AN HOMOGENEIZATION PROCEDURE FOR CARDOBORAD AND STITCHED SANDWICHES USING RESPECTIVELY ANALYTICAL AND NUMERICAL SIMULATION

Z. Aboura, N. Talbi, R. Ayad, S. Allaoui, M.L. Benzeggagh

III European Conference on Computational Mechanics
Solids, Structures and Coupled Problems in Engineering
C.A. Mota Soares et.al. (eds.)
Lisbon, Portugal, 5–8 June 2006

AN HOMOGENEIZATION PROCEDURE FOR CARDOBORAD AND STITCHED SANDWICHES USING RESPECTIVELY ANALYTICAL AND NUMERICAL SIMULATION

Z. ABOURA[1], N. TALBI[2], R. AYAD[2], S. ALLAOUI[3], M. BENZEGGAGH[3]

[1] L3M, IUT de Tremblay en France Paris 8

Rue de la Râperie 93290 Tremblay-France
zoheir.aboura@utc.fr

[2] ESIEC, Université de Reims Champagne Ardenne
Esplanade de Roland Garros B.P 1029 – 51686 Reims Cedex 2
nabil.talbi@univ-reims.fr, rezak.ayad@univ-reims.fr

[3] UTC, Université de Technologie de Compiègne
Centre de Recherche de Royallieu B.P 20529 – 60205 Compiègne Cedex
malk.benzeggagh@utc.fr, samir.allaoui@utc.fr

Keywords: Homogenization, cardboard, Stitched Sandwich, Finite element.

Abstract. *We present in this paper two analytical homogenizations of corrugated cardboard and stitched sandwiches with emphasis on experimental procedures and numerical modeling.*

The interest of the use of sandwich structures is not any more to show. Their performances go from pairs with their complexities. As example: the new generation of stitched sandwich structures. The structural analysis using this type of materials becomes very quickly expensive in term of meshing and computing time, taking into account the complexity of the components. The idea used through this study consists, in the first step, in working out a procedure of homogenization in order to obtain an equivalent homogeneous material [1]. Then powerful elements are developed to reproduce the mechanical behaviors of these structures.

Two types of sandwich structures were the subjects of this study. The first is a corrugated cardboard considered as thin (4 mm thickness) and the second is a stitched sandwich with PU core and whose skins are out of woven glass. The total thickness is 22 mm.

The results of the two analytical methods are compared with the experimental results and finite elements. They are promising and show a rather good correlation.

1 INTRODUCTION

The sandwich structures are used more and more. The applications go from the corrugated cardboard used in the packaging to the new generations of stitched sandwich structures used in aeronautics. The association of two rigid skins to a light core makes that this type of structure is ideal for bending solicitation. However, the use of these materials as structural materials inevitably requires calculations, which can very quickly prove expensive in time and elements.
Thus, the mesh of corrugated cardboard sample required 900 elements and 1070 nodes (fig.1).

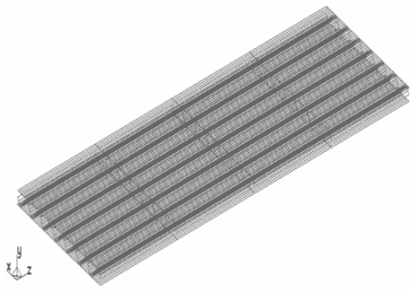


Figure 1 - 3D meshing of corrugated cardboard sample. 900 elements

The idea that this study proposes is in two steps:

- First make an homogenization of the elastic behavior of this type of structure. The methods used must be not very complex and inexpensive.
- Second, after obtaining these homogenized properties; powerful finite element is using in order to simulate the bending behavior of these structures.

Thus, in the case of the corrugated cardboard of which the thickness does not exceed 4mm, a shell element will be used. A modeling 2D will thus be made. Its formulation will have to take into account shearing. As regards the stitched sandwich structures, solid elements will be used in order to modeling this kind of structures.

