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Optimal Harvesting of Fish Stocks under a Time-varying Discount Rate

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Abstract

Optimal control theory has been extensively used to determine the optimal harvesting policy for renewable resources such as fish stocks. In such optimizations, it is common to maximise the discounted utility of harvesting over time, employing a constant time discount rate. However, evidence from human and animal behaviour suggests that we have evolved to employ discount rates which fall over time, often referred to as "hyperbolic discounting". This increases the weight on benefits in the distant future, which may appear to provide greater protection of resources for future generations, but also creates challenges of time-inconsistent plans. This paper examines harvesting plans when the discount rate declines over time. With a declining discount rate, the planner reduces stock levels in the early stages (when the discount rate is high) and intends to compensate by allowing the stock level to recover later (when the discount rate will be lower). Such a plan may be feasible and optimal, provided that the planner remains committed throughout. However, in practice there is a danger that such plans will be re-optimized and adjusted in the future. It is shown that repeatedly restarting the optimization can drive the stock level down to the point where the optimal policy is to harvest the stock to extinction. In short, a key contribution of this paper is to identify the surprising severity of the consequences flowing from incorporating a rather trivial, and widely prevalent, non-rational aspect of human behaviour into renewable resource management models. These ideas are related to the collapse of the Peruvian anchovy fishery in the 1970's.

Keywords. Resource management, Bio-economics, Optimal control, Dynamical systems.

1 Introduction

In this paper, we consider the optimal harvesting of a renewable resource when the discounting of the utility of harvesting varies over time. The analysis is based upon a standard model of the harvesting of a sole owner fishery (Clark, 1990; Sethi & Thompson, 2000), but the results might also be applied to other renewable resources such as forestry, water resources and agricultural land use. We consider a system where the rate of change of the fish stock level is the difference between the natural growth rate of the population and the harvesting rate. In the absence of any discounting, if the utility is a monotonically increasing function of the harvesting rate, then the optimal integrated utility over an infinite time horizon is obtained by following a harvesting rate that drives the stock to the saddle point equilibrium point corresponding to the maximum sustainable yield, which will coincide with the peak of the natural growth rate curve (Clark, 1990). As shown in Figure 1, when a constant discount factor is included in the objective function, so that the optimal harvesting rate maximizes the integrated discounted utility over an infinite time horizon, the optimal sustainable policy converges to an equilibrium at a stock level that is below the maximum sustainable yield associated with the undiscounted objective function. The harvesting rate follows the stable manifold of the saddle point equilibrium and for an initial stock level below the optimal sustainable yield, the stock level will increase monotonically with time. However, for very low initial stock levels, it can take a long time for the stock to recover to the optimal sustainable yield and because the discount factor falls during this period, the optimal solution in this case is to collapse the stock by following a trajectory that converges to a stable equilibrium where there is no remaining stock, resulting in the extinction of the species (Clark, 1990). There is a critical initial stock level above which the optimal policy converges to the sustainable equilibrium, but when the initial stock is below this critical level, the optimal policy harvests all of the existing stock. This can be regarded as a consequence of the fact that the discount factor reduces the benefits of harvesting in the future.

[Figure 1 about here.]

It is usual to assume that the discount factor is described by an exponentially decaying function with a constant discount rate. However, evidence from behavioral economics discussed in Section 2 suggests that time-declining discount rates are commonly observed in human behaviour (Thaler, 1981; Kirby, 1997; Harris & Laibson, 2001; Ainslie, 1992; Laibson, 1997; Harris & Laibson, 2001) and animal behaviour (Mazur, 1987; Green & Myerson, 1996). Such behaviour can have some surprising and unappealing results, discussed in Section 2 and exemplified by the analysis in this paper. However, using time-declining discount rates also puts greater weight on distant future events. For instance, Li & Löfgren (2000) have examined the effect of using a “hyperbolic discount” model[1] when determining the

[1] Although hyperbolic discounting was originally introduced to describe a discount rate that varied inversely with time, the term is now commonly used to describe any discounting with a declining discount rate (Dasgupta & Maskin, 2005).

optimal policy for achieving economic sustainability and they showed that it is necessary for the discount rate to decline with time in order to avoid the current generation having 'tyranny' over future generations.

An example of a failure to use a discounting policy that takes account of the benefits to future generations is the collapse of the stock of Peruvian anchovy, which was once the world's largest fishery (Idyll, 1973). The Peruvian fishing fleet expanded rapidly during the 1960's to such an extent that the authorities feared a collapse of anchovy stocks and they responded by restricting the length of the fishing season (Aguero, 1987). Experts such as the Instituto del Mar del Peru (Glantz, 1979) warned that the fishery should be closed to allow the stocks to recover so that it could survive the effect of the next El Niño. When the El Niño arrived in 1972-73, rather than closing the fishery, the Peruvian authorities continued to allow fishing, resulting in a collapse of the anchovy population, which had a worldwide effect on food prices (Idyll, 1973). Barrett *et al.* (1985) highlighted the difference between a discount rate based purely on economic factors and a discount rate that takes into account the long term benefit to society of the anchovy fishery (Cropper & Laibson, 1999). They compared the net present value of the anchovy harvest associated with a policy that allowed the fish stocks to recover over a period of ten years, to the short-term policy based on a market discount rate and showed that the decision not to impose a conservation policy that would allow fish stocks to recover, was based on a short-term policy.

When a decreasing discount rate is applied to the problem of determining the optimal harvesting of fish stocks, the effect is to reduce the benefit of the collapse solution, because unlike the constant discount rate case, the discount rate decays asymptotically to zero as time becomes large, increasing the benefit of allowing the stock to recover to the point corresponding to the optimal sustainable yield. This paper shows how the optimal trajectories that converge to both the sustainable and the collapse solutions can be calculated for the case where the discount rate decays exponentially over time. The optimal policy can lead to a reduction in stocks during the early stages of the policy when the discount rate is high, which is then offset by a relatively low harvesting rate in the later stages of the policy when the discount rate is reduced, allowing the fish stocks to recover to the sustainable equilibrium. However, declining discount rates generally give rise to time-inconsistent plans (Ramsey, 1928; Strotz, 1956), which implies that policies made today will not be carried out tomorrow unless a mechanism for ensuring commitment is in place. This paper shows that problems can occur if the planner does not remain committed to the optimal trajectory, but instead follows a policy leading to the sustainable equilibrium for only a finite time before recalculating the policy starting from the current stock level. If the stock level at which the re-optimization occurs is lower than the initial stock level and if the re-optimization is repeated, and even though the planner had been attempting to follow a policy that led to the sustainable solution, at some point in the future the collapse solution becomes optimal (Hepburn *et al.*, 2010). Although the factors leading to the collapse of the stocks of North Atlantic cod due to over-fishing are primarily the result of errors in estimating the existing stock level (Walters & Maguire, 1996), it is not impossible that repeated re-optimization of policies based on an initial high discount rate contributed to the collapse.

