Fuzzy Bipolar Conditions of Type ”or else”

Ludovic Lietard, Nouredine Tamani, Daniel Rocacher

Fuzzy Bipolar Conditions of Type "or else"

Ludovic Liétard
IRISA/IUT/Univ. Rennes 1,
Rue Edouard Branly, BP 30219
22302 Lannion Cedex, France
Email: ludovic.lietard@univ-rennes1.fr

Nouredine Tamani
IRISA/ENSSAT/Univ. Rennes 1,
6, rue de Kerampont, BP 80518,
22305 Lannion Cedex, France
Email: tamani@enssat.fr

Daniel Rocacher
IRISA/ENSSAT/Univ. Rennes 1,
6, rue de Kerampont, BP 80518,
22305 Lannion Cedex, France
Email: rocacher@enssat.fr

***Abstract*—Previously studied fuzzy bipolar conditions of type "and if possible" are made of a mandatory condition c and an optional condition w. They allow expressing complex preferences of a conjunctive nature. We define in this paper, a new kind of fuzzy bipolar conditions of the form "or else" which express complex preferences of a disjunctive nature. We show that the "or else" form can be used as a negation operator of the "and if possible" form and vice versa. We also show that these both forms are compatible and, therefore, fuzzy bipolar conditions of both types can be used together in the same bipolar query.**

***Keywords*—Flexible querying, bipolar conditions, negation of bipolar conditions, fuzzy sets theory.**

I. Introduction

Flexible querying allows expressing preferences in user queries, which are addressed to regular databases and deliver a set of ranked answers from the most to the least preferred. In this context, the fuzzy sets theory [10] provide a general framework for the expression and the interpretation of queries involving preferences modelled by fuzzy predicates (or conditions). It is also possible to consider fuzzy bipolar conditions to model complex preferences [4], [5]. In this context, a bipolar condition is made of two components: a constraint and a wish. More precisely, for the expression of user preferences, we rely on fuzzy bipolar conditions in which the constraint and the wish are defined by fuzzy sets. We define then a bipolar query as a query that involves bipolar conditions.

Several interpretations have been introduced for the evaluation of queries involving fuzzy bipolar conditions of type "and if possible" (see [4], [5], [9], [11], [12]). The algebraic operators (selection, projection, join, union, intersection) have also been extended to fuzzy bipolar conditions [7], [2] of type "and if possible". It is therefore possible to express the usual query statements over relational bipolar relations, but in order to build a bipolar relational algebra, it stills to define a global operator for the negation of bipolar conditions. Several operators have been proposed in the literature but none of them fits in well with the framework of flexible querying.

We introduce in this paper a new kind of fuzzy bipolar conditions noted "or else", which expresses fuzzy bipolar conditions of a disjunctive nature. Its definition and properties allow us to define a negation operator of fuzzy bipolar conditions of the form "and if possible".

We show that each of these forms of bipolarity expresses a negation form of one another. These forms of bipolarity are also compatible in the sense that it is possible to mix in the same bipolar queries an "and if possible" fuzzy bipolar condition with an "or else" fuzzy bipolar condition.

The remainder of this paper is organized as follows. In section II, a short reminder about fuzzy bipolar conditions of type "and if possible" is introduced. In section III, we sum up different negation operators of fuzzy bipolar conditions of the form "and if possible", which have been proposed. Section IV is dedicated to our contribution; we introduce, firstly, the definition of fuzzy bipolar conditions of type "or else" and we show, secondly, that such fuzzy bipolar conditions can be used to define a negation operator for fuzzy bipolar conditions of type "and if possible". In section V, we show that these forms of fuzzy bipolar conditions are mutually compatible and can be used together in the same bipolar query. Section VI recalls our contribution and draws some lines for future works.

II. Fuzzy Bipolar Conditions

A bipolar condition is an association of a negative condition (negative pole) and positive condition (positive pole). In this paper, a bipolar condition is made of two conditions defined on the same universe: i) a constraint c, which describes the set of acceptable elements, ii) a wish w which defines the set of desired or wished elements. The negation of c is the set of rejected elements since it describes non-acceptable elements. Since it is not coherent to wish a rejected element, the following property of coherence holds: $w \subseteq c$.

In addition, condition c is mandatory since an element which does not satisfy c is rejected; $\neg c$ is then considered as the negative pole of the bipolar condition. Condition w is optional because its non-satisfaction does not automatically mean the rejection; w is then considered as the positive pole of the bipolar condition. In this paper, a bipolar condition is noted (c,w) and means, *"to satisfy c and if possible to satisfy w"* [4], [5].

