Diagnosis of Three-Phase Electrical Machines Using Multidimensional Demodulation Techniques
Vincent Choqueuse, Mohamed Benbouzid, Yassine Amirat, Sylvie Turri

To cite this version:

HAL Id: hal-00654230
https://hal.archives-ouvertes.fr/hal-00654230
Submitted on 21 Dec 2011
Diagnosis of Three-Phase Electrical Machines Using Multidimensional Demodulation Techniques
Vincent Choqueuse, Member, IEEE, Mohamed El Hachemi Benbouzid, Senior Member, IEEE, Yassine Amirat, and Sylvie Turri

Abstract—This paper deals with the diagnosis of three-phase electrical machines and focuses on failures that lead to stator-current modulation. To detect a failure, we propose a new method based on stator-current demodulation. By exploiting the configuration of three-phase machines, we demonstrate that the demodulation can be efficiently performed with low-complexity multidimensional transforms such as the Concordia transform (CT) or the principal component analysis (PCA). From a practical point of view, we also prove that PCA-based demodulation is more attractive than CT. After demodulation, we propose two statistical criteria aiming at measuring the failure severity from the demodulated signals. Simulations and experimental results highlight the good performance of the proposed approach for condition monitoring.

Index Terms—Condition monitoring, electrical machines, principal component analysis (PCA), signal processing.

I. INTRODUCTION
THREE-PHASE electrical machines such as induction motors or generators are used in a wide variety of applications. To increase the productivity and to reduce maintenance costs of these systems, condition monitoring and diagnosis are often desired. A wide variety of condition monitoring techniques have been introduced over the last decade. Among them, motor current signature analysis (MCSA) [1] has several advantages since it is usually cheaper and easier to implement than other techniques. In steady-state configurations, MCSA based on stationary spectral analysis techniques is commonly used (fast Fourier transform (FFT) and multiple-signal classification [2]). However, in practice, the steady-state assumption is often violated due to nonconstant-supply-frequency or adjustable-speed drives. In these situations, several authors have investigated the use of nonstationary techniques such as time–frequency representations [1], [3]–[6], time-scale analysis [7]–[10], and polynomial-phase transform [11] (see [12] for a more complete review). The main drawback of these methods relies on their computational complexity. Furthermore, these representations usually suffer from poor resolution and/or artifact (cross terms, aliasing, etc.), which can lead to misleading interpretations.

Other investigations on machine modeling have recently demonstrated that many types of failure lead to stator-current modulation with a modulation index which is directly proportional to the failure severity [5], [13]–[17]. In particular, it has been proved that load torque oscillations lead to stator-current phase modulation (PM) [9], [13]–[15], whereas air-gap eccentricity and rotor asymmetry lead to stator-current amplitude modulation (AM) [5], [14]. Therefore, a straightforward technique to monitor the behavior of an electrical machine is based on stator-current demodulation.

Classical demodulation techniques include the square-law demodulator, the Hilbert transform (HT) [18], the energy separation algorithm [19], and other approaches. Applications to failure detection are available in [7], [14], and [20]–[27]. Interestingly, for a balanced system, it has been shown in [14], [21], [22], [24], and [28] that the Concordia transform (CT), which has been used for failure detection purposes in [14], [21]–[24], and [28]–[38], can also be interpreted as a demodulating tool. As compared to classical demodulation tools, CT exhibits interesting properties such as lower complexity and lack of end effect problems or other artifacts [28]. However, in practice, this approach can lead to poor performance since a real machine usually presents a small degree of imbalance [22].

Once the demodulation has been performed, demodulated signals must be further analyzed to measure failure severity. In the literature, many criteria and/or techniques have been proposed to perform this task. In [14], [21], [22], [24], [25], and [29], failure severity is measured through statistical criteria. However, these criteria require knowledge of the fault frequencies, which also depend on other parameters (speed or slip information). To overcome this problem, more sophisticated approaches have been proposed for failure detection. These include neural networks [31], [36], [39], Bayesian classifiers [23], fuzzy logic classifiers [33], [36], genetic algorithms [38], and other classifiers [34]. However, these approaches are computationally demanding, and their performances highly depend on the representativeness of the training set.

In this paper, we address the condition monitoring problem from a signal processing point of view. As failure severity is proportional to the modulation index, we propose to use the modulation-index estimate as a failure severity indicator. The proposed approach is composed of two steps: a stator-current
amplitude/frequency demodulation followed by a modulation-index estimation. These steps are described in Fig. 1. To perform demodulation, we propose to exploit the multidimensional nature of three-phase systems through low-complexity linear transforms. Then, we propose to estimate the modulation indexes from the demodulated signals with two original estimators.

This paper is organized as follows. Section II describes the signal model of the stator current under healthy and faulty conditions. Section III investigates the use of CT and principal component analysis (PCA) to perform multidimensional current demodulation, and Section IV describes the proposed failure severity criteria. Finally, Section V reports on the performance of the proposed approach with synthetic and experimental signals.

II. SIGNAL MODEL

In the presence of a fault, it has been shown in [5], [13], [14], and [16] that the stator current is amplitude modulated and/or phase modulated. For AM and/or PM, the instantaneous amplitude $a(t)$ and phase $\phi(t)$ can be expressed respectively as

$$a(t) = \alpha (1 + m_a \cos(2\pi f_a t))$$  \hspace{1cm} (1)

$$\phi(t) = 2\pi f_\phi t + m_\phi \sin(2\pi f_\phi t)$$  \hspace{1cm} (2)

where $\alpha$ is a scaling coefficient, $f_0$ is the supply current frequency, and $f_a$ ($f_\phi$) is the AM (PM) modulating frequency. The scalars $m_a$ and $m_\phi$ correspond to the AM and PM indexes, respectively. For a faulty system, the modulation indexes are directly proportional to the failure severity. In particular, without any fault, the instantaneous amplitude and frequency do not vary with time, i.e., $m_a = m_\phi = 0$.

