Safe Incremental Type Checking
Matthias Puech, Yann Régis-Gianas

To cite this version:
Matthias Puech, Yann Régis-Gianas. Safe Incremental Type Checking. TLDI 2012 - Seventh ACM SIGPLAN Workshop on Types in Language Design and Implementation, Jan 2012, Philadelphia, United States. <hal-00650341>

HAL Id: hal-00650341
https://hal.archives-ouvertes.fr/hal-00650341
Submitted on 9 Dec 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Safe Incremental Type Checking

Matthias Puech
Department of Comp. Sci., Univ. of Bologna,
PPS, Team πr² (Univ. Paris Diderot, CNRS, INRIA)
puech@cs.unibo.it

Yann Régis-Gianas
PPS, Team πr² (Univ. Paris Diderot, CNRS, INRIA)
yr@pps.jussieu.fr

Abstract
We study the problem of verifying the well-typing of terms, not in a batch fashion, as it is usually the case for typed languages, but incrementally, that is by sequentially modifying a term, and re-verifying each time only a smaller amount of information than the whole term, still ensuring that it is well-typed.

2. Sharing-based incrementality
As a first example, let us consider a purportedly simplistic sorted language of boolean and arithmetic expressions:
\[ e, e' ::= n \mid e + e' \mid e \wedge e' \mid e \leq e' \]
The algorithm to determine in a batch fashion whether the term \( e_1 = (1 + 3 \leq 2 + 4) \land (8 \leq 3) \)
is well-sorted is trivial (we don’t care about its evaluation here, just its well-sortedness). But what if I then change subterm \( 2 + 4 \) in \( e_1 \) into \( 7 \leq 2 + 4 \), to obtain \( e_2 \)? Clearly, it should be verified that context \( 7 \leq [] \) is well-sorted (it is), that \( 2 + 4 \) “fits” into its hole (it does), that the whole expression “fits” into its new context \( (1 + 3 \leq []) \land (8 \leq 3) \) (it does not); but the other, unchanged subterms need not be verified again. To achieve this incremental verification, the system would have to “remember” the states of the verifier in some way.

1. Introduction
As programs grow and type systems become more involved, writing a correct program in one shot becomes quite difficult. On the other hand, writing a program in many correct programs is the usual practice when the time for verification is negligible; the verification tool then rechecks the entire development at each step. But this gets more tedious especially when the language in question contains proof aspects, and verification involves proof search. Some mechanisms already exist to cope with the incrementality of proofs or program development: separate compilation, interactive toplevel with undo, tactic languages; they all provide in different ways a rough approximation of the process of modifying and checking incrementally a large term.

We propose here an architecture for a generic and safe incremental type checker, a data structure for repositories of typed proofs and a language for describing proof deltas. It is based on the simple idea of sharing common subterms to avoid rechecking, and exploits encoding a derivation in a metalanguage to guarantee the well-typing of the result. This way, given a signature declaring the typing rules and an (untrusted) typing algorithm for my language of choice, I get an incremental type checker for that language. The metalanguage approach gives us the ability to encode all the aforementioned usual incrementality mechanisms in a type-safe way, and more, making our system akin to a typed version control system.

Categories and Subject Descriptors D.3.3 [Language Constructs and Features]: Data types and structures; F.3.1 [Logics and Meanings of Programs]: Specifying and Verifying and Reasoning about Programs — Logics of programs

General Terms Theory, Languages

Keywords incrementality, type checking, logical framework, version control

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.
TLDI ’12 January 28 2012, Philadelphia, USA.
Copyright © 2012 ACM [to be supplied] . . $10.00
and check terms against this signature with a generic algorithm. We’ll use an increasing fragment of it. The so-called intrinsic style of LF signature for our expression language is:

\[
\begin{align*}
\text{tp} & : * , \\
\text{nat} & : \text{tp} , \\
\text{bool} & : \text{tp} , \\
\text{exp} & : \text{tp} \to \text{nat} , \\
\text{atom} & : \mathbb{N} \to \text{exp nat} , \\
\text{plus} & : \text{exp nat} \to \text{exp nat} \to \text{exp nat} , \\
\text{leq} & : \text{exp nat} \to \text{exp nat} \to \text{exp bool} ,
\end{align*}
\]

In this style, both the encoding of an expression and its sort are terms in the metalanguage, but the sort appears in the type of the encoded expression. As an example, the repository associated with expression e₁ is:

\[
T₁ = \begin{pmatrix}
X & \text{plus} & 1 & 3 & \text{exp nat} \\
Y & \text{plus} & 2 & 4 & \text{exp nat} \\
Z & \text{leq} & 3 & 3 & \text{exp bool} \\
T & \text{and} & (\text{leq} X Y) & Z & \text{exp bool}
\end{pmatrix}
\]

