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Safe Incremental Type Checking

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Abstract
We study the problem of verifying the well-typing of terms, not
in a batch fashion, as it is usually the case for typed languages,
but incrementally, that is by sequentially modifying a term, and
reverifying each time only a smaller amount of information than
the whole term, still ensuring that it is well-typed.

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1. Introduction
As programs grow and type systems become more involved, writing
a correct program in one shot becomes quite difficult. On the
other hand, writing a program in many correct steps is the usual
practice when the time for verification is negligible; the verifica-
tion tool then rechecks the entire development at each step. But this
gets more tedious especially when the language in question con-
tains proof aspects, and verification involves proof search. Some
mechanisms already exist to cope with the incrementality of proofs
or program development: separate compilation, interactive toplevel
with undo, tactic languages; they all provide in different ways a
rough approximation of the process of modifying and checking in-
crementally a large term.

We propose here an architecture for a generic and safe in-
cremental type checker, a data structure for repositories of typed
proofs and a language for describing proof deltas. It is based on
the simple idea of sharing common subterms to avoid rechecking,
and exploits encoding a derivation in a metalanguage to guarantee
the well-typing of the result. This way, given a signature declaring
the typing rules and an (untrusted) typing algorithm for my lan-
guage of choice, I get an incremental type checker for that lan-
guage. The metalanguage approach gives us the ability to encode
all the aforementioned usual incrementality mechanisms in a type-
safe way, and more, making our system akin to a typed version
control system.

2. Sharing-based incrementality
As a first example, let us consider a purposefully simplistic sorted
language of boolean and arithmetic expressions:

\[ e, e' ::= n \mid e + e' \mid e \land e' \mid e \leq e' \]

The algorithm to determine in a batch fashion whether the term

\[ e_1 = (1 + 3 \leq 2 + 4) \land (8 \leq 3) \]

is well-sorted is trivial (we don’t care about its evaluation here,
just its well-sortedness). But what if I then change subterm \(2 + 4\)
in \(e_1\) into \(7 \leq 2 + 4\), to obtain \(e_2\)? Clearly, it should be verified
that context \(7 \leq \) is well-sorted (it is), that \(2 + 4\) “fits” into its
hole (it does), that the whole expression “fits” into its new context
\((1 + 3 \leq \) \& \((8 \leq 3)\) (it does not); but the other, unchanged
subterms need not be verified again. To achieve this incremental
verification, the system would have to “remember” the states of
the verifier in some way.

If only we had names (memory addresses, hashes) for enough
subterms of our initial term,

\[ e_1 = (\text{X} \leq \text{Y}) \land \text{Z} \],

we could express concisely the change as a delta

\[ \delta_1 = (\text{X} \leq \text{Y}) \land \text{Z} \],

using these names to refer to unchanged subterms. If only we had
annotated our initial term with the states of the verifier,

\[ e_1 = (\text{X} \leq \text{Y}) \land \text{Z} \],

we would have a simple process to verify \(e_2\) taking advantage
of \(e_1\)’s derivation, in \(O(|\delta_1|)\): verify \(\delta_1\) as a term, retrieving
the sort of names from a stored map. This suggests a data structure
for a repository of names, annotated, verified subexpressions: a
monotonously growing map, from names, or metavariables, to
terms and types, together with a head metavariable identifying the
top of the expression:

\[ R := X, \Delta \quad \text{where} \quad \Delta := (X \mapsto M : A) \]

3. A metalanguage to encode derivations
What language are terms \(M\) and types \(A\) written into? Terms
should encode our expressions with metavariables, and types \(A\)
should encode the whole state of the batch verifier (here the sort).
An obvious choice is to take \(M := (e \text{ with metavariables})\) and
\(A ::= B \mid \mathbb{N}\), but switching to another language, we’d have to
redefine another repository language. Moreover, this choice is no
longer expressive enough when introducing binders. Another more
modular choice for this is the metalanguage \(\text{LF}\) [2]: it allows to
specify syntax and rules of an object language as a signature \(\Sigma\).
and check terms against this signature with a generic algorithm. We’ll use an increasing fragment of it. The so-called intrinsic style of LF signature for our expression language is:

- \( \text{tp} : *, \text{nat} : \text{tp}, \text{bool} : \text{tp}, \exp : \text{tp} \rightarrow \text{nat} \)
- \( \text{atom} : \mathbb{N} \rightarrow \exp \text{nat} \)
- \( \text{plus} : \exp \text{nat} \rightarrow \exp \text{nat} \rightarrow \exp \text{nat} \)
- \( \text{leq} : \exp \text{nat} \rightarrow \exp \text{nat} \rightarrow \exp \text{bool} \)

In this style, both the encoding of an expression and its sort are terms in the metalanguage, but the sort appears in the type of the encoded expression. As an example, the repository associated with expression \( e_1 \) is:

\[
T_1 = \left( \begin{array}{l}
X \rightarrow \text{plus} 1 3 : \exp \text{nat} \\
Y \rightarrow \text{plus} 2 4 : \exp \text{nat} \\
Z \rightarrow \text{leq} 8 3 : \exp \text{bool} \\
T \rightarrow \text{and} (\text{leq} X Y) Z : \exp \text{bool}
\end{array} \right)
\]

The dependent nature of types in LF allows to express more complex languages. We can for example add functions, applications and variables to our expressions in a purely first-order style (using de Bruijn indices for variables). If we annotate them not only with sorts but with an environment of free variables:

- \( \exp : \text{env} \rightarrow \text{tp} \rightarrow * \)
- \( \text{atom} : \text{II} : \text{env} \rightarrow \exp \text{nat} \)
- \( \text{var} : \text{II} : \text{env} \rightarrow \text{II} : \text{tp} \rightarrow \exp \text{E} \rightarrow \exp \text{E} \)
- \( \text{leq} : \text{II} : \text{exp} \rightarrow \text{nat} \rightarrow \text{exp} \rightarrow \text{E} \rightarrow \exp \text{bool} \)
- \( \text{lam} : \text{II} : \text{env} \rightarrow \text{II} : \text{tp} \rightarrow \exp \rightarrow \exp (\text{cons} \text{E} \text{E}) \rightarrow \exp \rightarrow \exp (\text{arr} \text{E} \text{E}) \)

The encoded expressions are however very verbose: each term constructor takes as arguments all these annotations. We can nonetheless make these information implicit in terms (but explicit in types) and let a reconstruction algorithm infer them, as in [3]. This reconstruction is language-dependent, user-provided but does not impair the safety of the system for the whole term is still checked afterwards.

LF promotes the use of lambda-tree syntax to represent binders: instead of encoding the syntax first-order, it uses the \( \lambda \) binder built in LF to encode binders in the object language. This style of encoding has the advantage of making the manipulation of the environment (weakening, exchange...) implicit in the deltas, but raises new challenges for the delta language and the verification process: how to share a subterm underneath a lamba? How to efficiently verify that such a delta is well-typed?

4. Expressivity

Aside from enabling to encode a large class of deductive systems safely and generically, the metalanguage approach allows to express incrementality features usually implemented in an ad-hoc manner, simply by adding new constants to the signature.

Version control

Suppose we want to implement an undo system, storing successive versions of a closed expression of sort bool and able to rollback to a previous version. We add constants

- \( \text{version} : *, \text{vnill} : \text{version} \)
- \( \text{vcons} : \exp \text{nil bool} \rightarrow \text{version} \rightarrow \text{version} \)

to the signature. The empty repository is now represented as \( \text{vnill} \). Each time we have pushed a full expression \( M \), and if \( S \) was the previous head (a version called its ancestor), we push \( \text{vcons} M S \). This gives us a data structure for an undo stack, and a commit algorithm. But the sharing inherent to our repositories lets us actually represent trees of versions, by sharing common stack tails, each list head being a branch. Reconciling two branches’ changes into a unique head is called merging in version control system’s terminology: a merge is a version with several ancestors. We can represent merges by revising our previous addition to the signature into

- \( \text{version} : *, \text{ancestors} : *, \text{anil} : \text{ancestors} \)
- \( \text{acons} : \text{version} \rightarrow \text{ancestors} \rightarrow \text{ancestors} \)
- \( \text{vcons} : \exp \text{nil bool} \rightarrow \text{ancestors} \rightarrow \text{version} \)

This defines a data structure to represent (acyclic) graphs of versions; it is the exact data structure of repository used by version control systems Git, Monotone and Mercurial (see e.g. [1]) except that where they have directories and text files we have arbitrary typed terms.

Top-down construction

While our system is based on bottom-up term construction, we can encode top-down construction common to some programming environments (e.g. Agda) and tactic-based proof assistants (e.g. Coq). The user constructs terms by successively filling holes with terms containing other holes. To add (linear) holes to our expressions, add constant

- \( \text{hole} : \text{IIE} : \text{env} \rightarrow \text{II} : \text{tp} \rightarrow \exp \text{E} \)

to the signature. To instantiate a hole with an expression, commit the substituted term preserving sharing of subterms.

5. Architecture

We can implement this system following a layered architecture.

The kernel is the component in charge of verifying terms against a signature and a repository, and updating this repository. It supports two basic operations:

- \( \text{push}_{\text{h}}(\text{R}, M) \) checks a small part \( M \) of a larger term against \( \Sigma \) in \( \text{R} \), synthesizes its type \( A \), chooses a fresh metavariable \( X \) for \( M \) and returns \( \text{R}[X \rightarrow M : A] \) and \( X \).

- \( \text{pull}_{\text{h}}(\text{R}, X) \) returns the term \( M \) associated with \( X \) in \( \text{R} \) recursively; all metavariables are unfolded to their definitions.

The slicer is the component in charge of slicing a term \( M \) into many terms, pushing them to the repository to enable future sharing, and adding version markers. It supports operations:

- \( \text{commit}_{\text{h}}(\text{R}, M) \) pushes \( \text{vcons} M (\text{acons} X \text{anil}) \) to \( \text{R} \) in several push() operations, where \( X \) is the current head.

- \( \text{merge}_{\text{h}}(\text{R}, M, Y) \) returns \( \text{vcons} M (\text{acons} X (\text{acons} Y \text{anil})) \) to \( \text{R} \). Note that it doesn’t actually perform the merge, it simply commits a previously computed merge node with value \( M \).

The reconstructor performs the reconstruction of the derivation (an \( M \)) from the initial expression (an \( e \)), given the derivations for its metavariables (an \( \text{R} \)), and the expected type (\( \exp \text{nil bool} \)), and commits \( M \).

Finally, the compressor computes a delta \( e' \) from a metavariable-free expression \( e \) by recognizing equal subterms in \( \text{R} \). This can be achieved by hash-consing.

References

