Recent progress in spin electronics have demonstrated that owing to the spin transfer torque (STT) [1, 2], biasing magnetic hybrid nanostructures by a direct current can lead to microwave emission. These spin transfer nano-oscillators (STNOs) [3–5] offer decisive advantages compared to existing technology in tunability, agility, compactness and integrability. In view of their applications in high-frequency technologies, a promising strategy to improve the coherence and increase the emitted microwave power of these devices is to mutually synchronize several of them [6–10].

The synchronization of the STNO oscillations to an external source has already been demonstrated [11, 12]. In particular, it has been shown that symmetric perturbations to the STNO trajectory favor even synchronization indices (ratio of the external frequency to the STNO frequency) [13, 14]. But so far, the influence of the spatial symmetry of the spin-wave (SW) mode which auto-oscillates on the synchronization rules has not been elucidated.

To address this open question, the spectroscopic identification of the auto-oscillating mode is crucial. It is usually a challenge, as a large variety of dynamic modes can be excited in STNOs, and their nature can change depending on the geometry, magnetic parameters and bias conditions. In this work, we study a STNO in the most simple configuration: a circular nanopillar saturated by a strong magnetic field applied along its normal. It corresponds to an optimum configuration for synchronization, since it has a maximal nonlinear frequency shift, which provides a large ability for the STNO to lock its phase to an external source [8]. Moreover, the perpendicular configuration coincides with the universal oscillator model, for which an exact analytical theory can be derived [15]. Last but not least, this highly symmetric case allows for a simplified classification of the SW eigenmodes inside the STNO [16].

We shall use here a magnetic resonance force microscope (MRFM) to monitor directly the power emitted by this archetype STNO vs. the bias dc current and perpendicular magnetic field. In the autonomous regime, these quantitative measurements allow us to demonstrate that the mode which auto-oscillates just above the threshold current is the fundamental, spatially most uniform SW mode. By studying the forced regime, we then show that this mode synchronizes only to an external source sharing the same spatial symmetry, namely, a uniform microwave magnetic field, and not the common microwave current passing through the device.

For this study, we use a circular nanopillar of nominal diameter 200 nm patterned from a (Cu60/Py15/Cu10/Py4/Au25) stack, where thicknesses are in nm and Py=Ns0/Fc20. A dc current \( I_{dc} \) and a microwave current \( i_{mt} \) can be injected through the STNO using the bottom Cu and top Au electrodes. A positive current corresponds to electrons flowing from the thick Py layer to the thin PyFe layer. This STNO device is insulated and an external antenna is patterned on top to generate a spatially uniform microwave magnetic field \( h_{mt} \) oriented in the plane of the magnetic layers. The bias magnetic field \( H_{ext} \), ranging between 8.5 and 11 kOe, is applied at \( \theta_H = 0^\circ \) from the normal to the sample plane.

The room temperature MRFM setup [17] consists of a spherical magnetic probe attached at the end of a very soft cantilever, coupled dipolarly to the buried nanopillar (see inset of Fig.1) and positioned 1.5 \( \mu \)m above its center. This mechanical detection scheme [18, 19] sensitively measures the variation of the longitudinal magnetization \( \Delta M_y \) over the whole volume of the magnetic body [20], a quantity directly proportional to the normalized power.
p emitted by the STNO [15]:

$$ p = \frac{\Delta M_z}{2M_s}, $$

where $M_s$ is the saturation magnetization of the precessing layer.

First, we measure the phase diagram of the STNO autonomous dynamics as a function of $I_{dc}$ and $H_{ext}$, see Fig.1. In this experiment, $I_{dc}$ is fully modulated at the cantilever frequency, $f_c \approx 12$ kHz, and the mechanical signal represents $\Delta M_z$ synchronous with the injection of $I_{dc}$ through the STNO. This quantitative measurement [21] is displayed using the color scale indicated on the right of Fig.1.

Three different regions can be distinguished in this phase diagram. At low negative or positive current (region (5)), $\Delta M_z$ is negligible, because in the subcritical region, the STT is not sufficient to destabilize the magnetization in the thin or thick layer away from the perpendicular field direction. As $I_{dc}$ is reaching a threshold negative value (from $-3$ to $-7$ mA as $H_{ext}$ increases from 8.5 to 10.7 kOe, see pink solid line in Fig.1), the MRFM signal starts to smoothly increase in region (6). It corresponds to the onset of spin transfer driven oscillations in the thin layer, which will be analyzed in details below. As $I_{dc}$ is further decreased towards more negative values, the angle of precession increases in the thin layer, until it eventually reaches $90^\circ$: at the boundary between regions (5) and (7) (see black dashed line) $4\pi \Delta M_z$ equals the full saturation magnetization in the thin layer, $4\pi M_s = 8$ kG.