Let us consider a three-phase system. In the presence of a fault, all three line currents $i_1(t), i_2(t)$, and $i_3(t)$ are simultaneously modulated, and the currents can be expressed as

$$i_1(t) = a(t) \cos(\phi(t))$$  \hspace{1cm} (3a)

$$i_2(t) = a(t) \cos(\phi(t) - 2\pi/3)$$  \hspace{1cm} (3b)

$$i_3(t) = a(t) \cos(\phi(t) + 2\pi/3).$$  \hspace{1cm} (3c)

In the literature, most studies assume a perfect balance configuration. However, healthy electrical systems are rarely perfectly balanced. Furthermore, the balance assumption usually does not hold when a failure introduces some asymmetry. In this study, balanced and unbalanced three-phase systems are considered. Let us denote $s(t) = [s_1(t), s_2(t), s_3(t)]^T$ the $3 \times 1$ vector which contains the stator currents, where $(\cdot)^T$ corresponds to the matrix transposition. In this paper, we investigate the two following systems.

1) A balanced three-phase system, where the stator currents are given by

$$s(t) = \mathbf{i}(t) = [i_1(t), i_2(t), i_3(t)]^T.$$  \hspace{1cm} (4)

In particular, by using (3), one can easily verify that $s_1(t) + s_2(t) + s_3(t) = 0$.

2) A three-phase system with unbalanced currents, where the stator currents are given by

$$s(t) = D\mathbf{i}(t) = [\alpha_1 i_1(t), \alpha_2 i_2(t), \alpha_3 i_3(t)]^T$$  \hspace{1cm} (5)

where $D$ is a nonscalar $3 \times 3$ diagonal matrix which contains the “nonequal” diagonal entries $\alpha_1$, $\alpha_2$, and $\alpha_3$. Without loss of generality, we assume that the overall energy of the system is conserved, i.e., $\sum_{k=1}^{3} \alpha_k^2 = 3$.

In this study, the modulation indexes are employed as failure severity indicators. From a signal processing viewpoint, the condition monitoring problem is therefore translated into an estimation problem. One should note that the estimation of the modulation indexes can be simplified by using a demodulation preprocessing step. In the following, the demodulation is performed by using a linear transformation of the stator currents $s(t)$.

III. AM/FM DEMODULATION USING MULTIDIMENSIONAL TRANSFORM

In this section, we prove that the use of the three-phase current can expedite the demodulation step. In particular, we show that the CT and the PCA can be considered as low-complexity techniques for current demodulation. Furthermore, we demonstrate that the PCA has a larger domain of validity than the CT.

A. CT

CT is a linear transform which converts the three-component $s(t)$ into a simplified system composed of two components. By denoting $y^{(c)}(t) = [y_1^{(c)}(t), y_2^{(c)}(t)]^T$ the two Concordia components, CT can be expressed into a matrix form as

$$y^{(c)}(t) = \begin{bmatrix} y_1^{(c)}(t) \\
 y_2^{(c)}(t) \end{bmatrix} = \sqrt{2/3} C s(t)$$  \hspace{1cm} (6)

where $C$ is the $2 \times 3$ Concordia matrix which is equal to

$$C = \begin{bmatrix} \sqrt{2/3} & -1/\sqrt{2} & -1/\sqrt{2} \\
 0 & -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}.$$  \hspace{1cm} (7)

One can verify that the Concordia matrix is an orthogonal matrix since it satisfies $CC^T = I_2$, where $I_2$ is a $2 \times 2$ identity matrix.

One should note that the overall energy can be absorbed into the coefficient $\alpha$ in (1).
By using (3), (4), and (6), it can be demonstrated that the Concordia components of a balanced system are equal to
\[
\begin{align*}
    y_1^{(c)}(t) &= a(t) \cos(\phi(t)) a \\
    y_2^{(c)}(t) &= a(t) \sin(\phi(t)) a.
\end{align*}
\]

(8a)
(8b)

These components \(y_1^{(c)}(t)\) and \(y_2^{(c)}(t)\) are called in-phase and quadrature components in the signal processing community.

Let us define the complex signal \(z^{(c)}(t)\) as
\[
    z^{(c)}(t) = y_1^{(c)}(t) + jy_2^{(c)}(t).
\]

(9)

By using (8) and (9), one can verify that \(z^{(c)}(t)\) is the analytical signal of \(s_1(t)\), i.e., \(z^{(c)}(t) = a(t)e^{j\phi(t)}\). Therefore, the instantaneous amplitude and frequency can be obtained from the modulus and the derivative of the argument of \(z^{(c)}(t)\), respectively, i.e.,
\[
\begin{align*}
    a(t) &= |z^{(c)}(t)| \\
    f(t) &= \frac{1}{2\pi} \frac{d}{dt} \arg[z^{(c)}(t)].
\end{align*}
\]

(10a)
(10b)

where \(\cdot\) and \(\arg[\cdot]\) correspond to the modulus and the argument, respectively. It is important to note that (10) only holds for a balanced system, i.e., for \(s(t) = i(t)\).

