Square-wave oscillations with different duty cycles

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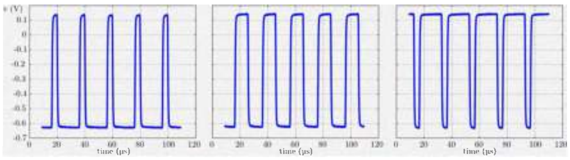

Figure 1: Square-wave oscillations. By changing the feedback phase Φ, the plateau lengths can be tuned.

A fundamental property of nonlinear dynamical systems controlled by a delayed feedback is their tendency to exhibit 2τ -periodic square-wave oscillations of equal plateau lengths (τ is the delay) [1]. The question was recently raised whether an optical system could exhibit stable square-wave oscillations with different plateau lengths [2]. We have experimentally found these regimes using a single optoelectronic oscillator (OEO) with a bandpass feedback [3]. See Fig. 1. The period is close to τ (and not 2τ). The response of the OEO is accurately described by the following delay differential equations (time $s = t/\tau$) [4]

$$y' = x,\ \varepsilon x' = -x - \delta y + \beta \left[\cos^2\left(x(s-1) + \Phi\right) - \cos^2(\Phi)\right]. \tag{1}$$

Here $\varepsilon = 10^{-3}$ and $\delta = 8.43 \times 10^{-3}$ are fixed parameters. Stable long time 1-periodic square-wave solutions have

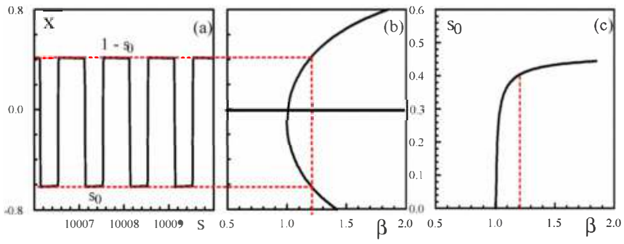

Figure 2: Numerical solution for $\Phi = -\pi/4 + 0.1$, $\beta = 1.2$ and analytical bifurcation diagrams.

been obtained numerically (see Fig. 2a). Taking advantage of the small values of ε and δ, we have determined analytically the bifurcation diagram of these square-waves. Fig. 2b and Fig. 2c show the extrema of x and the length of the shortest plateau as a function of β. As $s_0 \to 0$, the solution disappears through a bifurcation point. We have found numerically that this point is not connected to the Hopf bifurcation points of the zero solution but is an isolated bifurcation point.

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