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Evaluation of a Steady-State Test of Foam Stability

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ABSTRACT

We have evaluated a steady-state test of foam stability which is based on the steady-state height of a foam produced by a constant velocity of gas flow. This test is mentioned in the book by Bikerman [1] and an elementary theory was developed for it by Verbist et al. in 1996 [2]. For the study we used an aqueous solution of the cationic surfactant dodecyl trimethylammonium bromide, C_{12}TAB, at a concentration of 2 times the critical micelle concentration (2 cmc). During foam generation bubbles collapse at the top of the column which, in turn, eventually counterbalances the rate of bubble production at the bottom. The resulting balance can be described mathematically by an appropriate solution of the foam drainage equation under specified boundary conditions. Our experimental findings are in agreement with the theoretical predictions of a diverging foam height at a critical gas velocity and a finite foam height in the limit of zero velocity. We identify a critical liquid fraction below which a foam is unstable as an important parameter for characterising foam stability.

Furthermore we deduce an effective viscosity of the liquid which flows through the foam. Currently unexplained are two experimental observations, namely sudden changes of the steady-state foam height in experiments that run over several hours and a reduction in foam height once an overflow of the foam from the containing vessel has occurred.

Keywords: Bikerman foam test, drainage equation, foams, cationic surfactants
1. Introduction

The formation of a foam requires the introduction of a gas into a liquid. The gas can, for example, be blown into the liquid through a nozzle, it can be mixed into the liquid via shaking, or it can be released from a super-saturated solution as a result of a pressure drop. The resulting foam is generally unstable and its volume decays with time. The issues of foamability and foam stability are vital for many industrial applications, ranging from foams in the brewing industry to foams in healthcare products, such as shampoos, and foams in froth flotation plants for the separation of mineral ores.

A variety of (empirical) standard foam tests are in use which specify conventional procedures and deliver a certain value that characterizes the foam in question. In the Rudin test [3], used in the brewing industry, the flow of liquid out of a decaying foam is determined. A specified amount of degassed beer is foamed up in a specified container to a specified height within a specified time. After the gas flow has been switched off, the time is measured for the liquid level at the bottom of the foam to go from a 50 mm mark to a 75 mm mark. In the NIBEM method [4], beer is foamed up in a cuvette using CO$_2$ gas and standardized flow rate and orifice opening, and then the foam/gas interface is monitored as a function of time.

In the Bikerman test [1], which recently attracted renewed interest for the characterisation of foams used in froth flotation [5,6], gas is sparged at a constant flow rate through a foam solution and the foam is collected in a vessel. After an initial increase, the height of the foam column reaches a maximum, whose value depends on the flow-rate $Q$ of the gas. In his experiments, which included the foaming of urine, Bikerman [7] found that within a certain range of flow rates the steady-state height scales linearly with the flow-rate. Thus in this range a quantity $\Sigma$ can be defined as $\Sigma = V_{\text{foam}}/Q = H/V$, where $V_{\text{foam}}$ is the volume of foam produced, $H$ is the height of the foam column, and $V$ is called the “gas superficial velocity” with units of meter per second. Bikerman referred to the parameter $\Sigma$, which he chose as it is the first letter in the Greek word for “lather”, as “the unit of foaminess” [7]. It has the physical dimensions of time.

There are also a number of more archaic foam tests still in use in industry, which are based on agitation of the liquid. They include the simple shaking of a liquid in a closed vessel or the pouring of liquid, followed by the determination of the resulting foam volume [1]. In all the above tests, the three main mechanisms that operate during and after the production of a foam are drainage, coalescence and coarsening (Ostwald ripening). Liquid drains out of the foam due to gravity, foam films burst (bubbles coalesce) e.g. due to evaporation, and coarsening
takes place due to pressure differences between bubbles, which drives gas diffusion between them. The detailed specification of vessel parameters, gas flow rates, time intervals for measurements etc. may also be vital in foam tests, since they crucially determine the outcome of the measurement. While a foam test may provide for comparing empirically and thus qualitatively different samples, the lack of theoretical interpretation obscures the significance of the results, which may be important in tuning foamability for practical purposes. In this paper we examine a steady-state foam test with an analysis based on a physical model of foam drainage. This leads to both an interpretation (by giving a physical meaning to the parameter $\Sigma$) and a generalization of Bikerman’s test.

2. Theoretical Model of a Steady-State Foam Test

The last two decades saw enormous progress in the understanding of the physics of foam drainage. A model in which drainage is restricted to the flow of liquid in the Plateau border network (channels at the intersections of liquid films) and where flow in the films is neglected, has been very successful in interpreting many different types of drainage experiments [2,8]. The corresponding mathematical formulation leads to the so-called foam drainage equation which describes the variation of liquid fraction with time and vertical position (see eqn.(A1) in Appendix A for more details).