The dependent nature of types in LF allows to express more complex languages. We can for example add functions, applications and variables to our expressions in a purely first-order style (using de Bruijn indices for variables) if we annotate them not only with sorts but with an environment of free variables:

\[
\begin{align*}
\text{exp} & : \text{env} \to \text{tp} \to * , \\
\text{atom} & : \Pi_1 \text{ env} \times \mathbb{N} \to \text{exp nat} , \\
\text{var} & : \Pi_1 \text{ env} \times \text{IA} \times \text{tp} \to \text{var} E A \to \text{exp E A} , \\
\text{leq} & : \Pi_1 \text{ env} \times \text{E nat} \times \text{E nat} \to \text{exp bool} , \\
\text{lam} & : \Pi_1 \text{ env} \times \text{IA} \times \text{B} \times \text{tp} \times \text{exp (cons A E)} \times \text{B} \\
& \to \text{exp E (arr A B)}
\end{align*}
\]

The encoded expressions are however very verbose: each term constructor takes as argument all these annotations. We can nonetheless make these information implicit in terms (but explicit in types) and let a reconstruction algorithm infer them, as in [3]. This reconstruction is language-dependent, user-provided but does not impair the safety of the system for the whole term is still checked afterwards.

LF promotes the use of lambda-tree syntax to represent binders: instead of encoding the syntax first-order, it uses the λ binder built in LF to encode binders in the object language. This style of encoding has the advantage of making the manipulation of the environment (weakening, exchange...) implicit in the deltas, but raises new challenges for the delta language and the verification process: how to share a subterm underneath a lambda? How to efficiently verify that such a delta is well-typed?

4. Expressivity

Aside from enabling to encode a large class of deductive systems safely and generically, the metalanguage approach allows to express incrementality features usually implemented in an ad-hoc manner, simply by adding new constants to the signature.

Version control Suppose we want to implement an undo system, storing successive versions of a closed expression of sort bool and able to rollback to a previous version. We add constants

version : *, 
vnil : version,

vcons : exp nil bool \to version \to version

to the signature. The empty repository is now represented as vnill. Each time we have pushed a full expression $M$, and if $S$ was the previous head (a version called its ancestor), we push $\text{vcons} M S$. This gives us a data structure for an undo stack, and a commit algorithm. But the sharing inherent to our repositories lets us actually represent trees of versions, by sharing common stack tails, each list head being a branch. Reconciling two branches’ changes into a unique head is called merging in version control system’s terminology: a merge is a version with several ancestors. We can represent merges by revising our previous addition to the signature into

version : *, ancestors : *, anil : ancestors, 

vcons : version \to ancestors \to ancestors,

vcons : exp nil bool \to ancestors \to version

This defines a data structure to represent (acyclic) graphs of versions; it is the exact data structure of repository used by version control systems Git, Monotone and Mercurial (see e.g. [1]) except that where they have directories and text files we have arbitrary typed terms.

Top-down construction While our system is based on bottom-up term construction, we can encode top-down construction common to some programming environments (e.g. Agda) and tactic-based proof assistants (e.g. Coq). The user constructs terms by successively filling holes with terms containing other holes. To add (linear) holes to our expressions, add constant

hole : \Pi_1 \text{ env} \times \Pi_1 \text{ IA} \times \text{tp} \times \text{exp E A} 

to the signature. To instantiate a hole with an expression, commit the substituted term preserving sharing of subterms.

5. Architecture

We can implement this system following a layered architecture. The kernel is the component in charge of verifying terms against a signature and a repository, and updating this repository. It supports two basic operations:

- push: \( R(M) \) checks a small part $M$ of a larger term against $\Sigma$ in $R$, synthesizes its type $\lambda$, chooses a fresh metavariable $X$ for $M$ and returns $R[X \mapsto M : \lambda]$ and $X$.
- pull: $R(X)$ returns the term $M$ associated with $X$ in $R$ recursively; all metavariables are unfolded to their definitions.

The slicer is the component in charge of slicing a term $M$ into many terms, pushing them to the repository to enable future sharing, and adding version markers. It supports operations:

- commit(A(M), R(M)) pushes $\text{vcons} M (\text{acons} X \text{ anil})$ to $R$ in several push operations, where $X$ is the current head.
- merge(A(M), X) pushes $\text{vcons} M (\text{acons} X (\text{acons} Y \text{ anil}))$ to $R$. Note that it doesn’t actually perform the merge, it simply commits a previously computed merge node with value $M$.

Finally, the compressor computes a delta $\delta$ from a metavariable-free expression $\epsilon$ by recognizing equal subterms in $R$. This can be achieved by hash-consing.

References