### B. PCA

PCA is a statistical tool that transforms a number of correlated signals into a smaller number of principal components. In [40], PCA is employed after a CT to detect a failure; however, no mathematical analysis has been performed to give a physical interpretation to the principal components. In [41], the PCA is applied on the three-phase stator currents directly, but only two principal components are extracted, without any mathematical justification. In this section, we give a deep theoretical analysis of PCA for balanced and (static) unbalanced three-phase systems. We show why the PCA can be applied on the stator-current signals directly. Furthermore, we demonstrate why the PCA can only extract two principal components and why principal components are strongly linked to in-phase and quadrature components. Finally, as opposed to CT, we prove that the PCA can be employed for signal demodulation whatever the balance assumption.

Let us define the 3 × 3 covariance matrix as
\[
    R_s = E\left[ss^T(t)\right]
\]

(11)

where \(E[\cdot]\) denotes the mathematical expectation. Using (3), one can remark that \(s_3(t) = -i_1(t) - i_2(t)\). Therefore, each component of \(s(t)\) can be rewritten as a linear combination of the two components \(i_1(t)\) and \(i_2(t)\) whatever the balance assumption.² Using an eigenvector decomposition, it follows that the 3 × 3 symmetric matrix \(R_s\) contains one zero eigenvalue. Therefore, \(R_s\) can be decomposed under the following form:
\[
    R_s = U \Lambda U^T
\]

(12)

²For unbalanced systems, one can verify that \(s_1(t) = \alpha_1 i_1(t)\), \(s_2(t) = \alpha_2 i_2(t)\), and \(s_3(t) = -\alpha_3(i_1(t) + i_2(t))\).

where \(U\) is a 3 × 2 orthogonal matrix \((U^TU = I_2)\) containing the two eigenvectors and \(\Lambda = \text{diag}(\lambda_1, \lambda_2)\) is a diagonal matrix containing the two nonzero associated eigenvalues \(\lambda_1\) and \(\lambda_2\). The two principal components of \(s(t)\), denoted \(y^{(p)}(t) = [y_1^{(p)}(t), y_2^{(p)}(t)]^T\), are given by
\[
    y^{(p)}(t) = \beta_3 \Lambda^{-\frac{1}{2}} U^T s(t)
\]

(13)

where \(\beta_3\) is a scaling term which is equal to
\[
    \beta_3 = \sqrt{\text{trace}[R_s]/3}
\]

(14)

with \(\text{trace}[\cdot]\) being the sum of the diagonal elements. Using (12) and (13), one can verify that the PCA components are uncorrelated.

Under the assumptions that \(\phi(t)\) is uniformly distributed in \([0, 2\pi]^3\) and that \(a(t)\) and \(\phi(t)\) are independent, it is demonstrated in the Appendix that the PCA components are equal to
\[
\begin{align*}
    y_1^{(p)}(t) &= a(t) \cos(\phi(t) - \theta) \\
    y_2^{(p)}(t) &= a(t) \sin(\phi(t) - \theta)
\end{align*}
\]

(15a)
(15b)

where \(\theta \in \mathbb{Z}\) whatever the balance assumption.

Let us define the complex signal \(z^{(p)}(t)\) as
\[
    z^{(p)}(t) = y_1^{(p)}(t) + jy_2^{(p)}(t).
\]

(16)

By using (15) and (16), one can verify that \(z^{(p)}(t)\) is a rotated version of the analytical signal of \(s_1(t)\), i.e., \(z^{(p)}(t) = a(t)e^{j\phi(t) - \theta}\). Therefore, the instantaneous amplitude and frequency can be obtained from the modulus and the derivative of the argument of \(z^{(p)}(t)\), respectively, i.e.,
\[
\begin{align*}
    a(t) &= |z^{(p)}(t)| \\
    f(t) &= \frac{1}{2\pi} \frac{d}{dt} \arg[z^{(p)}(t)].
\end{align*}
\]

(17a)
(17b)

As opposed to (10), it is interesting to note that (17) holds whatever the balance assumption. Therefore, the PCA-based demodulation is less restrictive than the Concordia one.

### IV. MODULATION-INDEX ESTIMATION

After demodulation, the analytical signal and the instantaneous amplitude and/or frequency must be properly analyzed to assess failure severity. Many papers propose to monitor the deviation of the analytical signal \(z(t)\) from a circle in the complex plane [29]–[32], [34]–[37], [40]. This solution is perfectly valid if the failure leads to stator-current AM since the radius \(|a(t)|\) varies with time. However, if the failure leads to PM, this solution is no longer correct since the failure only affects the rotational speed in the complex plane. In this section, we propose to estimate the AM and PM indexes to assess the failure severity. Using the demodulated signals, two original estimators

³From a decision viewpoint, the uniform probability density function (pdf) for the phase represents the most ignorance that can be exhibited by the fault detector. This is called the least favorable pdf for \(\phi(t)\) [42].
of the modulation indexes are provided. These estimators are based on the method-of-moments (MoM) technique. Although the MoM estimation technique has no optimal properties, it produces an estimator that is easy to determine and simple to implement [43].

A. Estimation of \( m_a \)

Let us consider the AM signal model in (1). Under the assumption that \( 2\pi f_a t \mod 2\pi \) is uniformly distributed in \([0, 2\pi]\), it is demonstrated in the Appendix that the variance of the instantaneous amplitude is given by

\[
\sigma_a^2 = E \left[ (a(t) - \mu_a)^2 \right] = \frac{\mu_a^2 m_a^2}{2} \tag{18}
\]

where \( \mu_a = E[a(t)] \) is the statistical average of \( a(t) \). Therefore

\[
m_a = \frac{\sigma_a \sqrt{2}}{\mu_a}. \tag{19}
\]

The corresponding MoM estimator, denoted \( \hat{m}_a \), is obtained by replacing the theoretical moments \( \mu_a \) and \( \sigma_a \) by their natural estimators.