If c and w are boolean conditions, the satisfaction with respect to (c,w) is an ordered pair from $\{0,1\}^2$. When querying a database with such a condition, tuples satisfying the constraint and the wish are returned in priority to the user. If such answers do not exist, tuples satisfying only the constraint are delivered.

If c and w are fuzzy conditions (defined on the universe U), the property of coherence becomes: $\forall u \in U, \mu_w(u) \leq \mu_c(u)$.

The satisfaction with respect to (c,w) is an ordered pair of degrees from the unit interval $[0,1]^2$. Each element u from U is then attached with a pair of grades $(\mu_c(u), \mu_w(u))$ that expresses the degrees of its satisfaction respectively to the constraint and the wish.

When querying a relation R with a fuzzy bipolar condition, each tuple t from R is then attached with a pair of grades $(\mu_c(t), \mu_w(t))$ that expresses the degrees of its satisfaction respectively to the constraint c and the wish w (and a so-called fuzzy bipolar relation is obtained). A tuple t is then denoted $(\mu_c, \mu_w)/t$. Any tuple u such that $\mu_c(u) = 0$ does not belong to the fuzzy bipolar relation.

In such a context, tuples cannot be ranked from the most preferred to the least preferred using an aggregation of μ_c and μ_w because the constraint and the wish are not commensurable. However, they can be ranked using the lexicographical order: t_1 is preferred to t_2 if and only if:

$\mu_c(t_1) > \mu_c(t_2)$ or $(\mu_c(t_1) = \mu_c(t_2) \wedge \mu_w(t_1) > \mu_w(t_2))$,
which is noted $(\mu_c(t_1), \mu_w(t_1)) > (\mu_c(t_2), \mu_w(t_2))$.
In this case, the satisfaction with respect to the constraint is firstly used to discriminate among answers (the constraint being mandatory). The satisfactions with respect to the wish being optional, they can only be used to discriminate among answers having the same evaluation with respect to the constraint. A total order is then obtained on μ_c and μ_w (with $(1,1)$ as the greatest element and $(0,0)$ as the least element).

Based on the lexicographical order, the *lmin* and *lmax* operators [6], [2] are introduced in order to define the conjunction (resp. intersection) and the disjunction (resp. union) of bipolar conditions (resp. relations). They are respectively defined as follows:

$$\begin{array}{rcl} ([0,1]\times[0,1])^2 & \rightarrow & [0,1]\times[0,1] \\ ((\mu,\eta),(\mu',\eta')) & \mapsto & lmin((\mu,\eta),(\mu',\eta')) = \\ & & \begin{cases} (\mu,\eta) & if\, \mu < \mu' \vee (\mu = \mu' \wedge \eta < \eta'), \\ (\mu',\eta') & else. \end{cases} \end{array} \quad (1)$$

$$\begin{array}{rcl} ([0,1]\times[0,1])^2 & \rightarrow & [0,1]\times[0,1] \\ ((\mu,\eta),(\mu',\eta')) & \mapsto & lmax((\mu,\eta),(\mu',\eta')) = \\ & & \begin{cases} (\mu,\eta) & if\, \mu > \mu' \vee (\mu = \mu' \wedge \eta > \eta'), \\ (\mu',\eta') & else. \end{cases} \end{array} \quad (2)$$

The *lmin* (resp. *lmax*) operator is commutative, associative, idempotent and monotonic. The pair of grades $(1,1)$ is the neutral (resp. absorbing) element of the operator *lmin* (resp. *lmax*) and the pair $(0,0)$ is the absorbing (resp. neutral) element of the operator *lmin* (resp. *lmax*).

Remark 1: Fuzzy bipolar conditions generalize fuzzy conditions since a fuzzy condition C can be rewritten (C,C) to express *"to satisfy C and if possible to satisfy C"*. As a consequence, a fuzzy relation R is a particular case of a fuzzy bipolar relation such that $\forall t \in R, \mu_R(t) = \mu_c(t) = \mu_w(t)$. Therefore, it is easy to demonstrate that the *lmin* (resp. *lmax*) generalizes the triangular norm *min* (resp. the triangular conorm *max*).

More formally, bipolar conditions of the form "C and if possible W" can be defined with the following properties:

1) $\neg C$ corresponds to the rejection (C denotes acceptable elements),
2) W corresponds to the optimal values,
3) The acceptability condition C is more important than the optimality (the condition W),
4) The set of optimal values is included in the set of acceptable values ($W \subseteq C$).