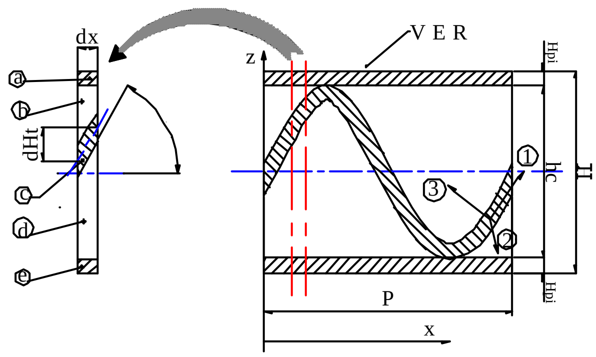
2 HOMOGENEISATION OF CORRUGATED CARDBOIARD

2-1 ANALYTICAL METHOD

To analyze the elastic behavior of the corrugated cardboard, the proposed technique is a point-wise lamination approach using the classical laminate theory. This approach is inspired from Ishikawa et al [2], Aboura [3, 4] and Scida et al. [5] related to the elastic behavior modeling of woven composites materials. The core consists of in-plane sinusoidal cells extending vertically between top and bottom face laminates. A unit cell representative of the corrugated cardboard is defined as in figure 2 and considered the skins and the undulated fluting as an assembling of many infinitesimal elements "*dx*" of unidirectional lamina oriented at different angles.

The classical laminate theory is then applied to each element. The relationship between the in-plane stress and moment resultants N_i and M_i, and in-plane strains, curvature e_j and k_j is given by:

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} e_j \\ k_j \end{Bmatrix} \quad \text{(i, j =1,2 and 6)} \tag{1}$$



(a) superior skin - (b) emptiness - © fluting - (d) emptiness - (e) inferior skin

Figure 2 - Unit cell representative of the corrugated cardboard.

in which A_{ij}, B_{ij} and D_{ij} are the in-plane stiffness for each infinitesimal element dx. These are defined by

$$(A, B, D) = \int_{-h/2}^{h/2} (1, z, z^2) Q_{ij} dz \tag{2}$$

Q_{ij} is evaluated for each constituent of the corrugated cardboard unit cell. This means that the superior and inferior skins and the fluting undulation are taken into account. The local stiffness of each infinitesimal element depends on the constituent elastic properties as well as on the fluting orientation defined by the local off-axis angle $q(x)$. This angle is calculated from the fluting median fiber function $H_t(x)$:

$$q(x) = \tan^{-1}\left(\frac{dHt(x)}{dx}\right) \tag{3}$$

$H_t(x)$ is assumed to be of sinusoidal form with a maximum thickness of hc:

$$Ht(x) = \frac{hc}{2} \sin\left(2p\,\frac{x}{P}\right) \tag{4}$$

A detail of this approach is explained on previous work (see Aboura & al 2004).

2-2 FINITE ELEMENTS METHOD

The cell has been modeled using the finite element method (figure 3). Numericals tests are conducted on it for extract the equivalent elastic constants for corrugated cardboard. The unit cell is first subjected to uni-axial and equal bi-axial extension tests for determination of Poisson's ration and the tensile elastic moduli. The equivalent shears moduli G_{LT} is determined by constraining the displacement of the upper and lower faces and imposing a horizontal displacement for the nodes of the upper face.

The sinusoidal and flat laminates of the corrugated cardboard are modeled with rectangular 4-node isoparametric shell elements. The homogenized elastic analytical model result and

their good correlations with FE analyses and experimental [1] predictions are presented in table 1.

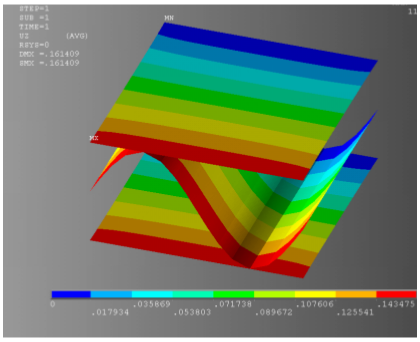


Figure 3 - Finite elements homogenization of cell unit of corrugated cardboard

2-3 COMPARISON BETWEEN ANALITICAL AND FINITE ELEMENTS METHODS

The two methods have results close to the experimental results (tab.1), with an advantage to the analytical method. This advantage is accentuated if the computing time is taken into account. On the other hand the advantage of finite elements the method is in the possibility of obtaining the properties in the third direction (these results are not presented for this study).