B. Estimation of \( m_\phi \)

Let us consider the PM signal model in (2). Taking the derivative of \( \phi(t) \) leads to the instantaneous frequency, which is equal to

\[
f(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt} = f_0 + m_\phi f_\phi \cos(2\pi f_\phi t). \tag{20}
\]

Under the assumption that \( 2\pi f_\phi t \mod 2\pi \) is uniformly distributed in \([0, 2\pi]\), it can be demonstrated that the variance of the instantaneous frequency is (see the Appendix for a similar proof)

\[
\sigma_f^2 = E \left[ (f(t) - \mu_f)^2 \right] = \frac{m_\phi^2 f_\phi^2}{2} \tag{21}
\]

where \( \mu_f = E[f(t)] \) is the statistical average of \( f(t) \). Therefore

\[
m_\phi = \frac{\sigma_f \sqrt{2}}{f_\phi}. \tag{22}
\]

The corresponding MoM estimator \( \hat{m}_\phi \) is obtained by replacing the theoretical moments by their natural estimators. One should note that the criterion \( m_\phi \) depends on the modulating frequency \( f_\phi \). If this frequency is unknown, it can be replaced by its estimate \( \hat{f}_\phi \). This estimate can be obtained, for example, by maximizing the periodogram of \( f(t) \) [44], [45].

V. PERFORMANCES

This section reports on the performances of the proposed approaches. Experiments were performed with a supply frequency equal to \( f_0 = 50 \) Hz. Signals were sampled with a sampling period of \( T_s = 10^{-4} \) s, and the proposed technique was applied offline in Matlab. For discrete signals, a straightforward adaptation of the proposed techniques is given by Algorithms 1 and 2, respectively. As compared to the continuous case, \( s(t) \) is replaced by its discrete counterpart \( s[n] = s(n T_s) \), where \( T_s \) is the sampling period and \( n = 0, 1, \ldots, N-1 \). Furthermore, the instantaneous frequency is approximated by replacing the phase derivative with a two-sample difference, and the statistical moments are replaced by their natural estimators. In particular, \( f(n) \), \( R_s \), \( m_f \), \( \sigma_a^2 \), and \( \sigma_f^2 \) are respectively given by

\[
f(n) = \frac{\arg[z(n)] - \arg[z(n-1)]}{2\pi T_s}, \tag{23}
\]

\[
\hat{R}_s = \frac{1}{N} \sum_{n=0}^{N-1} s[n] s^T[n], \tag{24}
\]

\[
\hat{\mu}_a = \frac{1}{N} \sum_{n=0}^{N-1} a[n],\tag{25}
\]

\[
\hat{\mu}_f = \frac{1}{N} \sum_{n=0}^{N-1} f[n], \tag{26}
\]

\[
\hat{\sigma}_a^2 = \frac{1}{N} \sum_{n=0}^{N-1} (a[n] - \hat{\mu}_a)^2, \tag{27}
\]

\[
\hat{\sigma}_f^2 = \frac{1}{N} \sum_{n=0}^{N-1} (f[n] - \hat{\mu}_f)^2. \tag{28}
\]

The next sections present the performances of the proposed algorithms with synthetic and experimental signals, respectively.

---

Algorithm 1 Concordia-based failure severity criteria

1) Extract \( N \)-data samples \( s[n] \).
2) Compute \( y^{(o)}[n] \) with (6).
3) Compute the analytical signal \( z^{(o)}[n] \) with (9).
4) Extract the AM demodulated signal \( a[n] \) with (10a).
5) Extract the FM demodulated signal \( f[n] \) with (23).
6) Compute \( \hat{m}_a \) with (19), (25), and (27).
7) Compute \( \hat{m}_\phi \) with (22), (26), and (28).

Algorithm 2 PCA-based failure severity criteria

1) Extract \( N \)-data samples \( s[n] \).
2) Compute \( R_s \) with (24).
3) Perform eigenvalue decomposition of \( R_s \) as in (12).
4) Compute \( \beta_s \) with (14).
5) Compute \( y^{(p)}[n] \) with (13).
6) Compute the analytical signal \( z^{(p)}[n] \) with (16).
7) Extract the AM demodulated signal \( a[n] \) with (17a).
8) Extract the FM demodulated signal \( f[n] \) with (23).
9) Compute \( \hat{m}_a \) with (19), (25), and (27).
10) Compute \( \hat{m}_\phi \) with (22), (26), and (28).

---

A. Synthetic Signals

Synthetic signals \( s[n] \) were simulated by using the signal model in (3). Analysis of the algorithm performances with amplitude- and phase-modulated signals is investigated independently.
Fig. 2. AM demodulated signals for a healthy ($m_a = 0$) and a faulty ($m_a = 0.5$) balanced system.

Fig. 3. AM demodulated signals for a healthy ($m_a = 0$) and a faulty ($m_a = 0.5$) unbalanced system.