The foam test that we are presenting and analysing in this paper dates back to Bikerman [1,7] and was originally modelled by Verbist et al. [2] in their comprehensive review of the mathematics of the channel dominated foam drainage equation ($k = 1$ in eqn.(A1)). It deals with the determination of the steady-state height of a foam, produced by the continuous bubbling of gas at a fixed flow rate into a surfactant solution. The growth of the foam is accompanied by the drainage of liquid due to gravity, and the rupture of films. In the continuum model of Verbist et al. [2] rupture takes place at a critical value of the (local) liquid fraction $\Phi(x)$. Since the liquid fraction decreases with height due to drainage, film rupture is confined to the upper foam surface (foam-atmosphere interface) in this model. The column arrives at a steady-state of constant foam height, in which the rate of bubble production at the bottom equals the rate of bubble disappearance at the top. The distance between bottom (where the bubbles are in contact with the surfactant solution) and top, i.e. the height of the foam column, is a function of bubble/gas velocity. Within the present model the functional relationship can be arrived at as follows.
The bubble velocity \( V \) in the rising column is the same everywhere; the small correction due to the variation of liquid fraction along the column is neglected. (This is justified as most of the variation occurs over a length scale of the order of the capillary length, which is only about 2 mm in our experiments described in section 3, and thus much shorter than the height of the foam column.) A steady-state is defined as having a zero liquid velocity throughout the sample. One may employ the usual form of the drainage equation by using the frame of reference in which the bubbles are stationary, as explained in Appendix A. The steady-state height of the column as a function of bubble velocity is determined by applying appropriate boundary conditions for the values of liquid fraction at both bottom \((x = 0)\) and top \((x = H)\) of the foam column. If the bubbles at the bottom are in contact with a pool of liquid (as in the experiments described below), their packing resembles that of a dense packing of spheres, leading to a liquid fraction \( \Phi \) of

\[
\Phi(x = 0) = \Phi_0 \approx 0.36. \tag{1}
\]

The liquid fraction at the top of the column

\[
\Phi(x = H) = \Phi_H \tag{2}
\]

corresponds to some finite critical liquid fraction \( \Phi_H \) below which the films collapse and whose value needs to be determined experimentally.

Using these two boundary conditions, an analytical expression for the variation of the foam height \( H(V) \) with bubble velocity \( V \) can be obtained (eqn.(A7) of Appendix A). Figure 1 illustrates this variation for three different values of the critical liquid fraction at the top of the column. \( H(V) \) is a monotonically increasing function of \( V \) which diverges at a critical velocity \( V_c \), i.e. at this velocity the height of the foam column increases to infinity. \( V_c \) is given by

\[
V_c = \frac{1}{5.35} \frac{3\rho g}{\eta} \Phi_H^{3/2}, \tag{3}
\]

where \( \rho \) and \( \eta \) are liquid density and viscosity, respectively, \( V_b \) is the (mean) bubble volume, \( g \) is the gravitational acceleration and \( f \) is a numerical factor that characterizes the boundary conditions (cross-section and surface mobility) for the flow of liquid through a Plateau border (see also appendix A and B).
Experimental evidence of this critical velocity $V_c$ was found by Neethling et al. in measurements of foam height as a function of time [9]. In follow-up work Grassia et al. [10] also computed the extremely slow approach to the steady-state height in the case of bubble velocities just below $V_c$.

Figure 1

How is the divergence of the foam column at the critical bubble velocity $V_c$ to be interpreted physically? As Appendix A and Figure A1 explain, for a given bubble velocity $V$ the steady-state profile of $\Phi(x)$ is described by a function which leads to a limiting value in the limit of $x \to \infty$. If this value is greater than $\Phi_H$, it cannot be obtained. In the steady-state, the amount of liquid carried upwards by the bubbles balances the amount of liquid that drains downwards. If one increases $V$ this balance can only be achieved by having a taller column. However, no such balance is possible for $V > V_c$. In the limit of small gas velocities $V$ the height of the foam column is finite and given by

$$H(V = 0) = \frac{\gamma}{\rho g} V_b^{1/3} (\Phi_H^{-1/2} - \Phi_0^{-1/2}), \quad (4)$$

where $\gamma$ is surface tension. This foam height may be directly obtained from the equilibrium profile for $V = 0$ (see Appendix A). Since the foam is generally much drier at the top than at the bottom, where it is in contact with a liquid pool, it holds $\Phi_H / \Phi_0 << 1$ and eqn.(4) can be simplified to give the following expression for the critical liquid fraction for foam collapse,

$$\Phi_H \approx \left( \frac{\gamma}{\rho g} \right)^2 V_b^{2/3} H(V = 0). \quad (5)$$