The paper is organized as follows. Section 2 provides a review of the literature on discounting and hyperbolic discounting and its relationship to renewable resource management. In Section 3, the solution to the problem of determining the optimal policy for a time-varying discount rate is presented (the details of the derivation are given in the Appendix). Although this closely follows the solution for the constant discount rate case (Clark, 1990), the version presented here explicitly includes the effects of the time-varying discounting and the requirement that the cost of harvesting must remain finite as the stock level approaches zero, which is relevant when determining the collapse solution associated with the extinction of the stock. Section 4 discusses the results and Section 5 concludes the paper.

2 Literature Review

A discount factor, $D(t)$, is used in management of renewable resources to reflect the fact that the utility derived from a harvest may be worth more at one point in time than another. Normally, humans prefer utility now rather than later, so $D(t)$ falls over time. An important distinction is drawn between discounting *utility* on the on hand, and *goods* on the other. In a realistic economy, where the rates of return on assets in the economy are not identical, the appropriate discount factor will be specific to the particular good being evaluated, and will also be specific to the individual or organisation performing the evaluation. The discount rate, $\delta(t)$, is defined as the rate of decline in the discount factor, $D(t)$, and it is commonly assumed for ease of analysis that the discount rate $\delta(t)$ is constant, such that the discount factor $D(t)$ falls exponentially.

Over the last couple of decades, debates over the theory and practice of discounting have occurred within the economic literature and even the mainstream media. One of the most significant areas of debate is whether the discounting behaviours implicit in real (but imperfect) human choices should be employed in economic analysis, or whether more "prescriptive" approaches should be employed, which might for instance explicitly take greater account of the long-term interests of future generations. We do not enter into these debates in this paper.[2] Instead, we examine the implications of incorporating imperfect but realistic human discounting behaviour into the management of a renewable resource. In other words, this paper provides a "descriptive" rather than a "normative" analysis, although we offer

[2]The interested reader is referred to the discussion and analysis on discounting following the Stern Review on the Economics of Climate Change (Stern, 2007; Beckerman & Hepburn, 2007; Weitzman, 2007; Nordhaus, 2007; Dasgupta, 2008; Heal, 2009).

some reflections on the normative and policy recommendations flowing from our analysis in Section 5.

Given our objectives, we start from various experiments from behavioural economists and psychologists into human and animal "time preference". This research shows robust evidence of "preference reversals", where subjects choose a today rather than b tomorrow, but will choose b in one year and a day ahead of a in one year from now. These preference reversals are not consistent with rational choice under a constant discount rate $\delta(t)$. They are consistent with $\dot{\delta}(t) < 0$ or "diminishing impatience". Indeed, many experiments suggest that impatience is higher in the present than with respect to trade-offs in the future (Ainslie, 1992; Frederick *et al.*, 2002; DellaVigna, 2009). This has come to be referred to as "hyperbolic discounting", in reference to the fact that one relevant functional form was hyperbolic. Interestingly, diminishing impatience (or "hyperbolic discounting") appears prevalent in both humans and animals, and while they may at first seem irrational, there are plausible ways in which they could confer evolutionary advantage. For instance, Dasgupta & Maskin (2005) note that in the presence of given uncertainty and waiting costs, evolutionary pressure may have generated such preferences.

While there may be other explanations for some of the observed behaviour,[3] careful reviews of the field (Frederick *et al.*, 2002; DellaVigna, 2009) conclude that the evidence for hyperbolic discounting is robust. Furthermore, recent evidence from functional imaging of the brain also supports a quasi-hyperbolic discounting model (McClure et al. 2007). Additionally, the hyperbolic discounting model provides explanations for otherwise baffling phenomena (Laibson et al. 2010) and there is robust evidence that people do have self-control problems of the sort implicated by hyperbolic discounting, that commitment mechanisms can be beneficial (Ariely and Wertenbroch 2002; Gine et al. 2009) and that some people are willing to effectively pay for commitment mechanisms to prevent time inconsistencies (Ashraf et al. 2006; DellaVigna and Malmendier 2004, 2006).

If hyperbolic discounting does provide an accurate description of the time preferences of humans and animals in some contexts, the policy implications are challenging. In particular, although hyperbolic discounting may place greater weight on the interests of future generations, there are problems with incorporating hyperbolic discounting of *utility* into social cost-benefit analysis (Pearce *et al.*, 2003; Groom *et al.*, 2005). Because hyperbolic discounting can explain a range of perplexing human behaviours, including drug addiction (Gruber & Koszegi, 2001), sub-optimally low savings rates (Laibson, 1997; Laibson *et al.*, 1998; Harris & Laibson, 2001), procrastination (ODonoghue & Rabin, 1999a,b; Benabou & Tirole, 2004) and various others (Akerlof, 1991), this paper does not make the normative recommendation that hyperbolic preferences be incorporated into optimal policy making.[4]

[3] For instance, there are potential confounding factors in some studies (Chabris et al. 2008), and other theories also appear consistent with the evidence for preference reversals (Read, 2001; Rubinstein, 2003).

[4] There are other reasons government policy may wish to employ a declining *consumption* discount rate over time, rather

On the other hand, it is important to understand the descriptive implications of what might happen if a planner does use hyperbolic discounting in making policy, if only so that we can be aware of potential problems before they arise. The contribution of this paper is to provide such an analysis in the context of renewable resource management.

Hyperbolic discounting has been considered, particularly by economists, in renewable resource management and more specifically climate change over the last decade.[5] In the context of climate change, Karp (2005), Karp & Tsur (2008) and Fujii & Karp (2008) develop models of hyperbolic discounting in which there are a succession of regulators developing plans which account for how future regulators will deviate from their plans. The paper thus finds a time-consistent Markov Perfect equilibrium. In contrast, in our model we consider the implications if the planner/regulator is even more imperfect (or realistic) such that, following the evidence outlined above, the planner determines a harvesting policy based on the erroneous assumption that they can commit future planners. The contribution of this paper is to show that where an (imperfect) planner is managing a renewable resource, such as a fishery, and they do not solve a sophisticated inter-temporal game but simply assume that future decisions will follow their plan, then the results can be dire. Of course, it is obvious that if a planner errs in anticipating their future behaviour, the outcome may be poor. The non-obvious aspect of the paper is that if the planner behaves much like as a normal human being, rather than as a hypothetical perfectly rational maximiser, or as a planner who can find Markov Perfect equilibria in intertemporal games, then there are conditions in which the erroneous assumption of future commitment can be catastrophic for the economic and biological survival of the fishery. In short, the surprising aspect of this paper is the potential severity of the consequences flowing from incorporating a rather trivial, and widely prevalent, non-rational aspect of human behaviour.