Let us now concentrate on the spin transfer dynamics in the thin layer at $I_{dc} < 0$. We first turn to the quantitative analysis of the subcritical region (6). We introduce $\mathcal{N} = V M_s/(g\mu_B)$, the number of spins in the thin layer ($V$ is its volume, $g$ the Landé factor, $\mu_B$ the Bohr magneton). The averaged normalized power $p$ in the subcritical regime ($\left| I_{dc} \right| < I_{th}$) is evaluated in the stochastic nonlinear oscillator model described in section VII of ref. [15]. Under the assumption that only one SW mode dominates the STNO autonomous dynamics, Eq.(1) follows the simple relationship:

$$ \frac{\Delta M_z}{2M_s} = \frac{k_B T}{\mathcal{N} \hbar \omega_\nu} \frac{1}{1 - I_{dc}/I_{th}}, $$

where $I_{th} = 2\alpha \omega_\nu \mathcal{N} c/\epsilon$ is the threshold current for auto-oscillation of the SW mode $\nu$ with frequency $\omega_\nu$ ($\alpha$ is the Gilbert damping constant in the thin layer, $\epsilon$ the electron charge, and $\epsilon$ the spin torque efficiency). In Eq.(2), the prefactor

$$ \eta \equiv \frac{k_B T}{\mathcal{N} \hbar \omega_\nu} $$

is the noise power: the ratio between the thermal energy ($k_B$ is the Boltzmann constant, $T$ the temperature) and the maximal energy stored in the SW mode $\nu$ ($\hbar$ is the Planck constant over $2\pi$).

From Eq.(2), the inverse power is linear with the bias current $I_{dc}$ in the subcritical region. A sample measurement at $H_{ext} = 10$ kOe (along the white dashed line in Fig.1) is shown in Fig.2a. From a linear fit, one can thus obtain the threshold current $I_{th}$ and the noise power $\eta$ at this particular field. The dependencies of $I_{th}$ and $\eta$ on the perpendicular magnetic field are plotted in Figs.2b and 2c, respectively.

The parameters $V$, $M_s$, $g$ (hence, $\mathcal{N} = 6.3 \times 10^6$) and $\alpha = 0.014$ of the thin layer have been determined from an extensive MRFM spectroscopic study performed at $I_{dc} = 0$ on the same sample and published in ref.[16]. This study also yields the dispersion relations $\omega_\nu = (\hbar^{-1} N \gamma / N \gamma) (H_{ext} - H_\nu)$ of the thin layer SW modes ($\gamma = g\mu_B / \hbar = 1.87 \times 10^6$ rad.s$^{-1}$ G$^{-1}$) is the gyromagnetic ratio, $H_\nu$ the so-called Kittel field associated to the mode $\nu$). By injecting $\omega_\nu$ in the expression of the threshold current, it is found that the latter depends linearly on the perpendicular bias field:

$$ I_{th} = \frac{2\mathcal{N} c}{\epsilon} \gamma (H_{ext} - H_\nu), $$

as observed in Fig.2b. The linear fit of $I_{th}$ vs. $H_{ext}$ using Eq.(4) yields $H_\nu = 6.80 \pm 0.15$ kOe and $\epsilon = 0.30 \pm 0.005$. The importance of the analysis of Fig.2b is that, first, it provides an accurate determination of the spin torque efficiency, found to be in agreement with the accepted value in similar STNO stacks [22]. Second, a comparison with the SW modes of the thin layer (see black symbols extracted from ref.[16] and mode profiles in Fig.2b) shows that the fitted value of $H_\nu$ precisely corresponds to the Kittel field of the $(\ell, n) = (0, 0)$ mode, $\ell$ and $n$ being respectively the azimuthal and radial mode indices. It thus
allows us to conclude about the nature of the mode that first auto-oscillates at $I_{dc} < 0$ as being the fundamental, most uniform precession mode of the thin layer.