The physical model of Verbist et al. [2] thus makes two non-trivial predictions, namely the divergence of the foam column at some critical gas velocity $V_c$ and a finite foam height $H(V=0)$ at zero gas velocity. Following the description of the experimental set-up and methodology in the next section, we show in section 4.1 that our experimentally obtained data, despite large scatter, adheres to these predictions. Our data is well described by the functional relationship for $H(V)$ as given by eqn.(A7). We discuss the advantages and
limitations of the experimental procedure as a steady-state foam test, which can be interpreted by an underlying theory of foam drainage. The foam test results in the determination of the critical liquid fraction $\Phi_H$ and of an effective viscosity of the liquid which flows through the foam. Finally we will suggest a variation of the test to extend its range of applicability.

3. Steady-state Foam Test Experiments

3.1. Experimental Configuration

The foam is produced in a 66 cm long perspex (Plexiglas) vessel with square cross-section (2.5 cm x 2.5 cm) containing 20 ml of surfactant solution at the bottom. Nitrogen gas, which was saturated with perfluorohexane \[11\], was introduced into a solution of C$_{12}$TAB (for details of the preparation of the surfactant solution see Appendix C) through a capillary hole at the bottom of the vessel. Note that the saturation with perfluorohexane drastically slows down coarsening of the foam. Since in our experiments the bubbles remained a maximum of 20 min in the foam, this is a purely precautionary measure because we found that coarsening is of marginal importance on this time-scale. Thus the use of perfluorohexane is not a general requirement for this foam test. The gas flow results in bubbles of equal volume which rise through the surfactant solution to form a foam in the vessel. Since the rising foam undergoes plug flow, the gas velocity $V$ – which equals the bubble velocity – can be determined by tracking the position of individual bubbles in contact with the wall. We determined the time it took a bubble to travel over a certain distance, which was 2 cm for low, 4 cm for intermediate, and 8 cm for high gas velocities. Bubble volumes were estimated from measurements of the lengths $L_W$ of Plateau borders in contact with the wall (indicated in Figure 2). The average length of a Plateau border of a bubble located in the interior of the foam $L_{PB}$ is then given by $L_{PB} = L_W/1.2$ for size dispersions (defined as the ratio of the standard deviation over the average) of the order of 30\% \[11,12\]. Since our foams are monodisperse and have very low liquid fractions (the dry foam seen in Figure 2 is representative of all foams studied), the bubble volume $V_b$ may be written as $V_b = 8\sqrt[3]{2L_{PB}}$, a relation which is exact for Kelvin cells \[13-15\] and a good approximation for other monodisperse foams. Using the above procedure, and averaging over 10 measurements of surface Plateau borders, we obtained a bubble volume of $V_b = 144 \pm 30$ mm$^3$. This corresponds to an equivalent sphere diameter of the

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bubbles of $6.5 \pm 0.5$ mm for all experiments (the error includes the very slight variation in bubble diameter over the range of flow rates used in our experiments).

Figure 2

3.2. Experimental Observations

3.2.1. Measured Variation of Foam Height with Gas Velocity

Our experimental findings essentially adhere to the scenario of the simple theoretical model described in section 2. Once the gas supply is switched on, a foam forms the height of which rises as a function of time at the given gas velocity $V$. The foam generation is accompanied by the bursting of bubbles, which mainly occurs at the top of the foam, i.e. at the foam-air interface, where the foam is driest. This observation is in accord with the theoretical assumptions. Thus foam generation and foam destruction take place simultaneously. After some time (see section 3.2.2) the foam height stops growing which corresponds to the establishment of a steady-state. Since the bursting of bubbles at the top often proceeds in terms of avalanches which involves many bubbles, the steady-state height fluctuates by about $\pm 3$ cm about an average value (fluctuations of only $\pm 1$ and of up to $\pm 5$ cm have been observed in some cases).

Figure 3 shows experimental measurements of the foam height $H(V)$ as a function of the gas velocity $V$. While there is experimental scatter in the data, which we comment on below, its two key features are as predicted by the theoretical model of Verbist et al. [2] (see section 2). Interpolation to a gas velocity $V$ of zero leads to a finite height $H$ of the foam, while the foam height diverges at a critical gas velocity $V_c$.

Figure 3

Before we describe the results of a least square fit of eqn.(A7) to the data we will discuss two unexpected features of the experiments, one related to the overflow of the foam containing vessel and the other to measurements at very long times.
3.2.2. Short- and Long-Term Measurements

Figure 4 displays two sets of measurements where the foam height is monitored as a function of time for two different constant gas velocities $V$.