Remark 2: The property 3 means that the non-rejection (or the acceptability) is more important than the optimality, therefore, the lexicographical order can be used to rank between elements.

III. The Negation of Fuzzy Bipolar Conditions

Many works have been carried out to define a negation operator of fuzzy bipolar conditions.

To be consistent, any negation operator must verify the following properties, which have been introduced in [1]:

Let (C', W') denotes the negation of a fuzzy bipolar condition (C, W).

a) Property 1. Order reversing: The negation operator must deliver tuples in reverse order in which they are delivered by the condition (C, W):

$$\begin{array}{r} \forall t_1, t_2, (\mu_C(t_1), \mu_W(t_1)) < (\mu_C(t_2), \mu_W(t_2)) \Leftrightarrow \\ (\mu_{C'}(t_2), \mu_{W'}(t_2)) < (\mu_C(t_1), \mu_W(t_1)). \end{array} \quad (3)$$

b) Property 2. Consistency condition: The negation operator must deliver a consistency bipolar condition, which means: $\forall t, \mu_{W'}(t) \leq \mu_{C'}(t)$.

This property means that the negation of a bipolar condition of the form "and if possible" must also be a bipolar condition.

c) Property 3. Involutivity: The negation operator must be an involutive operator, it means that: $\neg(\neg(C, W)) = (C, W)$.

Several negation operators have been proposed in the literature. Some of them are summarized in what follows.

A. The Negation Operator Inspired by Twofold Fuzzy Sets

This negation operator is proposed in [3]. In this context, the negation of the wish corresponds to the constraint and the negation of the constraint defines the wish of the obtained condition. More formally, let (C, W) be a bipolar condition, its negation is: $\neg(C, W) = (\neg W, \neg C)$.

This kind of negation is not intuitive and does not satisfy the property of order reversing as it is shown in the following example 1.

Example 1: Let (C, W) be a bipolar condition and R a relational table containing tuples t_1 and t_2 such that: $(\mu_C(t_1), \mu_W(t_1)) = (0.8, 0)$ and $(\mu_C(t_2), \mu_W(t_1)) = (0.6, 0.6)$, we obtain $t_1 > t_2$.

The negation of (C, W) delivers the following result: $(\mu_{C'}(t_1), \mu_{W'}(t_1)) = (1, 0.2)$ and $(\mu_{C'}(t_2), \mu_{W'}(t_2)) = (0.4, 0.4)$, we obtain, then $t_1 > t_2$. □

B. The Negation Operator Based on the Product and the Division

This operator has been proposed in [1]. It is defined as follows: $\neg(C,W) = (C', W')$, where:

$$\mu_{C'} = 1 - \mu_C, \tag{4}$$

and

$$\mu_{W'} = \begin{cases} (1-\mu_C) * (1 - \frac{\mu_W}{\mu_C}) & if\, \mu_C \neq 0, \\ 1 & else. \end{cases} \tag{5}$$

This operator is not involutive; indeed:
$\neg(\neg(1, b \neq 0)) = \neg(0,0) = (1,1) \neq (1, b \neq 0)$.

In order to make this operator involutive, it is necessary to accept ordered pairs of grades of the form $(0, b > 0)$ and to define the negation as follows:

$$\mu_{C'} = 1 - \mu_C \tag{6}$$

$$\mu_{W'} = \begin{cases} 1 - \mu_W & if\, \mu_C = \{0,1\}, \\ (1-\mu_C) * (1 - \frac{\mu_W}{\mu_C}) & else. \end{cases} \tag{7}$$

The above properties 1, 2 and 3 hold only if the following conditions are satisfied:

- do not substitute (C,W) by $(C, C \wedge W)$ when $\mu_C = 0$,
- substitute (C,W) by $(C, W \wedge (C \vee (\neg C \Rightarrow_{RG} true)))$, where $\Rightarrow_{RG}$ is the Rescher-Gaines fuzzy implication,
- do not discard from the bipolar relation, tuples which have an ordered pair of grades of the form $(0, b \neq 0)$.

Despite the satisfaction of properties $1, 2$ and 3 under above conditions, the following drawbacks are pointed out:

- this operator does not express a natural negation form from the user point of view,
- its implementation is difficult and complex,
- the three added conditions can be seen as an infringement to properties of fuzzy bipolar conditions.