To gain further insight in our analysis of the subcritical regime, we compare in Fig.2c the noise power determined as a function of $H_{ext}$ with the prediction of Eq.(3), in which the dispersion relation of the $\nu = (0,0)$ SW mode is used. It is found that the fluctuations of the STNO power are well accounted for by those of the previously identified auto-oscillating mode, which confirms that the single mode assumption made to derive Eq.(2) is a good approximation.

Using two different microwave circuits, we shall now compare the ability of the auto-oscillating SW mode to phase-lock either to the uniform microwave field $h_{rf}$ generated by the external antenna, or to the microwave current $i_{rf}$ flowing through the nanopillar. We know from previous studies that in the exact perpendicular configuration, the SW spectrum critically depends on the method of excitation [16]: $h_{rf}$ excites only the axially symmetric modes having azimuthal index $\ell = 0$, whereas due to the orthoradial symmetry of the induced microwave Oersted field, $i_{rf}$ excites only the modes having azimuthal index $\ell = +1$. The dependencies on $I_{dc}$ and $H_{ext}$ of the STNO dynamics forced respectively by $h_{rf}$ and $i_{rf}$ are presented in Figs.3a and 3b. The plotted quantity is $\Delta M_z$ synchronous with the full modulation of the external source power: $h_{rf} = 1.9 \text{ Oe}$ (a) and $i_{rf} = 140 \mu\text{A}$ (b). Although the $\ell = 0$ and $\ell = +1$ spectra are in principle shifted by 1.1 GHz from each other, a direct comparison of the phase diagrams (a) and (b) can be made by using different excitation frequencies for $h_{rf}$ (8.1 GHz) and $i_{rf}$ (9.2 GHz).

Below the threshold current (indicated by the pink lines in Fig.3), the observed behaviors of the $\ell = 0$ and $\ell = +1$ modes are alike: a small negative dc current slightly attenuates the SW modes $B_{\ell n}$ of the thick PyB layer, while it promotes quite rapidly the SW modes $A_{\ell n}$ of the thin PyA layer, in agreement with the expected symmetry of the STT [16]. On the contrary, there is a clear qualitative difference between the modes $A_{00}$ and $A_{10}$ beyond $I_{th}$. Although both peaks similarly shift towards lower field as $I_{dc}$ is decreased towards lower negative values, $A_{00}$ gets strongly distorted, with the appearance of a negative dip on its high field side, in contrast to $A_{10}$, which remains a positive peak.

The negative MRFM signal observed in Fig.3a in the region of spin transfer driven oscillations in the thin layer is striking, because it means that the precession angle can be reduced in the presence of the microwave excitation $h_{rf}$. As a matter of fact, the distortion of the peak $A_{00}$ is associated to the synchronization of the auto-oscillating mode to the external signal. Fig.4a illustrates the distortion of the STNO emission frequency induced by this phenomenon. These data were obtained by monitoring the fluctuating voltage across the nanopillar at $I_{dc} = -7 \text{ mA}$ with a spectrum analyzer as a function of.
the applied magnetic field [23]. The frequency shift of the forced oscillations with respect to the free running oscillations is plotted in Fig.4b, along with the MRFM signal. This demonstrates that in the so-called phase-locking range, the STNO amplitude adapts ($\Delta f > 0$: increases, $\Delta f < 0$: decreases), so as to maintain its frequency equal to the frequency of the source, here fixed at 8.1 GHz. This comparison also allows to estimate the phase-locking bandwidth, found to be as large as 0.4 GHz despite the small amplitude of the external signal. The nonlinear frequency shift is indeed the largest at 8.1 GHz. This comparison also allows to estimate the overlap integral between the external source and the circuitt configuration, we have unambiguously identified the auto-oscillating mode profile is crucial to synchronize to an external source. But we believe that this finding might be important for future strategies to synchronize large STNOs arrays.

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FIG. 4. (Color online). (a) Magnetic field dependence of the STNO frequency in the free and forced regimes (the external source at 8.1 GHz is $h_{\text{rf}}$). (b) Comparison between the STNO frequency shift deduced from (a) and the MRFM signal.
[23] Here, a slight tilt of the angle $\theta_H = 2^\circ$ is required. Indeed, no oscillatory voltage is produced in the exact perpendicular configuration, due to the perfect axial symmetry of the STNO trajectory.