In both cases the foam height is roughly constant for over an hour, but then starts to increase until it settles at an increased height after about two hours. The height then remains constant until the end of the experiment (180 min in one case, 150 min in the other case). Note that although the experiments run for such a long time, each of the bubbles lasts for only about 3 to 20 min (depending on gas flow rate), i.e. for the time it requires to reach the top of the foam, where it ultimately bursts. Thus the foam does not age in the experiment, since it is continuously renewed and coarsening is negligible. Also the surfactant does not decompose over the time of three hours. We have currently no explanation as to why the height can change after running the experiment for a long time and thus decided to record both plateau values for the height at a given gas velocity (provided no intermediate overflow of the vessel occurred, as discussed in the next section). This contributes to the scatter of the data in Fig.3.

3.2.3. Foam Stability and Foam Height

Relatively stable foams quickly lead to very long columns of foam, even for slow bubble velocities, and thus require long containers for this test. Preliminary tests \[16\] showed that for the chosen foam container (66 cm in height), foams generated by the cationic homologue tetradecyl trimethylammonium bromide, $\text{C}_{14}\text{TAB}$, at 2 cmc were much too stable. Even with the shorter chain $\text{C}_{12}\text{TAB}$ used in the present study, which is known to be a bad foamer [\[17,18\]], the generated amount of foam turned out to be too large at some high gas velocities. In this case the foam simply overflows the vessel.

Surprisingly, we found that after such an overflow, the foam volume can decrease enormously before finally settling to a plateau value. Figure 5 shows an example of such a behaviour. We decided to only record the gas velocity at which the overflow occurred and entered the corresponding foam height as maximal height (= length of the vessel) in Figure 3. We have discarded the data for the much lower heights after the overflow.
While the reason for the reduction in foam height after the foam overflow is not clear, we speculate that some sort of insoluble surface-active ingredient of low concentration is carried continuously to the surface and trapped there. It suppresses collapse when it is of sufficient concentration, but is “washed away” by the overflow of the vessel. The presence of such an ingredient could also explain the observed rise in foam height once there is sufficient time available for its accumulation (section 3.2.2). One is reminded of the important role of foams in fractionation, where rising foams are used to separate mixtures of solutions with different surface activity. The value of critical liquid fraction for foam collapse might thus also be influenced by the concentration of such ingredients at the top.

4. Data Analysis and Discussion

As already stated in section 3.2.1, our data adheres well in its general form to the predictions of the model of Verbist et al. as regards its asymptotic behavior \( V \to 0 \) (eqn.(4)) and \( V \to V_c \) (eqn.(3)). We have also performed a least square fit of the data to \( H(V) \) as given in eqn.(A7). The least-square fit was carried out in the open source program **Gnuplot** where the \( \text{arcoth} \) function was re-written as \( \text{arcoth}(x) = 0.5 \ln ((x+1)/(x-1)) \). The fit is shown as a solid line in Figure 3. It contains two free parameters, namely the value of the liquid fraction at the top of the column and the numerical flow factor \( f \). We find \( \Phi_H = 0.00010 \pm 0.00003 \) and \( f = 0.5 \pm 0.2 \). This corresponds to \( H(V=0) = 7 \pm 1 \) cm from eqn.(4) and \( V_c = 19.8 \pm 1.2 \) cm/min from eqn.(3). While the experimental data shows large scatter, the least square fit describes the general trend adequately. The scatter is particularly striking close to the critical velocity. In light of the theoretical prediction of a divergence of the height at this velocity, this is, however, to be expected.

As shown in Appendix B, the value of the flow factor can be used to estimate the surface viscosity of the surfactant solution. From [13] we obtain for \( f \approx 0.5 \), a value of about 100 for the dimensionless internal mobility \( M = \eta_r/\eta_s \), where \( \eta \) is the liquid viscosity and \( \eta_s \) is the surface viscosity. The Plateau border radius \( r \) can be estimated from eqn.(B2) as \( 0.05 \text{ mm} < r < 0.5 \text{ mm} \) for values of liquid fractions between \( 10^{-4} < \Phi < 10^{-2} \). From this data one can estimate a surface viscosity of \( 5 \times 10^{-10} \text{ kg/s} < \eta_s < 5 \times 10^{-9} \text{ kg/s} \), which is consistent with the value for SDS (sodium dodecyl sulfate), namely \( \eta_s = 10^{-8} \text{ kg/s} \) [20]. (The effective viscosity

\[ \text{Note that the determination of surface viscosities is subject to large errors and an established technique does not} \]
of the liquid flowing through the foam, as deduced from our experiments, is given by $3 f \eta \approx 1.5 \text{ mPa}$, see Appendix A.). The value of the liquid fraction $\Phi_H$ below which a foam is locally no longer stable may be expressed as a critical pressure difference $\Delta p_{\text{crit}}$ between a Plateau border and a neighbouring bubble, using the Laplace-Young equation, $\Delta p = 2\gamma/r$. For our values of $\Phi_H$ and the surface tension $\gamma$ we obtain $\Delta p_{\text{crit}} \approx 1400 \text{ Pa}$, where we have used eqn.(B2) to express the Plateau border radius $r$ in terms of liquid fraction. Again the value deduced from our analysis is in good agreement with data reported previously [17].