3 Solving the Optimization Problem

The analysis presented in this paper is based upon the mathematical model for the exploitation of a renewable resource that is presented in, for example, Clark (1990). Let $x(t) \geq 0$ denote the fish stock at time t, and $h(t) \geq 0$ denote the harvesting rate. If $U(h)$ denotes the social utility of harvesting and $c(x)$ denotes the unit cost of harvesting, the aim is to find the harvesting rate $h(t)$ that optimizes the

than a declining *utility* discount rate over time, relating to uncertainty. For instance, Sozou (1998) and Weitzman (1998) have shown that when the discount rate is constant but uncertain, the 'certainty-equivalent discount rate' will decline over time to the lowest possible rate. The UK Government Treasury guidelines on the appraisal of investments and policies requires the use of declining discount rates for these reasons (H M Treasury, 2003). Hepburn (2006) provides a review of approaches to discounting within OECD countries.

[5] See for instance Cropper & Laibson (1999), Shogren & Settle (2004), Voinov & Farley (2007) and the 2010 Special Issue 46:2 of *Environmental and Resource Economics* on behavioural economics and the environment.

integrated discounted net benefit of harvesting (utility less costs) over an infinite time horizon

$$\max_{h(t)} \left\{ \int_0^\infty D(t) \left[U(h) - c(x)h(t) \right] dt \right\} \tag{1}$$

subject to

$$\dot{x}(t) = F(x) - h(t) \quad \text{with } x(0) = x_0 \tag{2}$$

where x_0 is the initial stock level at $t = 0$ and $F(x)$ describes the natural growth rate of the fish stock in the absence of any harvesting (*i.e.* the reproductive rate minus the death rate) (Clark, 1990). The discount factor is defined as

$$D(t) = \exp \left\{ - \int_0^t \delta(\tau) d\tau \right\} \tag{3}$$

with $\delta(t) > 0$ being the (time-varying) discount rate.

Appendix A shows that the trajectories of $x(t)$ and $h(t)$ that optimize (1) for a time-varying discount rate satisfy

$$\begin{aligned} \dot{x}(t) &= F(x) - h(t) && (4) \\ \dot{h}(t) &= \frac{1}{U''(h)} \left\{ \left[\delta(t) - F'(x) \right] \left[U'(h) - c(x) \right] + c'(x)F(x) \right\} && (5) \end{aligned}$$

with $x(0) = x_0$. Note that when $\delta(t)$ is constant, (5) agrees with the expression given in Clark (1990). We consider the case where the discount rate decays over time according to

$$\delta(t) = \underline{\delta} + (\overline{\delta} - \underline{\delta})\mathrm{e}^{-\chi t} \tag{6}$$

which has the effect of reducing the discount rate from $\overline{\delta}$ at $t = 0$ to $\underline{\delta}$ as $t \to \infty$. From (3), the discount factor $D(t)$ associated with this discount rate is given by

$$D(t) = \exp \left\{ - \left[\underline{\delta} t + \frac{(\overline{\delta} - \underline{\delta})}{\chi} \left(1 - \mathrm{e}^{-\chi t} \right) \right] \right\} \tag{7}$$

and Figure 2 plots $D(t)$ for the parameters in Table 1. Because $\delta(t)$ is time-varying, the evolution equations in (4) and (5) are non-autonomous, but because

$$\dot{\delta}(t) = -\chi(\overline{\delta} - \underline{\delta})\mathrm{e}^{-\chi t} = -\chi(\delta(t) - \underline{\delta}) \tag{8}$$

combining (4), (5) and (8) gives a three state autonomous system

$$\begin{bmatrix} \dot{x}(t) \\ \dot{h}(t) \\ \dot{\delta}(t) \end{bmatrix} = \begin{bmatrix} F(x) - h(t) \\ \frac{1}{U''(h)} \left\{ \left[\delta(t) - F'(x)\right] \left[U'(h) - c(x)\right] + c'(x)F(x) \right\} \\ -\chi(\delta(t) - \underline{\delta}) \end{bmatrix} \tag{9}$$

It should be emphasized that this autonomous system is not used for analyzing the stability of the underlying non-autonomous system, but instead is introduced for the purpose of obtaining the optimal solution by numerical integration.

The evolution of $\delta(t)$ is independent of $x(t)$ and $h(t)$, so the autonomous system will only converge to an equilibrium in the limit $\delta(t) \to \underline{\delta}$, which occurs as $t \to \infty$, and equilibrium points will occur at values of x and h satisfying

$$F(x) - h = 0 \tag{10}$$

$$\frac{1}{U''(h)} \left\{ \left[\underline{\delta} - F'(x)\right] \left[U'(h) - c(x)\right] + c'(x)F(x) \right\} = 0 \tag{11}$$

This means that the equilibria associated with optimal harvesting in the presence of the time-varying discount rate in (7) are the same as the equilibria associated with optimal harvesting using a constant discount factor of $\underline{\delta}$ (Clark, 1990).

[Table 1 about here.]

[Figure 2 about here.]

3.1 Sustainable Solution

Assume that the natural growth rate of the fish population in the absence of any harvesting is described by a model with critical depensation that ignores the age structure of the population, so that

$$F(x) = rx \left(\frac{x}{K_0} - 1 \right) \left(1 - \frac{x}{K} \right) \tag{12}$$

where r is the net proportional growth rate of the population, K_0 is the minimum viable population and K is the environmental carrying capacity, with $0 < K_0 < K$ (Clark, 1990; Hanley *et al.*, 2007). The instantaneous utility of consumption is assumed to be isoelastic and is taken to have the form

$$U(h) = \frac{h^{(1-\gamma)}}{1-\gamma} \tag{13}$$

where $\gamma > 0$ is the Arrow-Pratt coefficient of relative risk-aversion (Pratt, 1964; Arrow, 1965). The unit cost of harvesting is taken to be

$$c(x) = \frac{C}{x^{\alpha}} \tag{14}$$

where C and α are constants and choosing $\alpha > 0$ ensures that the cost of harvesting increases as the stock level reduces. For simplicity, we assume that $\gamma \leq 1$ and $\alpha \leq 1$, although the analysis can be extended to handle larger values of these parameters.