	E_x (Mpa)	Ey (Mpa)	ν_{xy}	G_{xy}
Analytical Method	712	392	0,24	201
Finite Element Method	796	496	0,26	292
Experiment	656 ± 40	412 ± 25	$0{,}21 \pm 0{,}018$	211 ± 5

Table 1 - Comparison between analytical and finite elements methods

3 HOMOGENEISATION OF STITCHED SANDWICH

Advanced composite structures, such as three-dimensional woven and stitched stiffened composite laminates are used in many engineering applications. Stitching or other through the thickness reinforcement greatly reduces the driving force for propagation of the delamination crack. The stitching creates closing traction acting across the laminate thickness that limits damage extension.

3.1 Definition of unit cell

Elementary volume is defined by thickness H (figure 4) that is represented by the foam's dimension and the skin's thickness. The width py is determined by the spacing of the rows of seams, while the length px is variable and depends on the various structural parameters (step and angle of stitch).

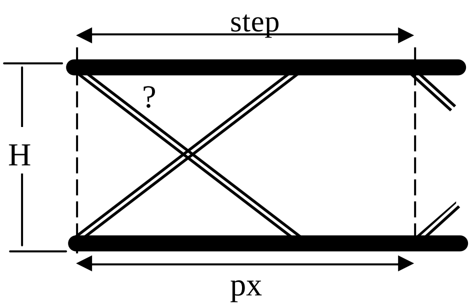


Figure 4 - Unit cell representative of stitched sandwich

Modeling procedure:

The stitched sandwich is considered as a stacking of three layers: higher skin, the reinforced core (foam and stitch) and lower skin. To simulate the behavior of the skins (composed by polyester matrix reinforced by woven glass fibers), MesoTex software [5] has been used.

Concerning the core, a homogenization procedure was carried out in order to consider both of foam and stitches. The used analytical model [5] consists in determining the global rigidity, starting from the basic rigidities of the components according to relation (1):

$$C_{ij,Global} = \frac{1}{V_t} \sum_{k=1}^{n} V_k \,.\, C'_{ij,k} \quad , \qquad \textit{i and j from 1 to 6} \tag{5}$$

C_{ij}: is the rigidity matrix of each constituent
V_k: volume fraction of each constituent

From the characteristics of each of the three layers, the theory of the conventional laminates is used to modelize the elastic behavior of the complete structure (2).

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_y \\ Q_x \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} & 0 & 0 \\ A_{21} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} & 0 & 0 \\ A_{31} & A_{32} & A_{33} & B_{31} & B_{32} & B_{33} & 0 & 0 \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} & 0 & 0 \\ B_{21} & B_{22} & B_{23} & D_{21} & D_{22} & D_{23} & 0 & 0 \\ B_{31} & B_{32} & B_{33} & D_{31} & D_{32} & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F_{44} & F_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & F_{45} & F_{55} \end{bmatrix} . \begin{bmatrix} e^0_{xx} \\ e^0_{yy} \\ ?^0_{xy} \\ ?_x \\ ?_y \\ ?_{xy} \\ ?^0_{yz} \\ ?^0_{xz} \end{bmatrix} \tag{6}$$

The properties in the third direction according to the procedure presented in reference [5]

E_1 (MPa)	E_2 (MPa)	E_3 (MPa)	$?_{12}$	$?_{23}$	$?_{13}$	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)
1776.9	1705.2	111.8	0.3	0.1	0.1	346.8	27.8	4.5

Table 2 – Material properties of the stitched sandwich

4 FINITE ELEMENTMODELING

The homogenized corrugated structure has been modeled using a new multilayer shell finite element, labeled DMTS (§4.1). A 3D volumetric finite element procedure using the new SFR Hexahedra solid element model has been used for the stitched structure. Performances of both finite element models are studied across some experimental and analytical problem tests.

