributions that lead to a stationary time series (4.16).

Even more general higher order models can be introduced through a generic extension

$$X_n = \sum_{k=1}^r \delta_{n,k} X_{n-k} + \varepsilon_n,$$

where $(\delta_{n,1}, \dots, \delta_{n,k})$ are independent of the $\{X_n\}$ and the $\{\varepsilon_n\}$. However, the dependence structure among the $\delta_{n,i}$, $i = 1, \dots, k$, needs to be defined, and can lead to essentially different models. For example, alternative model to (4.16) is obtained via

$$X_n = \sum_{k=1}^r \delta_{n-k} c_k X_{n-k} + \varepsilon_n,$$

where the $\{c_i\}$ are real numbers and the $\{\delta_i\}$ are independent Bernoulli variables (taking on the values 1 and 0 with probability p and $1 - p$, respectively). For $r = 1$, this reduces to the models discussed in this note.

Replacing δ_{n-k} by $\delta_{n,k}$, with $(\delta_{n,1}, \dots, \delta_{n,k})$ independent for different n , leads to a different model, in which $P(\delta_{n,k} = 1)$ may depend on k .

Under the same assumptions, one can also consider

$$X_n = \sum_{k=1}^r \delta_{n-1} \dots \delta_{n-k} c_k X_{n-k} + \varepsilon_n,$$

which also reduces to (1.2) if $r = 1$. Note that this model satisfies

$$X_n = \begin{cases} \varepsilon_n & \text{with probability } 1 - p, \\ \varepsilon_n + c_1 X_{n-1} & \text{with probability } p(1 - p), \\ \dots & \dots \\ \varepsilon_n + c_1 X_{n-1} + \dots + c_r X_{n-r} & \text{with probability } p^r. \end{cases}$$

A variation on this is obtained by replacing δ_{n-k} by $\delta_{n,k}$, which leads to arbitrary probabilities in the above and different type of dependence when n is varying.

Finally, another interesting extension of (1.2) is obtained by considering the time $N_p(n)$ of the first success in the Bernoulli sequence δ_{n-i} and taking

$$X_n = \sum_{k=1}^{N_p(n)} c_k X_{n-k} + \varepsilon_n.$$

We note that (1.2) is obtained by taking $c_k = 0$ for $k > 1$. Again, a variation of this is obtained if the $\{N_p(n)\}$ are independent for different values n .

Studying existence of stationary distributions for the above models is an interesting and non-trivial research topic, which is however beyond the scope of this note.

Acknowledgments We thank associate editor and two anonymous referees for useful comments that improved the organization of this note. Research of the second author was supported by the Swedish Research Council Grant 2008-5382.

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