Note that our theoretical analysis of the steady-state test was based on the equation for channel dominated drainage (fully rigid interfaces, i.e. $k = 1$ in eqn.(A1)), whereas the analysis of the experimental data indicates mobile interfaces, corresponding closer to node dominated drainage ($k = 1/2$ in eqn.(A1)). We justify our theoretical analysis at this stage by pointing out that both channel and node dominated drainage lead to similar mathematical solutions of the foam drainage equation, including solitary wave solutions, which were employed when developing the foam test. The numerical values for the critical liquid fraction $\Phi_H$ at the top of the column and the flow factor $f$ can thus only serve as an estimate. It is worth noting that quantitative predictions based on the node-dominated drainage equation are hindered by the current lack of a full quantitative description of the flow inside a node [21].

5. Assessment of the steady-state foam test

What are the advantages of the described foam test over the tests that are traditionally in use? The measured relation between foam height and gas velocity is interpreted by the standard model of foam drainage. The test is not just another purely empirical description of foam stability but is related to the underlying physics and provides an estimate of the liquid fraction $\Phi_H$ below which a foam is locally no longer stable, i.e. where bubbles collapse.

In any practical application of the foam test it is not necessary to perform a least square fit of the entire set of data in order to extract the parameters $\Phi_H$ (and thus $\Delta p_{\text{crit}}$) and $f$. $\Phi_H$ is easily obtained from a linear interpolation of the data in the limit $V \to 0$ and use of eqn.(4). The extrapolation also reduces the influence of eventual outliers, which can greatly disturb the result in the case of foam tests that rely on the measurement of a single value. The parameter $f$ can then be obtained from an estimate of the critical gas velocity $V_c$ and eqn.(3).

exist so far. Thus we refer to SDS as it is a soluble surfactant with a comparable cmc (cmc (SDS) = 9 $10^{-4}$ M). As SDS forms much more stable foams compared to the respective $C_{12}$TAB, a lower surface viscosity is expected for the latter – reliable experimental data do not exist for $C_{12}$TAB.
Moreover, please note that since the test concerns the steady-state of the foam, there is neither a need to specify certain gas rates for its production, nor time intervals for measuring its height. The parameters that are determined are specific to the surfactant solution studied and the (average) bubble volume. However, the test does not appear to be practical for surfactant solutions that produce very stable foams, as in this case the divergence of the foam column would already occur at very small gas flow rates.

Finally, our theoretical model of the test gives an interpretation of the empirical result by Bikerman. For sufficiently small gas velocities the steady-state height of the foam increases indeed linearly with velocity. This allows for an expression of Bikerman’s unit of foaminess $\Sigma$ in terms of relevant physical quantities and the (average) bubble volume as given by eqn.(A8). However, note that this expression contains the product of the flow factor $f$ and the liquid fraction $\Phi_H$, which complicates the interpretation of any measured value for $\Sigma$ in Bikerman’s test. Inserting our determined values for $f$ and $\Phi_H$ for the present system into eqn.(A8), we obtain $\Sigma \approx 11 \pm 3 \text{s}$, consistent with the slope of $H(V)$ in Figure 3 for small velocities.

6. Outlook

In his original description of the steady-state foam test, Bikerman noted the empirical finding of a finite value for the foam height $H$ in the limit of low gas velocity $V$ [1]. The paradox is that one may expect this to be zero, since a column left for sufficiently long times, without sparging, must surely collapse entirely. We are reminded that film rupture must be of a stochastic nature in reality and that there could well be different mechanisms with different time-constants. The reconciliation of this paradox therefore remains an intriguing topic for future investigation.

A simple variation of the test can be conceived that would make it applicable also to extremely poor foaming solutions. It consists of performing the test in the presence of forced drainage [22], whereby the same surfactant solution is continuously added to the top of the foam. This leads to an overall increase of the liquid fraction of the foam, and thus renders the foam more stable. There should be an analytical solution also for this case, but we will leave the exploration of this for the future.