Using these expressions for $U(h)$ and $c(x)$, the evolution of the optimal harvesting rate is

$$\dot{h}(t) = \frac{h}{\gamma}\left\{[F'(x) - \delta(t)]\left[1 - C\frac{h^{\gamma}}{x^{\alpha}}\right] + \alpha C \frac{h^{\gamma}}{x^{\alpha}}\frac{F(x)}{x}\right\} \tag{15}$$

For this model, there exists an equilibrium point of the system in (9) denoted by $[x_s, h_s, \underline{\delta}]^{\mathrm{T}}$, where $x_s > 0$ and $h_s > 0$ satisfy

$$\begin{aligned} F(x_s) - h_s &= 0 && (16)\\ \frac{h_s}{\gamma}\left\{[F'(x_s) - \underline{\delta}]\left[1 - C\frac{h_s^{\gamma}}{x_s^{\alpha}}\right] + \alpha C\frac{h_s^{\gamma}}{x_s^{\alpha}}\frac{F(x_s)}{x_s}\right\} &= 0 && (17)\end{aligned}$$

The Jacobian associated with this equilibrium is

$$\mathbf{J}_s = \begin{bmatrix} F'(x_s) & -1 & 0 \\ J_{21} & J_{22} & -\frac{h_s}{\gamma}\left(1 - C\frac{h_s^{\gamma}}{x_s^{\alpha}}\right) \\ 0 & 0 & -\chi \end{bmatrix} \tag{18}$$

where

$$\begin{aligned} J_{21} &= \frac{h_s}{\gamma}\left\{F''(x_s)\left[1 - C\frac{h_s^{\gamma}}{x_s^{\alpha}}\right] + \left[2\frac{F'(x_s)}{x_s} - \underline{\delta}\right]\alpha C\frac{h_s^{\gamma}}{x_s^{\alpha}} - \alpha(\alpha+1)C\frac{h_s^{\gamma}}{x_s^{\alpha}}\frac{F(x_s)}{x_s^2}\right\} && (19)\\ J_{22} &= C\frac{h_s^{\gamma}}{x_s^{\alpha}}\left[\alpha\frac{F(x_s)}{x_s} - (F'(x_s) - \underline{\delta})\right] && (20)\end{aligned}$$

By ignoring the higher order terms in (9), in the region close to the equilibrium point

$$\begin{bmatrix} x(t) - x_s \\ h(t) - h_s \\ \delta(t) - \underline{\delta} \end{bmatrix} \approx \alpha_1 \mathbf{v}_1 \mathrm{e}^{\lambda_1 t} + \alpha_2 \mathbf{v}_2 \mathrm{e}^{\lambda_2 t} + \alpha_3 \mathbf{v}_3 \mathrm{e}^{\lambda_3 t} \tag{21}$$

where $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$ are constants and $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbb{R}^3$ are the eigenvectors of the Jacobian associated with the eigenvalues λ_1, λ_2 and λ_3. One of the eigenvalues occurs at $\lambda_1 = -\chi$, which is associated with the evolution of $\delta(t)$ to $\underline{\delta}$. For a wide range of model parameters, including the values given in Table 1, one of the remaining two eigenvalues is positive, while the other is negative, indicating the presence of a saddle-point equilibrium. Given the structure of $\mathbf{J}_s$, it can be shown that the third element of both $\mathbf{v}_2$ and $\mathbf{v}_3$ are zero, and that the first two elements of both of these eigenvectors are equal to the eigenvectors associated with the saddle-point equilibrium for the case with a constant discount rate of $\underline{\delta}$. For values of $t \gg 0$, so that $\mathrm{e}^{-\chi t} \approx 0$, then $x(t)$ and $h(t)$ approach equilibrium along the eigenvector associated with the stable eigenvalue, which coincides with the stable trajectory of a system with constant discount rate, $\underline{\delta}$. This suggests that the following procedure can be used to find the trajectory of the time-varying system in (9) that approaches the sustainable equilibrium.

1. Substitute $h_s = F(x_s)$ into (17) and use a numerical root find routine to find x_s. The expression in (16) can then be used to find h_s.

2. Use numerical integration to determine the trajectory associated with the stable eigenvalue of the saddle point equilibrium for the system with constant discount rate $\underline{\delta}$ (Conrad & Clark, 1987; Clark, 1990).

3. Numerically integrate (9) backwards from $t = t_1 \gg 0$ to $t = 0$, where t_1 is chosen such that $\mathrm{e}^{-\chi t} \approx 0$, starting from $\delta(t_1) = \underline{\delta}$ and from $[x(t_1)\ u(t_1)]^{\mathrm{T}}$ that is a point on the trajectory obtained in step 2. In practice, $\delta(t)$ can be obtained directly from (6) rather than via numerical integration.

Figure 3 shows $h(t)$ plotted against $x(t)$ for a series of trajectories using a range of initial values $[x(t_1)\ u(t_1)]^{\mathrm{T}}$ along the constant discount rate trajectory for the parameters in Table 1. The simulations were carried out in MATLAB using the `fzero` routine to find x_s and the ode113 routine for numerically integrating (9). From the Figure, it can be seen that when the initial harvesting rate satisfies $h(0) > F(x(0))$, then $\dot{x}(t)$ is negative and the optimal policy to achieve a sustainable equilibrium is to harvest stock until $\dot{x}(t) = 0$, which occurs when the trajectory crosses the locus $F(x) = h$. After this point, the harvesting rate is sufficiently low to allow the stock to recover and increase towards the equilibrium level, x_s. This means that unlike the constant discount rate case, the optimal policy reduces stock in the initial period where the discount rate is high and then reduces the harvesting rate in the later stages when the discount rate is lower.

The objective function associated with a given trajectory can be expressed in the form

$$\int_0^{\infty} D(t) \left[\frac{h^{(1-\gamma)}}{1-\gamma} - \frac{C}{x^{\alpha}} h(t) \right] dt = \int_0^{t_1} D(t) \left[\frac{h^{(1-\gamma)}}{1-\gamma} - \frac{C}{x^{\alpha}} h(t) \right] dt + \int_{t_1}^{\infty} D(t) \left[\frac{h^{(1-\gamma)}}{1-\gamma} - \frac{C}{x^{\alpha}} h(t) \right] dt \quad (22)$$

where $D(t)$ is given in (7). The first integral on the right hand side is obtained by combining the numerical integration of the system in (9) backwards from $t = t_1$ to $t = 0$, with the integration of

$$\dot{V} = -D(t)\left[\frac{h^{(1-\gamma)}}{1-\gamma} - \frac{C}{x^{\alpha}}h(t)\right] \quad \text{with } V(t_1) = 0 \tag{23}$$

where the negative sign is included to allow for the integration backwards in time. The second integral is obtained by integrating forwards along the trajectory associated with the constant discount rate $\underline{\delta}$, from t_1 to $t_2 \gg t_1$, where t_2 is chosen to be sufficiently large such that $D(t_2) \approx 0$.

[Figure 3 about here.]