1) AM: Let us consider a discrete AM signal $a[n]$ with a modulating frequency equal to $f_a = 10$ Hz, i.e.,

$$a[n] = 1 + m_a \cos(2\pi n T_s)$$

(29)

where $m_a$ is the modulation index. Figs. 2 and 3 show the demodulated signal $\hat{a}[n]$ obtained with the Concordia and the PCA transform for balanced ($\alpha_k = 1$) and unbalanced systems ($\alpha_1 = 1.323$, $\alpha_2 = 0.5$, and $\alpha_3 = 1$), respectively. Demodulated signals are displayed for $m_a = 0$ and $m_a = 0.5$. In Fig. 2, one can note that both algorithms perform well since demodulation is near perfect. However, for the unbalanced system, PCA clearly outperforms CT since the latter exhibits interference terms which can lead to misleading interpretations. Table I presents the values of $\hat{m}_a$, $\hat{m}_\phi$, and $\gamma$ for $m_a = 0$ and $m_a = 0.5$. As expected, one could observe that the CT-based algorithm leads to a perfect modulation-index estimate for the balanced system ($\hat{m}_a = m_a$ and $\hat{m}_f = 0$) but leads to incorrect results for the unbalanced case ($\hat{m}_a \neq m_a$ and $\hat{m}_\phi \neq 0$). As opposed to the CT algorithm, the PCA algorithm leads to a perfect estimation whatever the balance assumption.

2) PM: Let us consider a discrete PM signal $\phi[n]$, with a modulating frequency equal to 10 Hz, i.e.,

$$\phi[n] = 2\pi f_0 n T_s + m_\phi \sin(2\pi n T_s)$$

(30)

where $m_\phi$ is the modulation index. Figs. 4 and 5 show the instantaneous frequency $f[n]$ extracted with the Concordia and the PCA transform for healthy ($m_\phi = 0$) and faulty ($m_\phi = 0.4$) systems. As expected, PCA-based demodulation clearly outperforms the CT-based one since it gives perfect
demodulation whatever the amount of current imbalance. Table II presents the values of the modulation-index estimates. For the balanced system, one can observe that the two techniques give perfect results since \( \hat{m}_a = 0.4 \). However, for the unbalanced case, Table II shows that the CT-based approach leads to incorrect results and makes the fault detection more difficult.

### B. Experimental Signals

The experimental setup is composed of a tachogenerator, a three-phase squirrel-cage induction motor, and a car alternator (see Fig. 6). The parameters of the induction motor are as follows: 0.75 kW, 220/380 V, 1.95/3.4 A, 2780 r/min, 50 Hz, and two poles. The tested motor has two 6204.2ZR-type bearings. The outside bearing diameter is 47 mm, and the inside one is 20 mm. Assuming the same thickness for the inner and the outer races leads to a pitch diameter equal to \( D_p = 31.85 \) mm. The bearing has eight balls \( (N = 8) \) with a diameter equal to \( D_B = 12 \) mm. The experimental tests have been performed with healthy and faulty bearings (inner race deterioration) under different motor load conditions [36].

As the faulty bearing introduces PM [15], the instantaneous frequency contains most of the information about the fault. Fig. 7 shows \( f[n] \) for healthy and faulty machines at a 300-W load condition. Additional processings show that the fault frequency is equal to \( f_\phi = 300 \) Hz whatever the load condition. Table III presents the values of the proposed criteria \( \hat{m}_a \) and \( \hat{m}_\phi \) when the motor operates under different motor load conditions.

From Table III, one can observe that the faulty bearings both increase \( \hat{m}_a \) and \( \hat{m}_\phi \). One can also note that \( \hat{m}_a \neq 0 \) for the healthy and faulty bearings. Indeed, signal \( s[n] \) is not perfectly sinusoidal and is composed of a small amount of AM even for the healthy bearing.

Comparing Algorithms 1 and 2, one can easily verify that the PCA-based criteria outperform the Concordia-based ones whatever the motor load. Indeed, in all experiments, healthy and faulty bearings are easier to distinguish with the PCA-based criteria. In particular, at a 200-W load motor condition, a faulty bearing leads to an increase in \( \hat{m}_\phi \) of 286% for Algorithm 1 and 340% for Algorithm 2. These results come from the fact that experimental signals contain a small amount of current imbalance, which makes PCA better suited than CT for current demodulation.

To compare the proposed approach with a conventional demodulation technique, Table IV presents the values of \( \hat{m}_a \) and \( \hat{m}_\phi \) obtained with an HT. The comparison of Table IV with Table III shows that the bearing fault is easier to detect with the criterion \( \hat{m}_\phi \) presented in Table III. Therefore, in our context, the multidimensional demodulation techniques seem to be better suited than the conventional HT. The differences can be explained by the intrinsic limitations of the HT: First, the domain of validity of this transform is restricted by the Bedrosian theorem [28]; then, the instantaneous amplitude and frequency obtained with HT can present overshoots at both ends [26]. Another advantage of the CT and PCA techniques over HT lies in the computational complexity. CT and PCA are linear transforms; therefore, they are simpler to implement than HT, which involves FFT and inverse FFT computations.