C. The Negation Operator of the Form $(\neg C, false)$

This operator is proposed in [2] and it is defined as follows:

$$\neg(C,W) = (\neg C, false) \tag{8}$$

This operator means that if tuples satisfying the constraint are discarded, then the wish becomes meaningless.

This form of negation is not involutive:
$\neg(\neg(C,W)) = \neg(\neg C, false) = (C, false) \neq (C,W)$.

D. The Negation Operator of the Form $(\neg C, \neg W)$

This operator is proposed in [7], [6]. The negation operator in this approach is defined as follows: $\neg(C,W) = (\neg C, \neg W)$.

This operator does not satisfy the consistency condition because $\neg W \nsubseteq \neg C$. The obtained operator is no more bipolar and can leads to non-consistent queries as in the following example 2.

Example 2: The negation of *"Train tickets which cost less than* 100 *\$ and if possible less than* 75 *\$"* is defined in this context as *"Train tickets which cost more than* 100 *\$ and if possible more than* 75 *\$"*.

This query is not consistent because the set of tickets which cost more than 75 \$ in not included in the set of tickets which cost more than 100 \$, but rather the opposite. □

IV. Fuzzy Bipolar Conditions of the Form "or else"

We introduce in this section a new kind of fuzzy bipolar conditions. They are of the form "or else". We, firstly, provide a formal definition for such bipolar conditions; then, we study semantic relationships and similarities between "or else" and "and if possible" fuzzy bipolar conditions. Finally, we show that these two forms can be used in a negation context to define a global negation operator for fuzzy bipolar conditions.

A. Definition

A bipolar condition of the form "E, or else F" is made of two parts: (i) a positive pole corresponding to the condition E, which expresses perfect values and (ii) a negative pole corresponding to the condition F that expresses acceptable values (any element does not satisfy F is discarded).

We define bipolar conditions of the form "E, or else F", denoted $[E,F]$, with the following properties:

1) $\neg F$ corresponds to the rejection (F denotes acceptable elements),
2) E corresponds to the optimal values,
3) The optimality condition E is more important than the acceptability condition F,
4) The set of optimal values is included in the set of acceptable values ($E \subseteq F$): $\forall x, \mu_E(x) \leq \mu_F(x)$.

The ordered pair of grades, denoted $[\mu_E(x), \mu_F(x)]$, expresses the satisfaction of an element x to the condition "E, or else F".

Since the satisfaction with respect to the optimality E is more important than the satisfaction with respect to the acceptability F, the lexicographical order can be used to rank between elements. Indeed, if an element x satisfies completely the optimal condition E then it gets a pair of grades $[1,1]$ because $\forall x, \mu_E(x) \leq \mu_F(x)$, and by definition, the lexicographical order shows that this element is optimal. If the satisfaction to the optimal condition E of an element x_1 is better than the satisfaction to the same condition by another element x_2, then x_1 is more preferred than x_2.

Finally, if both x_1 and x_2 have the same satisfaction to the optimal condition E, for want of anything better, the condition F is used to distinguish between them.

Remark 3: It is worth noticing that a fuzzy condition C can be expressed "C or else C" within the framework of bipolar conditions of the form "or else". This generalization of fuzzy conditions expresses that perfect and acceptable values are the same.

As is the case of fuzzy conditions of the form "and if possible", the *lmin* (resp. *lmax*) operator applied on bipolar conditions of the form "or else" defines an extended triangular norm (resp. co-norm).

B. Relationships Between "or else" and "and if possible" Fuzzy Bipolar Conditions

Let "E, or else F" and "C, and if possible W" be two fuzzy bipolar conditions. The former is denoted $[E,F]$, with $\forall x, \mu_E(x) \leq \mu_F(x)$ and the latter is denoted (C,W), with $\forall x, \mu_W(x) \leq \mu_C(x)$.

TABLE I
EXAMPLE OF THE BEHAVIOR OF THE (C,W) FORMALISM.

	$\mu_C(x_i)$	$\mu_W(x_i)$
t_1	1	1
t_2	0.8	0.2
t_3	0.7	0.5
t_4	0.4	0.3

TABLE II
EXAMPLE OF THE BEHAVIOR OF THE $[W,C]$ FORMALISM.

	$\mu_W(x_i)$	$\mu_C(x_i)$
t_1	1	1
t_3	0.5	0.7
t_4	0.3	0.4
t_2	0.2	0.8

We notice strong similarities between these two formalisms, since in both cases we have:

- C and F express the acceptable values (or a non rejected values),
- $\neg C$ and $\neg F$ correspond to the discarded values,
- W and E express the perfect or optimal values,
- the set of perfect values is included in the set of acceptable values.