It is thus possible to conceive a single apparatus with different protocols of operations, according to the surfactant solution that one is using. Very poor foamers are analysed using the steady-state foam test under forced drainage to determine the limiting values $H(V=0)$ and...
Poor foamers are analysed using the steady-state foam test under free drainage (as described in this paper) to determine these limiting values. Good foamers are characterised by the Rudin test, or a variation thereof.

Appendix A: Application of the foam drainage equation to a steady-state test of foam stability

The foam drainage equation is a non-linear partial differential equation which governs the variation of liquid fraction with position and time, throughout a foam [2,8]. Expressed in dimensionless quantities it takes the form

\[
\frac{\partial \alpha}{\partial \tau} = \frac{\partial}{\partial \xi} \left( \frac{1}{2k+1} \frac{\partial \alpha^{k+1/2}}{\partial \xi} - \alpha^{k+1} \right),
\]  

(A1)

where the dimensionless parameters \( \xi \) and \( \tau \) are related to vertical position \( x \) and time \( t \) by \( \xi = x / x_0 \) and \( \tau = t / t_0 \) and \( \alpha \) is proportional to the liquid fraction \( \Phi \) of the foam,

\[
\Phi = 5.35 x_0^2 / V_b^{2/3} \alpha,
\]  

(A2)

where \( V_b \) is the (average) bubble volume [23]. The constant \( k \) is \( k = 1 \) if the main contribution to dissipation is in the Plateau borders (rigid interfaces), while it is \( k = 1/2 \) if it is in their junctions (fully mobile interfaces). In the following we shall restrict ourselves to the case of \( k = 1 \). The material constants \( x_0 \) and \( t_0 \) are given by \( x_0 = \sqrt{C \gamma / \rho g} \) and \( t_0 = 3f \eta / \sqrt{C \gamma \rho g} \), respectively, with surface tension \( \gamma \), liquid density \( \rho \), and liquid viscosity \( \eta \). The product \( 3f \eta \) is called effective viscosity, where the numerical flow factor \( f \) is the ratio of the characteristic velocity scale \( \rho g A / \eta \) to the average velocity in a channel of cross-sectional area \( A \). For flow through a Plateau border under non-slip boundary conditions (Poiseuille flow) it is given by \( f \approx 49 \) (compared to \( f = 8\pi \approx 25 \) for flow through a circular cross-section). This value is reduced in the case of non-rigid boundaries [19,24], as discussed in Appendix B. (In the case of our experimental data we obtain a value for \( f \) from a least square fit to eqn.(A7).) The constant \( C = \sqrt{3} - \pi / 2 \approx 0.402 \) is a geometrical factor associated with the cross-sectional area of a Plateau border.
The foam drainage equation may be applied to the case at hand, as follows. A foam is continuously generated by bubbling gas through a surfactant solution and gathering the bubbles in a tube. The growth of the foam in height is accompanied by film bursts, which, in the simple model, occur only at the top of the foam column. Eventually a steady-state is reached in which the bubble generation at the bottom is counter-balanced by the bursting of bubbles at the top. This results in a foam column of a steady height, which depends on the velocity $v$ of the gas flow. In the steady-state the (average vertical) liquid velocity is zero throughout the sample. Changing to a frame of reference that moves with the bubbles thus leads to a flow of liquid in the downward direction. This corresponds to a solitary-wave solution of the foam drainage equation, that is, a solution of fixed profile that moves with constant velocity $v$. In the limit of $v \to 0$, i.e. a foam column where the bubbles do not move, the equilibrium solution for a foam under gravity, $\alpha(\xi, \tau) \sim (\text{const.} - \xi)^2$ should be recovered.

A solitary wave solution that fulfils the above requirement for $v \to 0$ is given by

$$\alpha(\xi, \tau) = v \cotanh \left( \frac{1}{\sqrt{v}} (\xi - \xi_a) \right).$$

(It appears that an equivalent solution of the node-dominated drainage equation ($\kappa = 1/2$) would still need to be worked out.) Note that in this dimensionless formulation the amplitude behind the advancing wave is given by $\alpha = v \sqrt{\cotanh(\xi - \xi_a)}$. Translated back into the laboratory frame this leads to the following time independent liquid profile

$$\alpha(\xi) = v \cotanh \left( \frac{1}{\sqrt{v}} (\xi - \xi_a) \right),$$

(A3)

where $\xi_a$ is a constant, which is fixed by the boundary conditions specified below. The foam height $h$ as a function of gas velocity $v$ can be inferred from eqn.(A3) by applying boundary conditions for $\alpha$ at bottom and top of the column. At the bottom, where the foam is in contact with a liquid pool, we may impose $\alpha(\xi=0) = \alpha_0$, where $\alpha_0$ is related (via eqn.(A2)) to the packing fraction of random sphere packing. At the top we set $\alpha(\xi=h) = \alpha_h$, which is related to the critical liquid fraction, below which the foam is unstable. The height $h$ of the foam column is then obtained as

$$h(v) = \frac{1}{\sqrt{v}} \text{arcoth} \left( \frac{1}{\sqrt{v}} \frac{\sqrt{\alpha_0 \sqrt{\alpha_h}} - \sqrt{\alpha_h}}{\sqrt{\alpha_0} - \sqrt{\alpha_h}} \right).$$