### TABLE II

<table>
<thead>
<tr>
<th>System</th>
<th>Modulation Index</th>
<th>Algorithm 1 Concordia</th>
<th>Algorithm 2 PCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balanced</td>
<td>( m_a )</td>
<td>( \hat{m}_a )</td>
<td>( m_a )</td>
</tr>
<tr>
<td></td>
<td>( m_\phi )</td>
<td>( \hat{m}_\phi )</td>
<td>( m_\phi )</td>
</tr>
<tr>
<td>Balanced</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Unbalanced</td>
<td>0.0</td>
<td>0.24</td>
<td>13.1</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.24</td>
<td>13.1</td>
</tr>
</tbody>
</table>

### TABLE III

<table>
<thead>
<tr>
<th>Load</th>
<th>Bearing</th>
<th>Algorithm 1 Concordia</th>
<th>Algorithm 2 PCA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( m_a )</td>
<td>( \hat{m}_a )</td>
<td>( m_a )</td>
</tr>
<tr>
<td></td>
<td>( m_\phi )</td>
<td>( \hat{m}_\phi )</td>
<td>( m_\phi )</td>
</tr>
<tr>
<td>No Load</td>
<td>Healthy</td>
<td>0.17</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Faulty</td>
<td>0.24</td>
<td>0.23</td>
</tr>
<tr>
<td>100W</td>
<td>Healthy</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>Faulty</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>200W</td>
<td>Healthy</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>Faulty</td>
<td>0.20</td>
<td>0.19</td>
</tr>
<tr>
<td>300W</td>
<td>Healthy</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Faulty</td>
<td>0.16</td>
<td>0.16</td>
</tr>
</tbody>
</table>

### TABLE IV

<table>
<thead>
<tr>
<th>Load</th>
<th>Bearing</th>
<th>Algorithm 1 Concordia</th>
<th>Algorithm 2 PCA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( m_a )</td>
<td>( \hat{m}_a )</td>
<td>( m_a )</td>
</tr>
<tr>
<td></td>
<td>( m_\phi )</td>
<td>( \hat{m}_\phi )</td>
<td>( m_\phi )</td>
</tr>
<tr>
<td>No Load</td>
<td>Healthy</td>
<td>0.17</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Faulty</td>
<td>0.24</td>
<td>0.23</td>
</tr>
<tr>
<td>100W</td>
<td>Healthy</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>Faulty</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>200W</td>
<td>Healthy</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>Faulty</td>
<td>0.20</td>
<td>0.19</td>
</tr>
<tr>
<td>300W</td>
<td>Healthy</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Faulty</td>
<td>0.16</td>
<td>0.16</td>
</tr>
</tbody>
</table>
### VI. Discussion

A standard approach for signal demodulation is based on the extraction of the analytical signal. An analytical signal is usually extracted from the HT for a monodimensional signal. In the case of balanced three-phase systems, the previous sections have shown that the analytical signal can be efficiently extracted with the CT or PCA. These transformations have several advantages over HT: First, they are simpler to implement; then, they are free from signal artifacts (overshoots).

The main drawback of CT relies on its domain of validity. Indeed, for an unbalanced system, this transform cannot be employed for current demodulation. On the contrary, the PCA transform, which is a data-driven approach, is well suited for an unbalanced system. Another interesting property of PCA relies on the fact that it can easily measure the amount of static imbalance. Indeed, (40) shows that the eigenvalues of $R_a$ are equal for balanced systems, i.e., $\lambda_1 = \lambda_2$. For unbalanced systems, this property does not hold, and $\lambda_1 \neq \lambda_2$. Therefore, an equality test can be employed to distinguish between balanced and unbalanced systems. This strategy has been previously employed to detect stator winding faults in [41], and an ad hoc criterion has been proposed to perform the equality test. Interestingly, this equality test is well known in the signal processing community and refers to a “sphericity test” [46].

In the field of multidimensional signal processing, another popular technique is the independent component analysis (ICA) [47]. Whereas PCA focuses on correlation (i.e., second-order statistics), the ICA focuses on statistical independence, a stronger property, which is usually measured through higher order statistics. Therefore, it can be tempting to use ICA to improve the performance of the proposed method. However, in our study, it can be proved that the ICA is useless. Indeed, most of the ICA algorithms are composed of two steps: a PCA preprocessing step followed by a demixing step. After the PCA, the demixing step searches for an orthogonal matrix that maximizes the statistical independence of the PCA components. Nevertheless, in our context,\(^5\) multiplying $y(t)$ with a $2 \times 2$ orthogonal matrix only modifies the value of $\theta$ in (15). As $\theta$ does not affect $a[n]$ and $f[n]$, the demixing step is useless, and the algorithm can therefore be limited to a PCA transform for demodulation.

\(^5\)The formal proof can be derived from (42) and (43).

### VII. Conclusion

This paper has focused on condition monitoring of three-phase electrical systems. A new method based on the amplitude and phase demodulation of the three-phase stator current with linear transforms has been proposed. The performances of two linear transforms have been investigated: the CT and the PCA. Based on a deep theoretical analysis, it has been proved that the PCA has a larger domain of validity than CT since it can deal with unbalanced currents.

Then, two original criteria have been described to assess the failure severity from the demodulated current. The performances of the proposed criteria have been corroborated by means of simulations with synthetic and experimental signals. In particular, the results have shown that the PCA-based demodulation outperforms the Concordia-based one since it can be employed for unbalanced stator currents.