3.2 Collapse Solution

As Clark (1990) points out, there appears to be no general results for determining the optimal approach to extinction, where $[x(t)\ h(t)]^{\mathrm{T}}$ approaches $[0\ 0]^{\mathrm{T}}$ in the limit $t \to \infty$. However, for the system considered here, when the stock level is below the minimum viable stock, so that $x(t) < K_0$, then from (12), $F(x) < 0$ and $\dot{x}(t) < 0$, so from (4) the stock will become extinct, although because $F(x) \to 0$ as $x \to 0$, extinction will occur in the limit $t \to \infty$. The difficulty is to determine the optimal harvesting as $x(t) \to \infty$. Using the expressions for $U(h)$ and $c(x)$ in (13) and (14), the given that $F(x)$ and $F'(x)/x$ are finite for finite x, the expression for $\dot{h}(t)$ in (15) can become infinite as $x(t) \to 0$. However, along any optimal trajectory that converges to the collapse solution, we require that $h(t) \to 0$ as $t \to \infty$, which will occur when $\lim_{t\to\infty} \dot{h}(t) \leq 0$. Given that $\lim_{t\to\infty} \delta(t) = \underline{\delta}$ and that

$$\lim_{x\to 0} F'(x) = \lim_{x\to 0} \frac{F(x)}{x} = -r \tag{24}$$

this condition will be met when

$$\lim_{t\to\infty} \frac{h^{\gamma}(t)}{x^{\alpha}(t)} \leq \frac{1}{C}\frac{r + \underline{\delta}}{(1-\alpha)r + \underline{\delta}} \tag{25}$$

Provided that this condition holds in the limit $h(t) \to 0$, then the cost of harvesting satisfies

$$c(x)h = h^{(1-\gamma)}C\frac{h^{\gamma}}{x^{\alpha}} \leq h^{(1-\gamma)}\frac{1}{C}\frac{r + \underline{\delta}}{(1-\alpha)r + \underline{\delta}} \tag{26}$$

which remains finite since it is assumed that $\gamma \leq 1$ and $\alpha \leq 1$.

[Figure 4 about here.]

The optimal trajectory will have the largest allowable value of h, which will occur when equality is achieved in (25) as both h and x approach zero. This can be used to find the optimal trajectory for a constant discount rate $\underline{\delta}$, by numerically integrating

$$\frac{dh}{dx} = \frac{h}{\gamma}\frac{1}{F(x)-\underline{\delta}}\left\{[F'(x)-\underline{\delta}]\left[1-C\frac{h^{\gamma}}{x^{\alpha}}\right]+\gamma C\frac{h^{\gamma}}{x^{\alpha}}\frac{F(x)}{x}\right\} \tag{27}$$

from $x = \varepsilon$ with ε close to zero, starting from a value of $h(\varepsilon)$ that achieves equality in (25). Once the trajectory for the constant discount rate has been found, the trajectories for the time-varying discount rate in (6) and the corresponding costs can be obtained using the same approach as in the sustainable case, where (9) is integrated backwards from $t = t_1 \gg 0$ to $t = 0$, starting from a point on the constant discount rate trajectory obtained from (27). Figure 4 shows the optimal trajectories for a range of initial stock levels, using the parameters in Table 1.

[Figure 5 about here.]

[Figure 6 about here.]

4 Discussion

Figure 5 shows the value of the objective function associated with the optimal sustainable and collapse trajectories plotted against initial stock level, $x(0)$. The two curves intersect at a critical initial stock level, x_c, and for any $x(0) < x_c$, the collapse solution is optimal, while for $x(0) > x_c$, the sustainable solution is optimal (Clark, 1990). As can be seen in Figure 3, for some initial stock levels that satisfy $x(0) > x_c$, the optimal sustainable trajectory takes the stock level below the initial critical stock level, x_c, before reducing the harvesting rate to allow the stock level to recover and converge to the sustainable equilibrium. This is shown in Figure 6, which plots the time evolution of a sustainable trajectory that starts with $x(0) > x_c$ and for a period of time, the stock level will be below x_c. This is acceptable provided that the policy is followed through to $t \to \infty$. However, if the harvesting policy is stopped during this period and then re-optimized, the optimal policy is to follow the collapse solution, illustrating the potential danger associated with periodically restarting an optimal policy. Even if the trajectory does not cross the critical stock level x_c, if the policy is restarted at time t_r where $x(t_r) < x(0)$, then the optimization is started from a lower initial stock level. Repeatedly restarting the optimization will drive the initial stock level down to the point where $x(0) < x_c$, making the collapse solution optimal. If the time between restarts becomes small, then the stock trajectory follows the "naive" policy shown by the dotted line in Figure 3, which drives down the stock level until it falls below x_c. Note that continually restarting the policy does not produce the same trajectory as discounting with a constant discount rate at level $\overline{\delta}$, because the (different) anticipated future harvesting trajectories change the optimal harvest rates at the present decision point.

5 Conclusion

This paper has examined optimal harvesting policies for fish stocks when hyperbolic discounting described by an exponentially decaying discount rate is applied to the utility of the harvesting. There are two possible solutions to the optimization problem; one solution converges to a sustainable equilibrium, while the other converges to the collapse solution where the stock is reduced to zero. Which of the two solutions is optimal depends upon whether the initial stock level $x(0)$ exceeds an initial critical stock level x_c. When $x(0)$ is greater than x_c, the optimal harvesting policy converges to the sustainable equilibrium, but when $x(0)$ is less than x_c, the optimal policy collapses the stock. With time-declining discount rates, the optimal policy leading to a sustainable solution may initially reduce the stock below the initial stock level during the period when the discount rate is high, which is then compensated by a low harvesting rate during the later stages when the discount rate is low. The problem is that a declining discount rate leads to time inconsistent policies, so that at some point in the future, it may appear that it is beneficial to stop the policy and recalculate the optimal harvesting rate, starting from the current stock level. If the policy is repeatedly restarted at times where the stock is below the initial stock, then this can result in the stock level being driven down to the point where the policy is restarted with an initial stock $x(0) < x_c$ and in this case, the optimal policy would be to drive the stock level to zero. The key conclusion is that care is required if declining utility discount rates, reflecting apparent human behaviour, are proposed to be used in renewable resource management.[6] While proposals to use hyperbolic discounting in this way might be motivated by a desire to preserve renewable resources for future generations, this paper has made clear that perverse results are possible without putting safeguards in place. These might include placing a side-constraint on the objective function to rule out paths which risk resource collapse. Alternatively, as noted in this paper, to prevent the stock level being inadvertently reduced by repeated re-optimizations, a commitment mechanism might be found to ensure that the optimal policy is followed at least to the point where the stock exceeds the initial level before any changes to the policy are considered.

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[6]Note the distinction between hyperbolic utility discounting and the application of declining *consumption* discount rates which reflect inherent economic uncertainty, as employed by H M Treasury (2003).

A Derivation of Optimal Harvesting Policies

For the discount factor in (3)

$$\frac{dD}{dt} = -\delta(t)D(t) \tag{A.1}$$

so that the state evolution in (2) can be written as

$$\frac{dx}{dD} = \frac{dx}{dt}\frac{dt}{dD} = -\frac{1}{\delta D}\left[F(x) - h\right] \tag{A.2}$$

where $D \in [1, 0]$ and the initial condition is $x(1) = x_0$. Using (A.1), the optimization in (1) becomes

$$\max_{h(t)}\left\{\int_1^0 -\frac{1}{\delta D}D\left[U(h) - c(x)h\right]dD\right\} = \min_{h(t)}\left\{\int_0^1 -\frac{1}{\delta}\left[U(h) - c(x)h\right]dD\right\} \tag{A.3}$$

and the aim is to minimise this objective function subject to the state evolution

$$\frac{dx}{dD} = f(x, h, D) \tag{A.4}$$

where

$$f(x, h, D) = -\frac{1}{\delta D}\left[F(x) - h\right] \tag{A.5}$$

with $x(1) = x_0$, together with $h(D) \geq 0$ and $x(D) \geq 0$.

