Intra Cytoplasmic Sperm Injection Simulator using Biomechanical Models
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To cite this version:
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Keywords: Finite element modeling, St Venant-Kirchhoff, Real-time interaction, Microinjection.

1. INTRODUCTION

Nowadays, a key challenge of deformable simulation is to satisfy the conflicting requirements of real-time interactivity and physical realism. Various models have been implemented, the soft tissue modeling are usually derived using either mass-spring approach or the linear finite element method (FEM). The linear elasticity is often used for the modeling of deformable materials, mainly because the equations remain quite simple and the computation time can be optimized. The physical behavior of soft tissue may be considered as linear elastic if its displacement and deformation remain small (typically less than 10 % of the mesh size). The objective of this paper is to develop and implement an interactive simulation techniques to facilitate training of biological cell injection operations. Using this tool, the operator can form, train and improve its control by developing a gesture similar to that performed in reality. The design of such a simulation environment requires a compromise between the realism of biomechanical models used, the accuracy and stability of algorithms and solution methods implemented and the computational speed required for real-time haptic rendering. Modeling Mechanical restraint involves the use of an hyperelastic model (St Venant Kirchhoff) and a specific dynamic finite element code (mass tensor formulation). The different results are compared to experimental data. This comparison shows the effectiveness of the proposed physically based model.

2. METHODS
A. Linear elastic mass-tensor model

In this section, we present the numerical development of the linear elastic mass-tensor model [2-3] and its extension to non linear elasticity based on the St Venant Kirchhoff hyperelastic model [4]

\[
\begin{bmatrix}
\sigma \\
\epsilon \\
\end{bmatrix} = \lambda \left( \begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\epsilon_3 \\
\end{bmatrix} + \frac{2}{3} \epsilon \right) \begin{bmatrix} 1 \end{bmatrix} + 2 \mu \left( \begin{bmatrix}
\epsilon \\
\end{bmatrix} \right) \tag{1}
\]

where \([I]\) is the identity matrix.

Using classical notations, inside each tetrahedron \(T^k\), the displacement field, defined by a linear interpolation \([N^k]\) of the nodal displacement vector \([u^k]\) of the four vertices of tetrahedron, is written as follows:

\[
\begin{bmatrix}
U(x) \\
u^k \\
\end{bmatrix} = \begin{bmatrix} [N^k(x)] \\
u^k \\
\end{bmatrix} \tag{2}
\]

The stiffness matrix is composed of a plurality of elementary submatrices each connecting the elementary force acting on the node, \(i\), to the displacement of the node, \(j\):

\[
K^k_{ij} = \frac{1}{36 V^k} \left( \lambda \left\{ m_i \right\} \left\{ m_j \right\}^T + \mu \left\{ m_i \right\} \left\{ m_j \right\}^T + \mu \left\{ m_i \right\} \left\{ m_j \right\}^T \begin{bmatrix} 1 \end{bmatrix} - 2 \mu \left\{ m_j \right\} \left\{ m_i \right\} \right) \tag{3}
\]

where \([m_i]\) are unit outward-pointing normals to triangular faces and \(V^k\) is the volume of the tetrahedron \(T^k\).

Taking into account the contribution of all adjacent tetrahedra, the global internal force acting on a node \(l\) can be expressed as follows:

\[
\left\{ F^l_{int} \right\} = \sum_{k \in V_l} \left( \sum_{j=1}^{4} \left[ K^k_{lj} \right] \left\{ u_j \right\} \right) \tag{4}
\]

where \(V_l\) is the neighborhood of vertex \(l\) (i.e. the tetrahedra containing node \(l\)).

The tensors \([K^k_{lj}]\), depending on the remaining geometry and Lame’s coefficients are constant. They can be pre-computed in an off-line phase. This is the essential advantage of the mass-tensor approach which makes it useful for real-time application.

B. Hyperelastic model of St Venant-Kirchhoff formulation

The algorithm is built the same way and the basic equation of energy remains the same as linear version. Writing complete strain tensor of Green-Lagrange function directly the displacement field can be written:

\[
E = \frac{1}{2} \left( \nabla U + \nabla U^T + \nabla U^T \cdot \nabla U \right) \tag{5}
\]

For the model of elasticity St Venant-Kirchhoff strain energy is a polynomial degree in four movements which can be rewritten:

\[
W_{NL} = \lambda \left( \nabla U \right)^2 + \mu \left( \nabla U \right)^2 - \frac{\mu}{2} \left( \nabla U \right)^2 + \frac{\lambda}{2} \left( \nabla U \right)^2 + \frac{\mu}{2} \left( \nabla U \right)^2
\]

(6)
the force applied at each vertex inside a tetrahedron is derived from the energy as follow:

\[ \{ F_i^T \} = \sum_{j=1}^{4} [B_{ij}] \{ u_j \} F_i^T + \sum_{j,k=1}^{4} \left( \{ u_k \} \{ u_j \}^T \{ C_{jki} \} + \frac{1}{2} \left( \{ u_j \}^T \{ u_k \} \right) \{ C_{ijk} \} \right) F_i^T + 2 \sum_{j,k,l=0}^{3} D_{ijkl} \{ u_l \} \{ u_k \}^T \{ u_j \} F_i^T \]

\[ F_i^{T} = \frac{F_i^{T}}{\text{Linear force}} + \frac{F_i^{T}}{\text{Nonlinear force}} \]

where \([B_{ij}], \{C_{ijk}\}, \{D_{ijkl}\}\) are respectively the matrix, the vector and the scalar.

To solve the dynamic system, we choose the explicit centered finite-difference scheme.

3. Results and Discussion

Based on the mechanical properties of the mouse oocyte Zona pellucida [1] The mechanical and geometrical properties of the cell are settled in Table 1. These properties are passed to the finite element simulations. The finite element simulations are performed using the developed FEM algorithms. In order to demonstrate the validity of our finite element method in real-time, we compare the simulation needle insertion using the physics-based FEM model versus the incremental data provided by AFM indentation tool.

### Table 1

**Mechanical and Geometrical Properties of Mouse Oocyte [1]**

<table>
<thead>
<tr>
<th>User Interface</th>
<th>Mechanical and Geometrical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young modulus</td>
<td>17.9 KPa</td>
</tr>
<tr>
<td>Poisson coefficient</td>
<td>0.49</td>
</tr>
<tr>
<td>Diameter</td>
<td>9 ( \mu )m</td>
</tr>
<tr>
<td>Cell volume</td>
<td>39,952 ( \mu )m (^3)</td>
</tr>
<tr>
<td>Mesh of the cell</td>
<td>115 vertices 332 Tetra</td>
</tr>
</tbody>
</table>

The simulations using the non linear finite element St-Venant-Kirchoff show quantitatively and qualitatively good agreement with experimental data (Fig.2, Fig.1), you can see clearly, the linear finite element is valid only for small displacement (less than 10 % of the mesh size).

4. Conclusion

The real-time haptics-enabled simulator is realistic since it is mainly based on experimental data (mechanical and geometrical property) to facilitate training of biological cell injection operations. We first investigated the challenging issues in the real-time modeling of the biomechanical properties of the needle insertion through linear and non linear finite element models (st-Venant Kirchooff). Compared to experimental results performed on oocyte cells, we can see clearly that the proposed physically-based Saint-Venant Kirchoff model is able to simulate the cell deformation through real-time simulation constraints. Currently, we are working on integrating different effects such as friction, viscosity and adhesion forces after puncture. All modalities will be merged in an ergonomic tool and intelligent biological simulator for intra cytoplasmic sperm injection.

**References**