### APPENDIX

Let us compute the correlation matrix $R_i = E[i(t)i^T(t)]$ for a balanced system. Using (3) and under the assumption that $a(t)$ and $\phi(t)$ are independent variables, one gets for all $u = 1, 2, 3$ and $v = 1, 2, 3$

$$E[i_u(t)i_v(t)] = E[a^2(t)]E_{uv}$$

where

$$E_{uv} = E[\cos(\phi(t) + \psi_u) \cos(\phi(t) + \psi_v)]$$

with $\psi_1 = 0, \psi_2 = -2\pi/3, \text{and} \, \psi_3 = 2\pi/3$. Let us assume that the random variable $x = \phi(t)$ is distributed in $[0, 2\pi]$ according to a uniform pdf, i.e., $f(x) = 1/2\pi \text{ if } x \in [0, 2\pi]$ and $f(x) = 0$ if elsewhere. It follows that

$$E_{uv} = E[\cos(\phi(t) + \psi_u) \cos(\phi(t) + \psi_v)]$$

$$= \int_{-\infty}^{+\infty} \cos(x + \psi_u) \cos(x + \psi_v) f(x) \, dx$$

$$= \frac{1}{2\pi} \int_{0}^{2\pi} \cos(x + \psi_u) \cos(x + \psi_v) \, dx.$$  \hspace{1cm} (33)

Using trigonometric identities and some simplifications, one can verify that

$$E_{uv} = \frac{1}{2} \cos(\psi_u - \psi_v).$$

Therefore, it follows that

$$E[i_u(t)i_v(t)] = \left(E[a^2(t)]/2\right) \cos(\psi_u - \psi_v).$$  \hspace{1cm} (35)

Finally, one gets

$$R_i = \frac{E[a^2(t)]}{2} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}.$$  \hspace{1cm} (36)

One can also verify that

$$\beta^2_i = \text{trace}[R_i]/3 = E[a^2(t)]/2.$$  \hspace{1cm} (37)
Using these equations and the definition of the Concordia matrix in (7), one can verify that
\[ R_i = \frac{3}{2} \beta^2 C^T C. \]  

(38)

Next, using an eigenvalue decomposition, \( R_i \) can also be decomposed as
\[ R_i = U \Lambda U^T \]  

(39)

where \( U \) is a 3 \times 3 orthogonal matrix (\( U^T U = I_3 \)) containing the two eigenvectors and \( \Lambda = \text{diag}(\lambda_1, \lambda_2) \) is a diagonal matrix containing the two nonzero associated eigenvalues \( \lambda_1 \) and \( \lambda_2 \).

By identifying (38) and (39), one gets
\[ U = (WC)^T, \]  

(40a)

\[ \Lambda = \frac{3}{2} \beta^2 I_2, \]  

(40b)

where \( W \) is a 2 \times 2 orthogonal matrix, i.e., \( W^T W = WW^T = I_2 \). This orthogonal matrix comes from the fact that the eigenvalue decomposition is not unique.

Using (40) in (13), one gets
\[ y^{(p)}(t) = \beta \Lambda^{-1/2} U^T i(t) \]
\[ = W \left( \sqrt{\frac{2}{3}} C i(t) \right) = W y^{(c)}(t). \]  

(41)

As \( W \) is a 2 \times 2 orthogonal matrix, \( W = \{W_1(\theta), W_2(\theta)\} \), where \( \theta \in \mathbb{R} \) and
\[ W_1(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \]
\[ W_2(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & -\cos(\theta) \end{bmatrix}. \]  

(42a)

(42b)

As \( y^{(c)}(t) = [a(t) \cos(\phi(t)), a(t) \sin(\phi(t))]^T \), it can be shown, using trigonometric identities, that
\[ W_1(\theta)y^{(c)}(t) = \begin{bmatrix} a(t) \cos(\phi(t) + \theta) \\ a(t) \sin(\phi(t) + \theta) \end{bmatrix}, \]
\[ W_2(\theta)y^{(c)}(t) = \begin{bmatrix} a(t) \cos(\phi(t) - \theta) \\ a(t) \sin(\phi(t) - \theta) \end{bmatrix}. \]  

(43a)

(43b)

Therefore, without loss of generality, the PCA components can be expressed as
\[ y_2^{(p)}(t) = a(t) \cos(\phi(t) - \theta), \]
\[ y_1^{(p)}(t) = a(t) \sin(\phi(t) - \theta). \]  

(44a)

(44b)

Let us consider an unbalanced system whose components are given by \( s(t) = D i(t) \). Using an eigenvalue decomposition, the covariance matrix, denoted \( R_s = E[s(t)s^T(t)] \), can be decomposed as
\[ R_s = E[s(t)s^T(t)] = U_d \Lambda_d U_d^T \]  

(45)

where \( U_d \) is a 3 \times 2 orthogonal matrix and \( \Lambda_d \) is a 2 \times 2 diagonal matrix. Using (5) and (39), \( R_s \) can also be expressed as
\[ R_s = DR_1D = DU \Lambda D^T D \]  

(46)

where \( R_1 = U \Lambda U^T \) is the covariance matrix for a balanced system. By identifying (45) and (46), one gets
\[ U_d \Lambda_d^{1/2} U_d^T = D U \Lambda^{1/2} D^T \]  

(47)

where \( U \) is a 2 \times 2 orthogonal matrix. This matrix comes from the fact that the equality \( U_d \Lambda_d U_d^T = D U \Lambda D^T D \) is satisfied for any orthogonal matrix \( V \). Taking the inverse of the previous equation leads to
\[ \Lambda_d^{1/2} U_d^T = V \Lambda^{1/2} U D^{-1}. \]  

(48)

Using these equations and (13), the PCA components of the unbalanced system, denoted \( g_d(t) \), can be expressed
\[ y^{(p)}(t) = \beta_s \Lambda^{-1/2} U_d^T s(t) \]
\[ = \beta_s V \Lambda^{-1/2} U D^{-1} s(t) \]
\[ = V \left( \beta_s \Lambda^{-1/2} U D^{-1} i(t) \right). \]  

(49)

(50)

(51)

Let us express \( \beta_s \) with respect to \( \beta \). By using (5), (35), and the normalization assumption \( \sum_{k=1}^{3} \alpha_k^2 = 3 \), one gets
\[ \beta_s = \text{trace}(R_s)/3 \]
\[ = \frac{3}{3} \sum_{k=1}^{3} \alpha_k^2 = \beta_s \]  