The Hamiltonian associated with the minimization problem is

$$H(x, h, D) = L(h, D) + \lambda(D)f(x, h, D) \tag{A.6}$$

where

$$L(h, D) = -\frac{1}{\delta}\left[U(h) - c(x)h\right] \tag{A.7}$$

and $\lambda(D)$ is the costate. The co-state satisfies

$$\frac{d\lambda}{dD} = -\frac{\partial L}{\partial x} - \lambda(D)\frac{\partial f}{\partial x} \tag{A.8}$$

$$= -\frac{1}{\delta}c'(x)h + \lambda(D)\frac{1}{\delta D}F'(x) \tag{A.9}$$

with $\lambda(0) = 0$, since there is no terminal cost at $D = 0$. The optimal $h(D)$ satisfies

$$\frac{\partial L}{\partial h} + \lambda(D)\frac{\partial f}{\partial h} = 0 \tag{A.10}$$

which reduces to

$$-\frac{1}{\delta}\left[U'(h) - c(x)\right] + \lambda(D)\frac{1}{\delta D} = 0 \tag{A.11}$$

which can be rearranged as

$$\lambda(D) = D\left[U'(h) - c(x)\right] \tag{A.12}$$

Instead of expressing the problem in terms of (x, λ, D), it is more convenient to consider the system in the space defined by (x, h, D). Differentiating (A.12) gives

$$\frac{d\lambda}{dD} = U'(h) - c(x) + D\,U''(h)\frac{dh}{dD} - Dc'(x)\frac{dx}{dD} \tag{A.13}$$

and after equating to (A.9) and simplifying, the system can be described by two coupled differential equations

$$\frac{dx}{dD} = -\frac{1}{\delta D}\left[F(x) - h\right] \tag{A.14}$$

$$\frac{dh}{dD} = -\frac{1}{\delta D}\frac{1}{U''(h)}\left\{\left[\delta - F'(x)\right]\left[U'(h) - c(x)\right] + c'(x)F(x)\right\} \tag{A.15}$$

with $x(1) = x_0$. Although the terminal condition for (A.9) is $\lambda(0) = 0$, from (A.12), the value of $h(D)$ is undefined when $D = 0$. However, as will be discussed below, we are interested in steady state solutions, which restricts the possible values of $\lim_{D\to 0} h(D)$. Transforming these equations back to the (h, x, t) domain gives the system described by (4) and (5).

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Table

Parameter	Value
K_0	1.0
K	3.0
r	0.06
γ	0.7
$\underline{\delta}$	0.025
$\overline{\delta}$	0.15
χ	0.1
C	1.0
α	0.5

Table 1: Values of parameters used in simulations

Figure Captions

Figure 1. Phase plane trajectories of optimal sustainable and collapse solutions (solid) for a natural growth rate with critical depensation that follows the curve shown by the dashed line and the discount factor is constant. The sustainable trajectory converges to the optimal sustainable yield (*) while the collapse solution converges to zero stock.

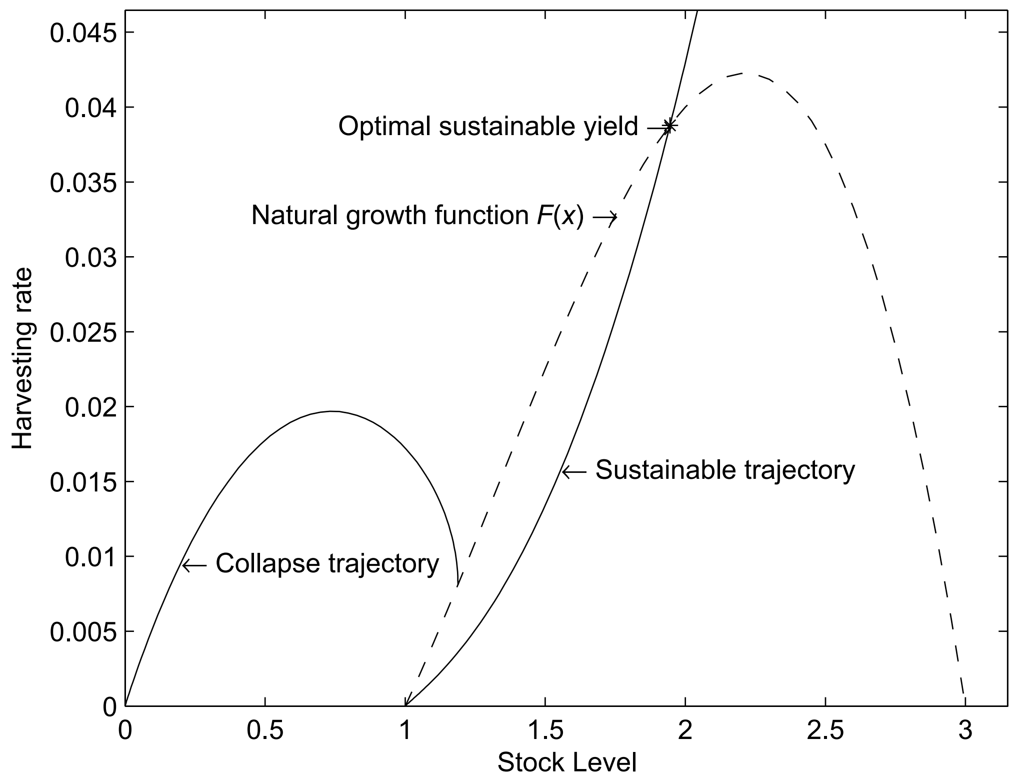
Figure 2. Plot of discount factor $D(t)$ (solid) for exponentially decaying discount rate in (6) with parameters in Table 1. The plot also shows the discount factor for the constant discount rates $\overline{\delta} = 0.15$ (dashed) and $\underline{\delta} = 0.025$ (dash-dotted).

Figure 3. Plot of trajectories (solid lines) of $\{x(t), h(t)\}$ that converge to the sustainable equilibrium, where each trajectory starts from a different initial stock level, $x(0)$, marked by a circle. The trajectories are obtained by numerically integrating (9). The dashed curve shows the locus of the natural growth function $F(x) = h$ and the location of the sustainable equilibrium, (x_s, h_s), is marked by an asterisk. The trajectory of the naive policy is shown by the dotted curve.