4.1 3-node shell element DMTS

The shell element DMTS (Discrete Mindlin Triangular for Shell) (figure 5) has three nodes and six degrees of freedom per node. It's obtained by combining the well-known membrane element CST (Constant Strain Triangle) and a bending/shear plate element DDMT (Displacement Discrete Mindlin Triangle) [6]. It has been developed for the analysis of isotropic and multilayered plate and shell structures.

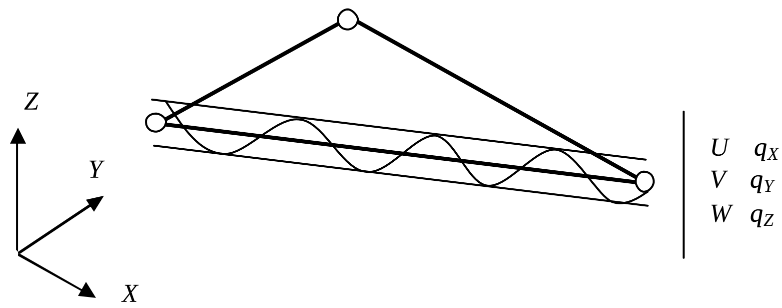


Figure 5 - Multilayered shell finite element model DMTS

4.2 8-node solid element SFR

The new 8-node solid element SFR [7] (figure 6) is based on a kinematical concept of a space fiber rotation. This concept allows to improve the accuracy of the displacement field {U}, getting a higher element with comparison to the 20-node quadratic hexahedral element.

$$\{U\} = \sum_{i=1}^{8} N_i \{u_i\} + \sum_{i=1}^{8} N_i \vec{q}_{z_i} \wedge (\vec{x} - \vec{x}_i); \quad \vec{q}_{z_i} = q_{z_i} \vec{k} \quad and \quad i = 1....8 \tag{7}$$

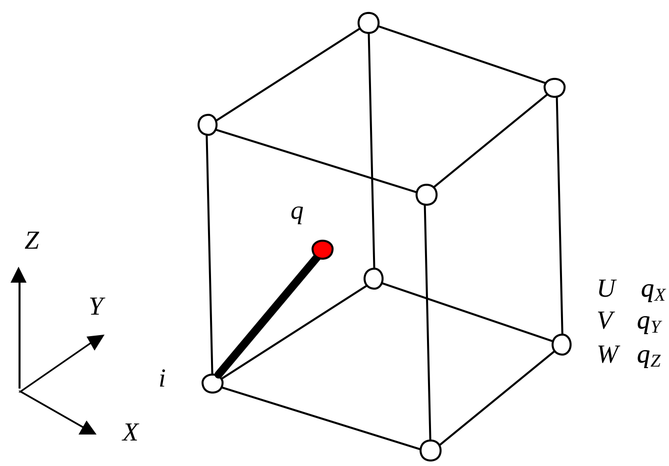


Figure 6 – Hexahedral 8-node Space Fiber Rotation model SFR

5 NUMERICAL VALIDATION

5.1 Cardboard sandwich (3 points bending)

The specimens of the 3-points bending tests do not require any particular preparation. Their dimensions are 200 x 60 x 4.01 mm. The span value is 160 mm giving the span-to-thickness ratio (L/h) equal to 39 minimizing therefore the shearing effect. The specimens are conditioned at 23°C and 50% RH for at least 24h.

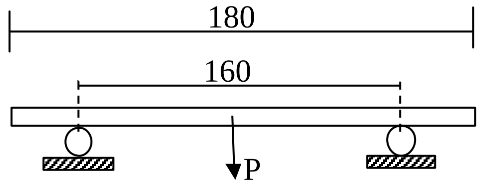


Figure 7 - Three points bending test

The FE equivalent shell model consists of 1-layer shell elements with material properties [8] defined by the equivalent orthotropic values. The overall dimensions of the models are the same with respect to the actual geometry models, and the same boundary condition is used.