(52)

By using (41), (51), and (52), one can conclude that
\[ y^{(p)}(t) = V \left( \beta_s \Lambda^{-1/2} U D^{-1} i(t) \right) = VW y^{(c)}(t). \]  

(53)

As \( V \) and \( W \) are orthogonal matrices, it follows that \( VW \) is also an orthogonal matrix. Therefore, \( y^{(p)}(t) \) can be expressed as
\[ y^{(p)}(t) = W_d y^{(c)}(t) \]  

(54)

where \( W_d = VW \) is a 2 \times 2 orthogonal matrix. As (54) is similar to (41), a similar development as in this Appendix leads to
\[ y_2^{(p)}(t) = a(t) \cos(\phi(t) - \theta) \]  

(55a)

\[ y_1^{(p)}(t) = a(t) \sin(\phi(t) - \theta). \]  

(55b)

Let us consider the AM signal in (1). Let us also assume that the random variable \( x = 2 \pi f_s t (\text{mod } 2\pi) \) is distributed in \([0 2\pi]\) according to a uniform pdf, i.e., \( f(x) = 1/2\pi \) if \( x \in [0, 2\pi] \) and \( f(x) = 0 \) is elsewhere. Under this assumption, one gets
\[ \mu_a = E[a(t)] \]
\[ = \int_{0}^{2\pi} a(t) (1 + m_a \cos(x)) f(x) dx \]
\[ = \mu_a. \]  

(56)

Therefore, it follows that
\[ \sigma_a^2 = E \left[ (a(t) - \mu_a)^2 \right] = \mu_a^2 m_a^2 E \left[ \cos^2(x) \right]. \]  

(57)

Using (32) and (34), one gets \( E[\cos^2(x)] = 1/2 \). It follows that
\[ \sigma_a^2 = \frac{\mu_a^2 m_a^2}{2}. \]  

(58)
Vincent Choqueuse (S’08–M’09) was born in Brest, France, in 1981. He received the Dipl.-Ing. and M.Sc. degrees from the University of Technology of Troyes, Troyes, France, in 2004 and 2005, respectively, and the Ph.D. degree from the University of Brest, Brest, in 2008.

Since September 2009, he has been an Associate Professor with the Institut Universitaire de Technologie de Brest, University of Brest, and a Member of the Brest Laboratory of Mechanics and Systems (LBMS) (EA 4325). His research interests focus on signal processing and statistics for diagnosis and multiple-input multiple-output systems.

Mohamed El Hachemi Benbouzid (S’92–M’95–SM’98) was born in Batna, Algeria, in 1968. He received the B.Sc. degree in electrical engineering from the University of Batna, Batna, in 1990, the M.Sc. and Ph.D. degrees in electrical and computer engineering from the National Polytechnic Institute of Grenoble, Grenoble, France, in 1991 and 1994, respectively, and the Habilitation à Diriger des Recherches degree from the University of Picardie “Jules Verne,” Amiens, France, in 2000.

After receiving the Ph.D. degree, he joined the Professional Institute of Amiens, University of Picardie “Jules Verne,” where he was an Associate Professor of electrical and computer engineering. Since September 2004, he has been with the Institut Universitaire de Technologie de Brest, University of Brest, Brest, France, where he is a Professor of electrical engineering. His main research interests and experience include the analysis, design, and control of electric machines, variable-speed drives for traction, propulsion, and renewable energy applications, and fault diagnosis of electric machines.

Dr. Benbouzid is a Senior Member of the IEEE Power Engineering, IEEE Industrial Electronics, IEEE Industry Applications, IEEE Power Electronics, and IEEE Vehicular Technology Societies. He is an Associate Editor of the IEEE TRANSACTIONS ON ENERGY CONVERSION, the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, and the IEEE/ASME TRANSACTIONS ON MECHATRONICS.

Yassine Amirat was born in Annaba, Algeria, in 1970. He received the B.Sc. and M.Sc. degrees in electrical engineering from the University of Annaba, Annaba, in 1994 and 1997, respectively. He is currently working toward the Ph.D. degree in wind turbine condition monitoring at the University of Brest, Brest, France.

He is currently a Lecturer with the Institut Supérieur de l’Électronique et du Numérique, Brest. His current research interests are the condition monitoring and the control of electrical drives and power electronics.

Sylvie Turri was born in France in 1972. She received the M.Sc. degree in electrical engineering from the University of Nancy, Nancy, France, in 1996 and the Ph.D. degree in electrical engineering from the University of Franche-Comté, Belfort, France, in 2000.

After receiving the Ph.D. degree, she joined the Systèmes et Applications des Technologies de l’Information et de l’énergie Laboratory (UMR CNRS 8029), Ecole Normale Supérieure de Cachan, Cachan, France, as a Teaching and Research Associate. In 2004, she joined the Institut Universitaire de Technologie de Aix-Marseille, University of Marseille III, Marseille, France, as an Associate Professor of electrical engineering. She was appointed to the Laboratoires des Sciences de l’Information et des Systèmes Laboratory (UMR CNRS 6168). In 2006, she joined the Institut Universitaire de Technologie de Brest, University of Brest, Brest, France, as an Associate Professor of electrical engineering and a Member of the Brest Laboratory of Mechanics and Systems (LBMS) (EA 4325). Her main research interests are in the field of electromechanical systems (power generation and fault diagnosis).