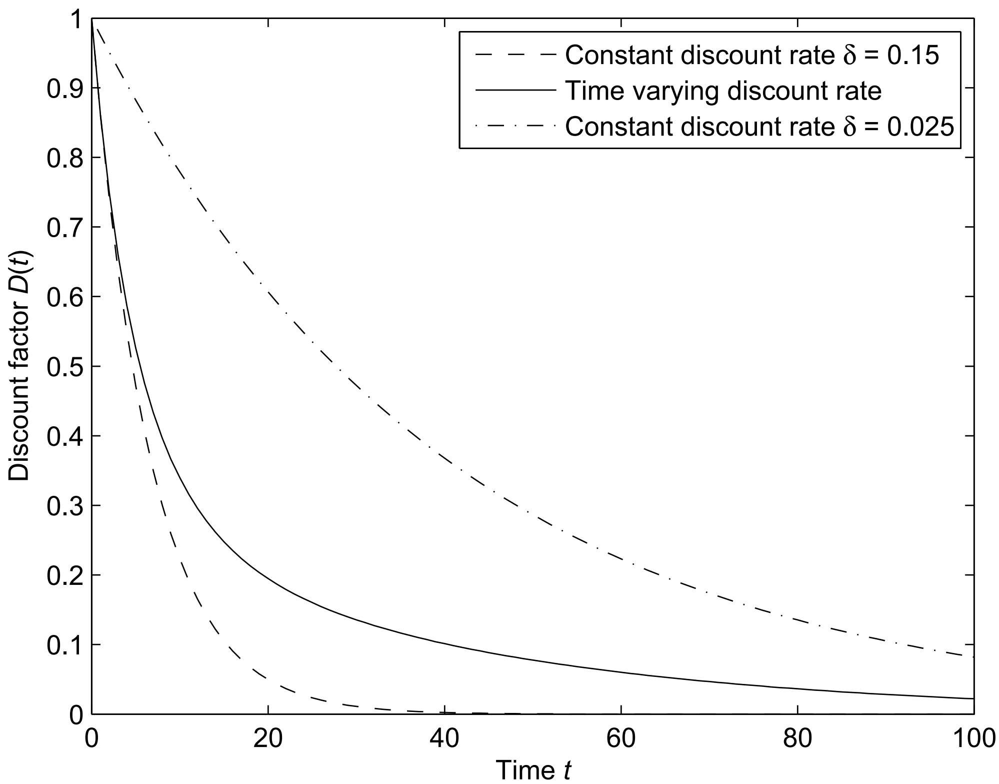
Figure 4. Plot of trajectories (solid lines) of $\{x(t), h(t)\}$ that converge to the collapse solution where each trajectory starts from a different initial stock level, $x(0)$, marked by a circle. The trajectories are obtained by numerically integrating (9).

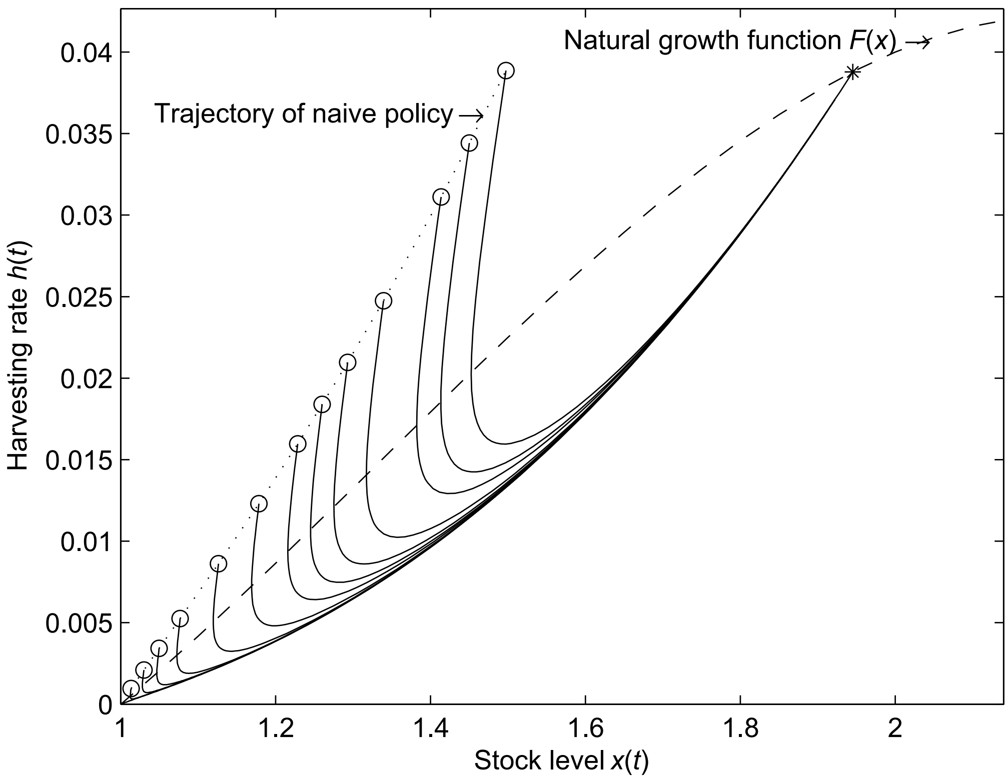
Figure 5. Optimal objective cost function for both sustainable ($\square$) and collapse ($\circ$) solutions, plotted against initial stock level, $x(0)$. The critical initial stock level, x_c, is shown by the vertical dashed line.

Figure 6. Plot of a single optimal sustainable trajectory against time for an initial stock level that satisfies $x(0) > x_c$. The critical initial stock level x_c is shown by the dashed line.