Moreover, these two conditions generalize fuzzy conditions in the same way: a fuzzy condition C is expressed (C,C) or $[C,C]$.

Since W (resp. C) plays the same role as E (resp. F), it is interesting to study the behavior of (C,W) and $[W,C]$. First of all, we notice that in the boolean case, both formalisms have the same meaning, as shown in the example 3.

Example 3: Let x, y and z be three elements attached respectively to the following pair of grades: $(1,1), (1,0)$ and $(0,0)$ with respect to conditions C and W.

The lexicographical order delivers the same order in both situations (C,W) and $[W,C]$: $x > y > z$. □

However, when C and W are fuzzy conditions, the two formalisms do not express the same semantic because the lexicographical order does not deliver the same order.

Tables I and II show that tuples t_1, t_2, t_3 and t_4 are not sorted according to the same order, depending on whether the formalism (C,W) or $[W,C]$ is used.

The basic difference between these two formalisms is the fact that the formalism "C, and if possible W" gives more importance to the non rejected elements (that means the satisfaction with respect to the condition of acceptance C is privileged), whereas the formalism "E, or else F" gives more importance to the optimal elements (i.e. the satisfaction with respect to the optimal condition E is privileged).

In this context, a fuzzy bipolar condition of the form (C,W) can be defined as a pair of fuzzy conditions, which define a set of optimal values (W) and a set of acceptable values (C) under the consideration that the non-rejection is more important than the optimality; and a fuzzy bipolar condition of the form $[E,F]$ is defined as a pair of fuzzy conditions, which define a set of optimal values (E) and a set of acceptable values (F) under the consideration that the optimality is more important than the non-rejection.

TABLE III
EXAMPLE OF TUPLES ATTACHED WITH THEIR ORDERED PAIRS OF GRADES W.R.T THE FUZZY BIPOLAR CONDITION $(Young, Young \wedge WellPaid)$.

#Employee	$\mu_{Young}(\#Employee)$	$\mu_{Young}(\#Employee) \wedge \mu_{WellPaid}(\#Employee)$
10	1	1
11	0.75	0.56
12	0.6	0.36
13	0.5	0.25

C. The Negation Context

In this subsection, we show that the negation of the fuzzy bipolar condition (C,W) is the fuzzy bipolar condition $[\neg C, \neg W]$ and, reciprocally, the negation of the fuzzy bipolar condition $[E,F]$ is the fuzzy bipolar condition $(\neg E, \neg F)$.

1) Negation of (C,W): We recall that a bipolar condition (C,W) is defined by the following properties:

1) $\neg C$ corresponds to the rejection (C denotes acceptable elements),
2) W corresponds to the optimal values,
3) The acceptability condition C is more important than the optimality condition W.

In addition, the set of optimal values is included in the set of acceptable values ($W \subseteq C$).

If we express these properties in the context of a negation $\neg(C,W)$, we obtain:

1) $\neg C$ corresponds to the optimal values (C contains rejected and acceptable elements),
2) W corresponds to the rejected values ($\neg W$ denotes acceptable elements),
3) The optimality condition $\neg C$ is more important than the acceptability condition $\neg W$.

In addition, since $W \subseteq C$, we have $\neg C \subseteq \neg W$.

Therefore, we obtain a fuzzy bipolar condition which fits within the definition of the "or else" fuzzy bipolar conditions and we denote it by $[\neg C, \neg W]$. That means:

$$\neg(C,W) = [\neg C, \neg W]. \quad (9)$$

It is important to notice that in (C,W), the importance is put on C and the concept attached to C stills the most important for $\neg(C,W)$. As an example, when considering *"young and if possible young and well-paid employees"*, the importance is put on the *age* (young). In its negation, the *age* stills the most important aspect to consider.

Example 4: Let $(Young, Young \wedge WellPaid)$ be a fuzzy bipolar condition of type "and if possible" which defines *young and if possible young and well-paid employees*. Let table III be a set of returned tuples from a relational table. The negation of the fuzzy bipolar condition $(Young, Young \wedge WellPaid)$ is defined as $[notYoung, not(Young \wedge WellPaid)]$, which corresponds to *employees which are not young, or else are not (young and well-paid)*. We notice that concepts used to define the fuzzy bipolar condition $(Young, Young \wedge$

TABLE IV
EXAMPLE OF TUPLES ATTACHED WITH THEIR ORDERED PAIRS OF GRADES W.R.T THE FUZZY BIPOLAR CONDITION $[notYoung, not(Young \wedge WellPaid)]$.