H (mm)	E_1 (MPa)	E_2 (Mpa)	N_{12}	G_{12} (Mpa)
4.01	719.44	390.53	0.24	203.11

Table 3 – Homogenized material properties of the corrugated cardboard

The central normalized value P/WC obtained from the present elements, together with those from experimental solution given [8] are presented in Table 4. It is obvious that the result from element DMTS are in good agreement with the experimental.

L/h	$\Delta_{\exp}$	$\Delta_{num} = P/W_{num}$	%
20	22.798	20.118	11.75

Table 4 – Results of the corrugated cardboard bending rigidity

5.2 Stitched sandwich (4 points bending)

The performance of the SFR element is tested on a homogeneous orthotropic beam. The length of the specimen was 550 mm length 50 mm width and 22 mm high. The outer span was 440 mm while the inner load span was 220 mm, see figure 8. It's analyzed with a mesh of 1x2x50. The SFR solution compared with H8 element solution is in good agreement with experimental data (table 5). The error on the bending module is less then 8%

	W(mm)	P/w N/mm)	*E_f (N/mm²)	%
SFR	6.2484	128.033	4301.901	7.81
H8	5.2909	151.203	5080.420	8.86
Experiments		138.89	4666.704	

Table 5 – Results of the stitched sandwich bending rigidities.

$$^{*}E_f = \frac{0.21L^3}{be^3}\frac{P}{W}$$

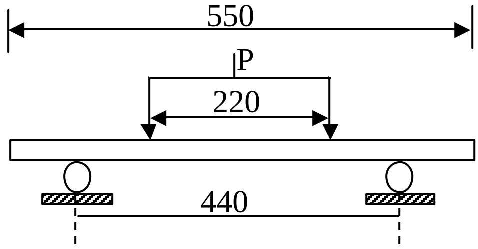


Figure 8 - Four point bending test

6 CONCLUSIONS

Our research and development is intended to contribute developing a fast and accurate tool for computing the homogenized behavior for a given periodic sandwich structures. Both analytical model and finite element predictions, for corrugated cardboard and stitched structure correlate reasonably well with experimental results.

The results of the two simulations using the new finite element are in good agreement with previous analytical and experimental and should increase modeling efficiency and diminish the numerical errors.

REFERENCES

[1] Z. Aboura, N. Talbi, S. Allaoui, A. Benzeggagh, Elastic behavior of corrugated cardboard: Experimental and modeling. Composite Structures 2004; 63:53-62.

[2] Ishikawa T, Chou TW. Stiffness and strength behavior of woven fabric composites. Jour. of Mat. Sci.. 1982, 17, 3211-3220.

[3] Aboura, Z., Etude du processus de délaminage Mode I, Mode II et Mode mixte (I et II) de matériaux composites à renforts tissés à différentes vitesses de sollicitation, Thèse de Doctorat, Université de Technologie de Compiègne 1993.

[4] Aboura, Z., Chouchaoui, C.S. and Benzeggagh, M.L., Analytical model of woven composite laminate. Superposition effect of two plies, ECCM 6, 1993, Bordeaux.

[5] Scida, D., Aboura, Z., Benzeggagh, M.L. and Bocherens, E., Prediction of elastic behavior of hybrid and non-hybrid woven composite, Composites, science and technology. 1997, 57, 1727-1740

[6] R. Ayad, N. Talbi, Un nouvel élément fini de coque triangulaire à 3 nœuds pour les coques composites multicouches, 6ème Colloque National en Calcul des Structures, Giens - France 2003.

[7] R. Ayad, A 3D 8-node solid finite element based upon the Space Fiber Rotation concept, Proceedings of the 16th French Mechanical Conference,1-5 September 2003, Nice, France.

[8] N. Talbi, R. Ayad, Z. Aboura, S. Allaoui, A. Validation du modèle DDM multicouches homogénéisé pour le carton ondulé à travers un test de flexion à trois points. 20th Canadian Congress Of Applied Mechanics, McGill University, Department Of Mechanical & Manufacturing Engineering, May 30th To June 2nd 2005, Montreal, Canada.