(A4)

(Note that in [2], eqn.(27) the “$v$” in the numerator was mistakenly written as “$1$”.) A plot of $h(v)$ is shown in Figure 1 for three different values of $\alpha_0$. As can be seen, $h(v)$ is a
monotonically increasing function of \( v \) which diverges at \( v = \alpha_h \), i.e. at this velocity the height of the foam column increases to infinity. At \( v = 0 \) the foam column has a finite, non-zero height, given by

\[
h(v = 0) = 1/\sqrt{\alpha_h} - 1/\sqrt{\alpha_0} ,
\]

and only depends on the boundary conditions at the top and the bottom. Linearisation of eqn.(A4) leads to

\[
h(v) = h(v = 0) + \frac{v}{3} (h(v = 0))^3 ,
\]

which results in the following expression for Bikerman’s unit of foaminess \( \Sigma \)

\[
\Sigma = \frac{\ell_0 (h(v) - h(v = 0))}{v} = \frac{\ell_0}{3} \left( \frac{h(v = 0)}{v} \right)^3
\]

Reinstating physical quantities, we obtain the following expression for the foam height as a function of bubble velocity,

\[
H(V) = \frac{C_V}{\sqrt{3 f \eta \rho g}} \frac{1}{\sqrt{V}} \arccosh \left\{ \frac{1}{\sqrt{V}} \left[ \frac{\rho g V_{f}^{2/3}}{3 f \eta} \sqrt{\frac{\beta_0 \beta_H - V}{\eta \rho g}} - V \right] \right\}^{1/2}
\]

where we now use capital letters \( H \) and \( V \) for the dimensional expressions for foam height and gas velocity (\( H = x_0 h \) and \( V = x_0/t_0 v \)).

Experimental measurements of \( H(V) \) in a foam test allows for the determination of the numerical value of \( f \) and the liquid fraction at the top of the sample \( \Phi_H \). Expressions for foam height at zero gas velocity and for the gas velocity at which the column diverges, expressed in physical parameters are given in the main text (eqns.(4) and (3), respectively). Bikerman’s unit of foaminess expressed in physical parameters follows from eqn.(A6) as

\[
\Sigma = \frac{1}{0.16 (\rho g)} \frac{\eta}{V_b} \left( \Phi_H^{-1/2} - \Phi_0^{-1/2} \right)^3
\]
Appendix B: Flow factor $f$ and interfacial mobility $M$

The value of the flow factor $f$, i.e. the ratio of characteristic velocity scale $\rho g A/\eta$ to the average velocity in a Plateau border of cross-sectional area $A$, is strongly dependent on the flow velocity at the boundary of the Plateau border. Leonard and Lemlich [24] and Koehler et al. [19] numerically computed $f$ as a function of the interface mobility $M$, defined as

$$M = \eta_s/\eta,$$  \hspace{1cm} (B1)

where $\eta$ is liquid viscosity, $\eta_s$ is the surface viscosity and $r$ is the Plateau border radius. The value of $f \approx 49$ for Poiseuille flow corresponds to the limit of $M \to 0$ (rigid boundaries). Since the foam test described in this paper gives an estimate of $f$, this allows for an estimate of $M$ using the numerical results of [19,24] and thus an estimate of the surface viscosity $\eta_s$. The required value of the Plateau border radius in eqn.(B1) is obtained from

$$\Phi = 5.35 \times 0.161 r^2/V_b^{2/3},$$  \hspace{1cm} (B2)

where $\Phi$ is the liquid fraction and $V_b$ is the average bubble volume [22].