#Employee	$\mu_{notYoung}(\#Employee)$	$\mu_{not(Young\ and\ WellPaid)}(\#Employee)$
13	0.5	0.75
12	0.4	0.64
11	0.25	0.44

$WellPaid)$ are the same ones used to express its negation $[notYoung, not(Young \wedge WellPaid)]$. More precisely, the concept *age* is used to define the most important fuzzy condition in both fuzzy bipolar condition $(Young, Young \wedge WellPaid)$ and $[notYoung, not(Young \wedge WellPaid)]$.

The obtained ordered pairs of degrees for the fuzzy bipolar condition $[notYoung, not(Young \wedge WellPaid)]$ are summed up in the table IV. The tuple #10 is completely discarded. □

2) Negation of $[E,F]$: We recall that a fuzzy bipolar condition $[E,F]$ is defined by the following properties:

- $\neg F$ corresponds to the rejection (F denotes the acceptable elements),
- E corresponds to the optimal values,
- The optimality condition E is more important than the acceptability.

In addition, the set of optimal values is included in the set of acceptable values ($E \subseteq F$).

If we express these properties in the context of a negation, we obtain:

- $\neg F$ corresponds to the optimal values,
- E corresponds to the rejected values ($\neg E$ denotes the acceptable elements),
- The acceptability condition $\neg E$ is the more important condition.

In addition, since $E \subseteq F$, we have $\neg F \subseteq \neg E$. This property states that, in the context of negation of a fuzzy bipolar condition of the form $[E,F]$, the set of optimal values is included in the set of acceptable values.

Therefore, we obtain a fuzzy bipolar condition, which fits within the definition of the "and if possible" fuzzy bipolar conditions and we denote it by $(\neg E, \neg F)$. That means:

$$\neg[E,F] = (\neg E, \neg F). \quad (10)$$

As for bipolar conditions of type "and if possible", we notice that in $[E,F]$, the importance is put on E and the concept attached to E stills the most important for $\neg[E,F]$.

This form of negation is involutive and reverses the lexicographical order (either for $\neg(E,F)$ or $\neg[E,F]$).

Proof: Reversing of the lexicographical order.

The proof is based on the following properties:

$$(x_1,y_1) > (x_2,y_2) \Rightarrow (1-x_1, 1-y_1) < (1-x_2, 1-y_2).$$

Let $[E,F]$ or (E,F) be a fuzzy bipolar condition. A tuple t_i is preferred to t_j means that:

$$(\mu_E(t_i), \mu_F(t_i)) > (\mu_E(t_j), \mu_F(t_j)),$$
$$\text{or } [\mu_E(t_i), \mu_F(t_i)] > [\mu_E(t_j), \mu_F(t_j)]$$

From the definition of the lexicographical order, we get:

$(t_i$ is preferred to $t_j) \Leftrightarrow \mu_E(t_i) > \mu_E(t_j)$ or $(\mu_E(t_i) = \mu_E(t_j)$ and $\mu_E(t_i) > \mu_F(t_j))$
$\Leftrightarrow (1-\mu_E(t_i) < 1-\mu_E(t_j))$ or $((1-\mu_E(t_i) = 1-\mu_E(t_j))$ and $(1-\mu_E(t_i) < 1-\mu_F(t_j)))$
$\Leftrightarrow t_j$ is preferred to t_i for $(\neg E, \neg F)$ or $[\neg E, \neg F]$, with $(\neg E, \neg F) = \neg[E,F]$ and $[\neg E, \neg F] = \neg(E,F)$. The order is then reversed for the negation. ■

Proof: Involutivity.

