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ABSTRACT
Burst-type signals constitute an important class of transient signals, being used especially in the investigation of various physical environments by electric or acoustic means. An important issue in the analysis of this type of signals is their detection in time. In this paper, we propose a detection method that is based on the histogram of the phase space distributed over time. The method consists in representing the analyzed signal in phase space and, then, quantifying the recurrences of the trajectory obtained in this space. In this way, we derive a time - recurrence radius representation for the signal, that allows identification of positions and durations of the transients. Afterwards, we propose a method to obtain a detection curve starting from this representation of the signal. We also present here some results concerning the performance of our method in the presence of noise on both synthetic and real signals.

Index Terms— Transient signals, phase space representation, recurrence plot, detection curve

1. INTRODUCTION

Transient signals are generally characterized by very short durations over the observation and they often indicate abrupt changes or unstable states in the operation of the system that is being studied. In this paper, we deal with a particular case of transient signals, namely burst signals consisting in pulses or short oscillations. Such signals are encountered in the investigation of various physical environments using ultrasonic or electric signals. The general approach of such applications consists in transmitting a short signal in the environment to be investigated. This signal usually consists in a pulse (e.g. Gaussian pulse) or an oscillation (e.g. sine, chirp). The propagated signal is then recorded and analyzed, in order to detect changes from the original.

The analysis of transient signals addresses two issues: detection (that consists in identifying the time intervals where transients are present in the analyzed signal) and characterization (that consists in identifying the characteristic parameters of these transients (e.g. envelope, frequency band, shape, zero crossing rate, energy)). In this work, only the detection issue is addressed. Some of the transient signal detection methods (e.g. based on spectrogram, wavelets, complex-time distribution [1] [2] provide the detection curve from a representation of the signal in a transformed space – that is represented, most often, by the time-frequency plane [3]. In this paper we propose a detection method based on a representation of the analyzed signal in the time - recurrence radius plane. We start (section 2) by discussing the idea of time-distributing the histogram of the signal and showing its potential in detecting transient signals. We continue (section 3) with improving this idea by first representing the signal in phase space and then computing the local recurrence rate, thus obtaining the time-distributed phase space histogram (TDPSH) of the signal. We present then (section 4) a method to obtain a detection curve from the TDPSH and then we show some results (section 5). We conclude the paper (section 6) by pointing out conclusions and directions for future research in this area.

2. TIME-DISTRIBUTED HISTOGRAM

The histogram of a signal is obtained by dividing the range of signal values in a chosen number of (non-overlapping) bins and by summing for each bin the number of samples whose values lie inside it. However, this gives only global information about the signal, without any time information. This can be solved by replacing every signal sample with the histogram value corresponding to the bin where the value of that sample lies. We obtain thus a time-distributed histogram (TDH):

\[ TDH^{(\varepsilon)}(t) = hist^{(\varepsilon)}(s(t)), \]

where \( s \) is the signal, and \( hist^{(\varepsilon)} \) is its histogram, computed using bins of size \( \varepsilon \).

The potential use of the TDH is illustrated in Figure 1.(a), for a test signal composed of two Gaussian pulses and an exponentially decaying sinusoidal oscillation (plus a 15dB noise). The figure illustrates the variation of \( TDH^{(\varepsilon)} \) for some range of the parameter \( \varepsilon \). During the time intervals where transients are present, the density of vertical black lines in the image \( TDH^{(\varepsilon)}(t) \) increases, which allows a fairly good visual detection of these transients. If, on the other hand, the
analyzed signal only contains noise (Figure 1(b)), the visual inspection of the TDH does not reveal any transient signals.

In the following section we show that extending the idea of TDH in phase space leads to a t-ε representation that has a better noise robustness, providing thus a more precise detection of transients.

3. TIME-DISTRIBUTED PHASE SPACE HISTOGRAM

Representing a time series (and, in particular, a signal) in phase space is discussed in various papers in literature [4, 5]. We will use a representation obtained by the method of delays, that consists in transforming the signal \( s \) into an ordered set of vectors (describing a trajectory in phase space) obtained by:

\[
\vec{v}_i = (s(t), s(t + d), ..., s(t + (m - 1)d)),
\]

(2)

where \( m \) is the dimension of the phase space used for representing the signal and \( d \) is the time delay between two successive components of such a vector. Discussion concerning these two parameters and the way they affect the appearance of the trajectory, as well as various methods for choosing them, can be found in [6]. An intensively studied method [7] to analyze this phase space trajectory is the recurrence matrix, that is computed with:

\[
R_{i,j}^{(m,d,ε)} = Θ (ε - ||\vec{v}_i - \vec{v}_j||),
\]

(3)

where \( Θ \) is the Heaviside step function, \( ε \) is the recurrence radius, and \( ||·|| \) is a norm that we associate to the phase space.