Appendix C: Surfactant solution

Foam stability is critically dependent on the chemical composition of the surfactant solution in use, hence we specify it precisely here for the present experiments. The cationic surfactant dodecyl trimethylammonium bromide C$_{12}$TAB (purity >98% AT) was purchased from Fluka and used as received. All measurements were carried out at 2 cmc (the cmc of C$_{12}$TAB in aqueous solution is $1.5 \times 10^{-6}$ M [25]) as this was found to be the optimal concentration for our experimental set-up. Preliminary tests showed that for 1 cmc foams as high as 60 cm could be generated but the height did not diverge at some critical gas velocity $V_c$ - a longer column would have been needed to measure the divergence. On the other hand, measurements at 4 cmc simply generated too much foam [16]. Acetone (p.a.) and ethanol (p.a.) were purchased from Aldrich. The solutions were prepared with Milli-Q® water. All glassware was cleaned with deconex® from Borer Chemie (as replacement for chromic sulphuric acid) and rinsed thoroughly with water before use. The foam column was cleaned after each run with a soft
brush and rinsed thoroughly with Milli-Q® water. For the theoretical calculations we assumed a surface tension of $\gamma = 38.3$ mN/m [25]. We note that the surfactant concentration in the foam is lower compared to that in the bulk solution as surface is generated. However, due to the fact that the generated foams were very stable, the surfactant concentration is expected to be above – or at least close to – the cmc. Thus the value of the surface tension is not affected by this concentration change. For both the density $\rho$ and the viscosity $\eta$ of our solution we used the corresponding values of pure water ($\rho = 998.2$ kg/m$^3$, $\eta = 1.002$ 10$^{-3}$ Pa s).

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References


Figure 1: Height $h(v)$ of a foam column as a function of gas velocity $v$ as given by the model of Verbist et al. [2] (eqn. (A4) of the appendix). The foam column diverges at $v = \alpha_h$, where $\alpha_h$ corresponds to the value of liquid fraction at the top of the column, where all films burst. Less stable foams are thus characterised by larger values of $\alpha_h$ and require a higher gas velocity to lead to a divergence of the foam column. Note that the column has a finite length at $v = 0$. Experimentally this can be obtained by interpolation of the data to $v = 0$. (The value of $\alpha_0$, corresponding to the liquid fraction at the foam-liquid interface at the bottom of the column, was set to $\alpha_0 = 0.36$ for all graphs in this figure.)
Figure 2: Photograph of a foam sample contained in a plexiglass vessel of dimensions 2.5 cm × 2.5 cm × 66 cm. The volume of the bubbles in the monodisperse foam was determined from measurements of the length of surface Plateau borders (see text for further details).
Figure 3: (top) Experimental data of the steady-state foam height $H(V)$, measured at a range of gas velocities $V$. The data was fitted to eqn.(A7) resulting in $\Phi_H = 0.00010 \pm 0.00003$ and $f = 0.5 \pm 0.2$. This corresponds to $H(V=0) = 7 \pm 1$ cm from eqn.(4) and $V_c = 19.8 \pm 1.2$ cm/min from eqn.(3). Data points shown as open symbols correspond to gas velocities that led to an overflow of the container. We have shown these merely as indicator but did not include them in the fits, since the actual foam heights for these flow rates could not be determined. (bottom) Blow-up of small velocity data. The slopes of the dashed lines are 8s, 11s, and 14s, respectively, and correspond to Bikerman’s foaminess $\Sigma$, which we calculated to be $11 \pm 3$ s.
Figure 4: Long term measurements of foam height $H$ and gas velocity $V$ as a function of time $t$ for two different experiments. The gas velocity $V$ was determined repeatedly to ensure it was kept constant throughout an experimental run ($V = 13.8 \pm 0.2 \text{ cm min}^{-1}$ (top), $V = 17.9 \pm 0.3 \text{ cm min}^{-1}$ (bottom)) and the foam height was measured until a plateau value was reached. Note that in both data sets the foam column started to increase in height after about 60 min before it settled again to a roughly constant value. We decided to include both plateau values, before and after such an increase, in our collected data shown in Figure 3.
Figure 5: Measurement of foam height $H$ and gas velocity $V$ as a function of time $t$ showing an overflow of the foam. The gas velocity was kept constant (here $20.2 \pm 0.2$ cm min$^{-1}$) and the foam height was repeatedly measured. In contrast to the measurements carried out at lower gas velocities (see Figure 4), a very steep increase of the foam height was observed at the beginning of the experiment, which eventually led to an overflow of the foam volume out of the vessel. Surprisingly, after the overflow the foam height shrinks enormously, before it settles at a much lower height. The measurement was recorded in Figure 3 (with height 66 cm, corresponding to the length of the vessel), but it was marked as an open symbol to indicate that the exact height could not be determined due to overflow. As we cannot make sense of the behaviour after the overflow and since the overflow lead to some foam loss, we decided to disregard the consecutive measurements.
Figure A1: The analytic function which describes the steady-state profile is of the form $\alpha(\xi) = v \coth^2 \left( \sqrt{v(\xi - \xi_0)} \right)$ (eqn. A3). The relevant segment is determined by the application of appropriate boundary conditions at the bottom and the top of the foam column.