$\neg(\neg(C,W)) = \neg[\neg C, \neg W] = (\neg\neg C, \neg\neg W) = (C,W)$ and $\neg(\neg[E,F]) = \neg(\neg E, \neg F) = [\neg\neg E, \neg\neg F] = [E,F]$. ■

V. MUTUAL COMPATIBILITY OF "AND IF POSSIBLE" AND "OR ELSE" FUZZY BIPOLAR CONDITIONS

It is possible to express fuzzy bipolar conditions of both forms "and if possible" and "or else" together in the same bipolar query. This means that it is possible to use *lmin* and *lmax* operators to compare and to handle ordered pairs of scores of these types of fuzzy bipolar conditions. This is due to the fact that both of them are semantically equivalent in the sense that they are defined similarly. Indeed, fuzzy bipolar conditions of types "and if possible" and "or else" consist of two parts: a first part which corresponds to the most important concept (the concept attached to C for the fuzzy bipolar condition (C,W) and the concept attached to E or the fuzzy bipolar condition $[E,F]$) and a second part which corresponds to the least important concept of the bipolar condition. Therefore, in order to rank between tuples, the first choice is made on the condition which corresponds to the most important concept in the fuzzy bipolar condition, and the second choice is made on the condition which corresponds to the least important concept of the same fuzzy bipolar condition. We show in the following example a bipolar query in which both forms of fuzzy bipolar conditions are used, and its returned tuples with their attached ordered pairs of degrees.

Example 5: Let R be a relational table about *journeys* from *Paris* to *Brest* (see table V), delivered from a multimodal transport information system.

One can express a bipolar query about preferred journeys in the following terms:

*"Find journeys from Paris to Brest which are **(fast, and if possible fast and cheap)**, or journeys with **(an early departure and a late arrival, or else an early departure)**".*

In the above query, the preferred journeys are those which have a short duration and cost as low as possible, or journeys in which the user takes the time to travel, which correspond to journeys having an early departure and a late arrival, otherwise journeys which have an early departure.

Based on the fuzzy predicates *fast*, *cheap*, *early* and *late*, we define from the relation R the following bipolar relations $Journey_{(Fast, Fast \wedge Cheap)}$ (see table VI), which corresponds to fast *and if possible* cheap journeys, and $Journey_{[Early \wedge Late, Early]}$ (see table VII), which corresponds journeys having an early departure and a late arrival *or else* an early departure. Tuples of these relations are ranked from the most satisfactory to the least satisfactory with regard to fuzzy bipolar conditions $(Fast, Fast \wedge Cheap)$ and $[Early \wedge Late, Early]$ respectively.

TABLE V
EXAMPLE OF EXTENSION OF THE RELATION R.

#Journey	Duration	Departure	Mode	Cost	...
10	2h	11h30 am	Plane	60	...
11	2h45	11h am	Plane	150	...
12	2h30	8h30 am	Plan	260	...
13	3h10	8h am	Train	80	...
14	3h30	10h am	Train	80	...
15	4h10	7h30 am	Train	90	...
16	5h15	7h15 am	Bus	85	...

TABLE VI
THE FUZZY BIPOLAR RELATION $Journey_{(Fast,Fast \wedge Cheap)}$.

#Journey	$\mu_{Fast}(\#Journey)$	$\mu_{Fast \wedge Cheap}(\#Journey)$
10	1	1
11	0.80	0.64
12	0.80	0.25
13	0.60	0.36
14	0.40	0.36
15	0.40	0.16
16	0.25	0.25

The ordered pair of grades of satisfaction, with regard to the combination of the fuzzy bipolar conditions $(Fast, Fast \wedge Cheap)$ and $[Early \wedge Late, Early]$, attached to each tuple is obtained by the application of the *lmax* operator on each tuple of both fuzzy bipolar relations as follows:

- #journey = 10: $lmax((1,1),[0.25,0.6]) = (1,1)$,
- #journey = 11: $lmax((0.8,0.64),[0.25,0.5]) = (0.8,0.64)$,
- #journey = 12: $lmax((0.8,0.25),[0.3,1]) = (0.8,0.25)$,
- #journey = 13: $lmax((0.6,0.36),[0.6,1]) = [0.6,1]$,
- #journey = 14: $lmax((0.4,0.36),[0.6,0.7]) = [0.6,0.7]$,
- #journey = 15: $lmax((0.4,0.16),[0.8,1]) = [0.8,1]$,
- #journey = 16: $lmax((0.25,0.25),[1,1]) = [1,1]$,

We notice that journeys are delivered depending on the grade of satisfaction w.r.t the most important fuzzy condition: *fast* for the fuzzy bipolar condition $(Fast, Fast \wedge Cheap)$ and *early* for the fuzzy bipolar condition $[Early \wedge Late, Early]$. In the case where the two most important conditions have the same satisfaction, the distinction is made on the least important conditions.

The maximum ordered pair of degrees is attached to tuples #10 and #16, because the corresponding fuzzy bipolar condition is completely satisfied (both are fully satisfactory and not distinguishable).