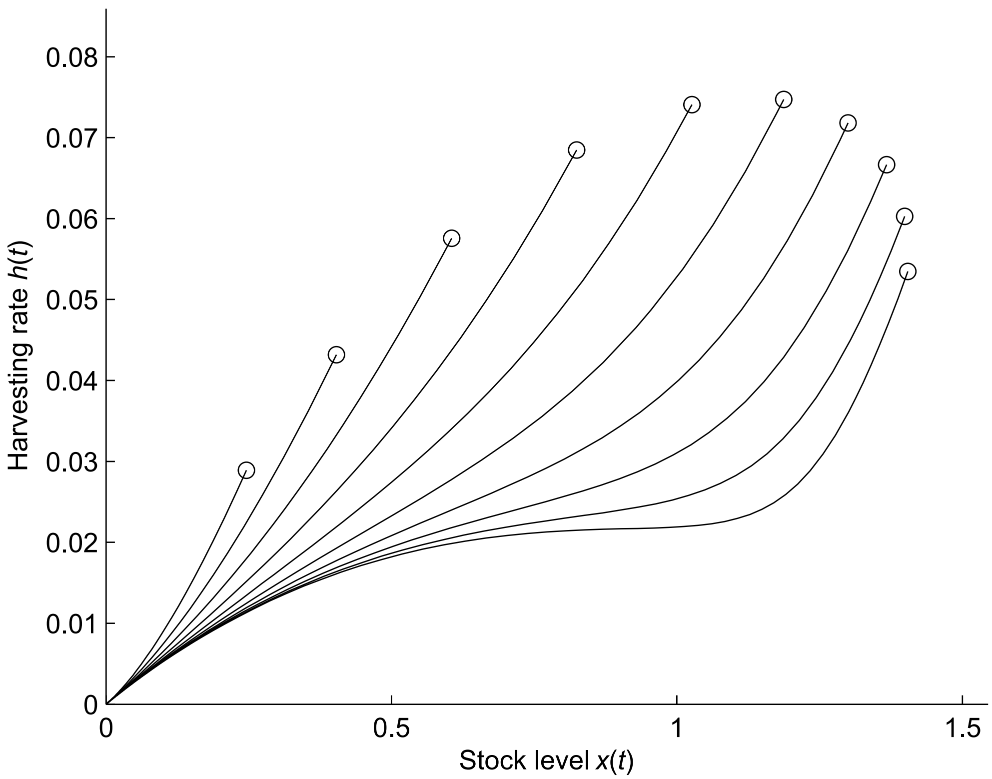


0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Harvesting rate
0
0.5
1
1.5
2
2.5
3
Stock Level
Optimal sustainable yield
Natural growth function $F(x)$
Sustainable trajectory
Collapse trajectory

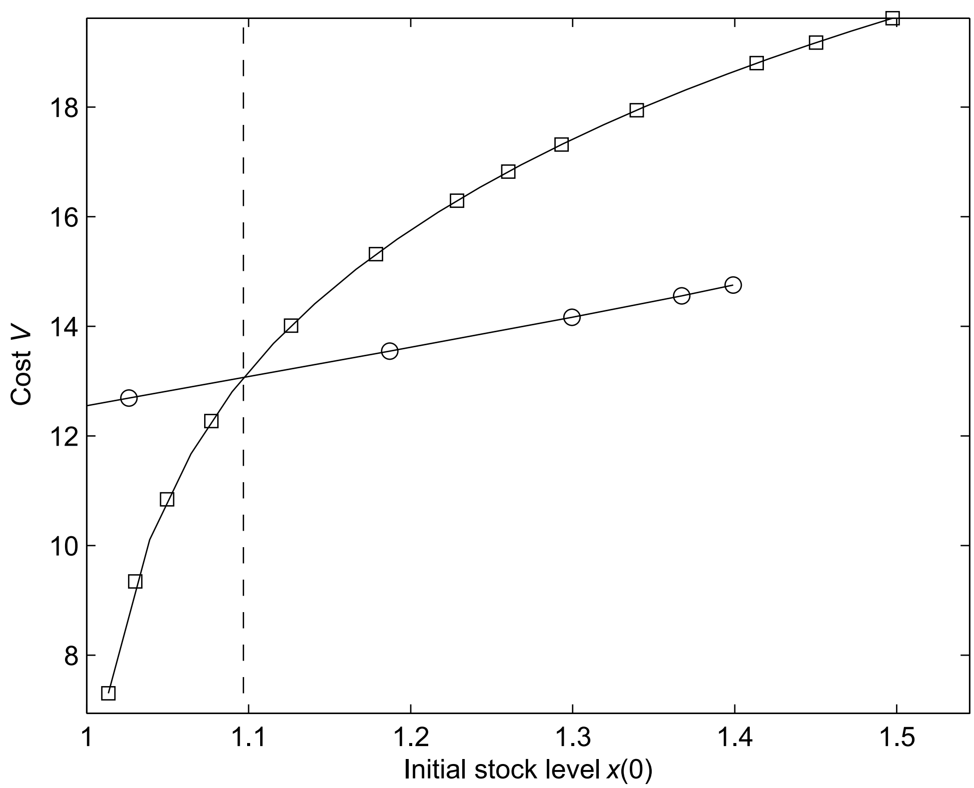

Constant discount rate δ = 0.15
Time varying discount rate
Constant discount rate δ = 0.025
Discount factor D(t)
Time t
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100



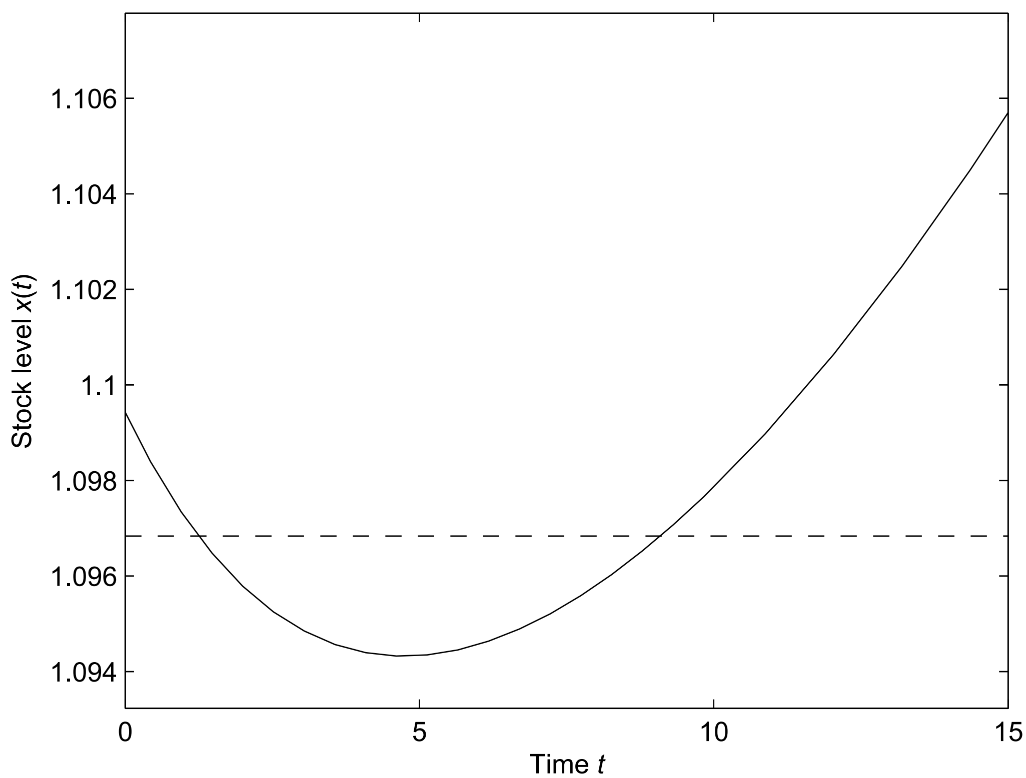
Natural growth function $F(x)$ →
Trajectory of naive policy →
Harvesting rate $h(t)$
Stock level $x(t)$
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
1
1.2
1.4
1.6
1.8
2



0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Harvesting rate $h(t)$
0
0.5
1
1.5
Stock level $x(t)$



18
16
14
12
10
8
Cost V
1
1.1
1.2
1.3
1.4
1.5
Initial stock level x(0)



1.106
1.104
1.102
1.1
1.098
1.096
1.094
Stock level x(t)
0
5
10
15
Time t