As a value of 1 in \( R_{i,j}^{(m,d,ε)} \) means that \( \vec{v}_j \) lies in the neighbourhood of \( \vec{v}_i \), summing after \( j \) the elements of the matrix \( R^{(m,d,ε)} \) would have for point \( i \) the signification of “the number of vectors that are in the neighbourhood of \( \vec{v}_i \)”. We call the function thus obtained (followed by a normalization) time-distributed phase space histogram (TDPSH). Its mathematical expression is:

\[
TDPSH^{(ε)}_{(m,d)}(t) = \frac{1}{M} \sum_{j=1}^{M} R_{i,j}^{(m,d,ε)},
\]

(4)

where \( M = N - (m - 1)d \), \( N \) being the number of samples in the original signal. (We must note that by representing the signal in phase space the time coordinate does not remain exactly the same, as each point \( t \) in \( TDPSH^{(ε)}_{(m,d)}(t) \) does not correspond to a certain time instant, but to an entire time interval, having a size of \( 1 + (m - 1)d \).)

In the case of representing the signal in a mono-dimensional phase space (i.e. with \( m = 1 \), \( R_{i,j}^{(1,d,ε)} \) becomes:

\[
R_{i,j}^{(1,d,ε)} = Θ (ε - |s(i) - s(j)|).
\]

(5)

Therefore, the neighbourhood of point \( s(i) \) is an interval of length \( 2ε \), having its center in the value \( s(i) \). Under these conditions, the similarity between \( TDH^{(ε)}(t) \) and \( TDPSH^{(ε)}_{(m,d)}(t) \) (for fixed values of \( m \) and \( d \) ) is clear. The second has the main advantage of being less sensitive to noise than the first one. Figure 2 illustrates this, for the same test signal used in Figure 1(a). This figure also shows that the choice of the \( m \) and \( d \) parameters is important. The bottom figure (Figure 2(b)) shows that for \( m = 3 \) and \( d = 15 \) the representation in the time - recurrence radius plane allows a better time localization of the transients.

We have shown so far that the time - recurrence radius representation given by the image of \( TDPSH^{(ε)}_{(m,d)}(t) \) that is obtained for fixed values of the parameters \( m \) and \( d \) allows a good visual detection of the transient signals. In the following section we propose a method to process this image in order to build a detection curve.
4. DETECTION USING TDPSH

The detection algorithm we propose involves the following steps:

- Choose from the image $TDPSH^{(\varepsilon)}_{(m,d)}(t)$ the horizontal line ($L$) corresponding to $\varepsilon = \varepsilon_{opt}$, where:

$$\varepsilon_{opt} = \frac{1}{M-1} \sum_{i=1}^{M-1} ||\vec{v}_{i+1} - \vec{v}_i||. \quad (6)$$

(We obtained the value of $\varepsilon_{opt}$ empirically.)

- Rescale $L$, by dividing it to its maximum value: $L = L/\max(L)$.

- Complement $L$: $C = 1 - L$.

- Compute the envelopes of $C$ (i.e. $Anv_{min}$ and $Anv_{max}$) by linear interpolation of the local minima and maxima, respectively.

- Compute the median-filtered version of $C$: $C_f = filt_{med}(C)$, using a window of size $w_{filt}$.

As can be noticed from this description, before using the method the following three parameters must first be chosen: $m$, $d$, and $w_{filt}$.

5. RESULTS

In Figure 3 we illustrate the application of the method we proposed in the previous section, for the signal in Figure 1.(a). Figure 4 shows what happens to this detection curve as the noise level increases (i.e the signal-to-noise ratio (SNR) takes values from 30 dB to -20dB).

A quantitative view on the method is offered by the receiver operating characteristic (ROC) curves shown in Figure 5. This figure shows, for comparison purposes, also the ROC curves that we obtained for a detection using the spectrogram, on the same signal.
Fig. 6. Some real signals and the corresponding detection curves. $m$ is 3 for all three figures, and $d$: (a) 15, (b) 5, (c) 2. $w_{filt} = 30$ for all three figures.

We have shown so far how our method behaves for a synthetic signal. Figure 6 shows its behavior when it comes to detecting transients in a real signal. These signals were measured during experiments conducted in collaboration with Electricité de France [2].

We point out that the $m$ and $d$ parameters have been chosen separately in each case, based on empirical criteria. We noticed that a value of 3 (or sometimes even 2 or 1) for $m$ is enough for detecting transients having a fairly simple shape, i.e., without highly irregular oscillations. This is because the trajectory that corresponds to such transients is usually a simple curve, without many loops and returns. The $d$ parameter has been chosen so that the trajectory be as wide as possible. In the same time, we wanted $d$ to be as small as possible (in order to allow a better time localization of the phase space vectors). Therefore, we have chosen $d$ by using visual criteria. For the median filtering involved by the algorithm we used a window of length $w_{filt} = 30$. The optimal choice for $w_{filt}$ should take into account the minimum expected duration of the transients to be detected.

6. CONCLUSION

We proposed in this paper a new method for transient signal detection. The method is based on representing the analyzed signal in phase space and counting the number of recurrences that correspond to each point of the trajectory. We obtained thus a bi-dimensional representation for the signal, by plotting in the time - recurrence radius plane the local recurrence rate of the trajectory. This representation, that is in fact a generalized and improved version of the time-distributed histogram of the signal, allows a good visual detection of the transients. We proposed an empirical method to perform this detection task automatically.

Further, we are planning to develop a technique for determining the optimum phase space representation parameters, as they affect the performance of our method. The optimum choice for the size of the window to be used in the filtering of the detection curve is also important. Another research direction consists in experimenting with other phase space representation techniques (e.g., singular value decomposition).

7. REFERENCES