In the case of tuples #11, #12, #14, #15, the returned ordered pair of grades corresponds to the ordered pair in which the satisfaction w.r.t the most important fuzzy condition in both used fuzzy bipolar conditions is the highest.

TABLE VII
THE FUZZY BIPOLAR RELATION $Journey_{[Early \wedge Late, Early]}$.

#Journey	$\mu_{Early \wedge Late}(\#Journey)$	$\mu_{Early}(\#Journey)$
16	1	1
15	0.80	1
13	0.60	1
14	0.60	0.70
12	0.30	1
10	0.25	0.60
11	0.25	0.50

In the case of tuple #13, the satisfaction is the same with regard to the most important fuzzy condition in both fuzzy bipolar conditions; therefore, the least important condition is used to determine which ordered pair of degrees to attach to the resulting tuple.

Finally, journeys are delivered in the following order: $(1,1)$/#10, $[1,1]$/#16, $[0.8,1]$/#15, $(0.8,0.64)$/#11, $(0.8,0.25)$/#12, $[0.6,1]$/#13, $[0.6,0.7]$/#14.

VI. CONCLUSION AND FUTURE WORKS

A new kind of fuzzy bipolar conditions is introduced in this paper. These conditions are of the form ”or else” and allow users to express complex preferences of a disjunctive nature in their queries. We showed that this form of fuzzy bipolar conditions are compatible with fuzzy bipolar conditions of the form ”and if possible” in the sense that (i) each one can be used as the negation form of the other and that (ii) it is possible to combine them in the same query and to handle them using the lexicographical order based operator *lmin* and *lmax*.

As future work, we aim at studying this form of negation within the scope of a bipolar relational language and at integrating it into the bipolar SQLf language, which is an extension of the SQLf language to fuzzy bipolar conditions.

ACKNOWLEDGMENTS

This work is funded by the Brittany region and the department of Côtes-d'Armor and is partially supported by the National Agency for Research (AOC Ref. ANR-08-CORD-009).

REFERENCES

[1] P. Bosc and O. Pivert. A propos de la négation de conditions bipolaires floues. on the negation of fuzzy bipolar conditions. In *Rencontre francophone sur la logique floue et ses applications*, pages 21–28, 2010.

[2] P. Bosc, O. Pivert, L. Liétard, and A. Mokhtari. Extending relational algebra to handle bipolarity. In *25th ACM Symposium on Applied Computing, SAC'10*, pages 1717–1721, 2010.

[3] D. Dubois and H. Prade. Twofold fuzzy sets and rough sets–some issues in knowledge representations. *Fuzzy sets and Systems*, 23(1):3–18, 1987.

[4] D. Dubois and H. Prade. Bipolarity in flexible querying. *LNAI*, 2522:174–182, 2002.

[5] D. Dubois and H. Prade. An introduction to bipolar representations of information and preference. *International Journal of Intelligent Systems*, 23:866–877, 2008.

[6] L. Liétard and D. Rocacher. On the definition of extended norms and co-norms to aggregate fuzzy bipolar conditions. In *IFSA/EUSFLAT*, pages 513–518, 2009.

[7] L. Liétard, D. Rocacher, and P. Bosc. On the extension of SQL to fuzzy bipolar conditions. In *The 28th North American Information Processing Society Annual Conference (NAFIPS'09)*, 2009.

[8] E. Trillas, C. Moraga, S. Guadarrama, S. Cubillo, and E. Castineira. Computing with antonyms. In M. Nikravesh, J. Kacprzyk, and L. Zadeh, editors, *Forging New Frontiers: Fuzzy Pioneers I*, Studies in Fuzziness and Soft Computing, pages 133–153. Springer Berlin / Heidelberg, 2007.

[9] G. D. Tré, S. Zadrozny, T. Matthé, J. Kacprzyk, and A. Bronselaer. Dealing with positive and negative query criteria in fuzzy database quering bipolar satisfaction degrees. *LNAI, FQAS*, 5822:593–604, 2009.

[10] L. Zadeh. Fuzzy sets. *Information and control*, 8(3):338–353, 1965.

[11] S. Zadrozny and J. Kacprzyk. Bipolar queries using various interpretations of logical connectives. *LNAI, IFSA*, 4529:182–190, 2007.

[12] S. Zadrozny and J. Kacprzyk. Bipolar queries: An approach and its various interpretations. In *In Proceedings of IFSA/EUSFLAT*, pages 1288–1293, 2009.