Local and global instabilities in the wake of a sphere

Benoît Pier

Local and global instabilities in the wake of a sphere

Benoît PIER*

The wake of a sphere is known to display the following behaviour at moderate Reynolds numbers: the steady axisymmetric wake is stable for $\mathrm{Re} < \mathrm{Re}_1 \simeq 210$; a steady planar-symmetric flow is observed for $\mathrm{Re}_1 < \mathrm{Re} < \mathrm{Re}_2 \simeq 270$; at Re_2 a periodic régime takes over with a vortex shedding Strouhal frequency of $\mathrm{St} \simeq 0.14$ near onset.

The present investigation revisits this flow and compares the global behaviour, reproduced by direct numerical simulations, to the local stability characteristics, computed for the basic flow under a quasi-parallel flow assumption. It is shown that the time-periodic régime may be interpreted as a nonlinear global mode and its naturally selected frequency derived from local stability considerations.

For a given Reynolds number, the local linear stability characteristics are determined using the corresponding basic wake flow, defined as the time-independent solution of the Navier–Stokes equations. The axisymmetric and non-axisymmetric base flows are obtained by direct numerical simulation or, when they are gobally unstable, by a Newton–Raphson iterative scheme.

After computation of these time-independent basic flow fields, the local linear dispersion relation is derived from two-dimensional eigenproblems based on the velocity fields prevailing at each streamwise station z. The essential feature of the frequency selection mechanism is the complex local absolute frequency $\omega_0(z)$ shown in Figure 1 for $\mathrm{Re} = 300$. According to the theory of "elephant global modes"[1], a self-sustained nonlinear oscillating structure is triggered by a front at the transition station z^{ca} from convective ($\omega_{0,i} < 0$) to absolute ($\omega_{0,i} > 0$) local instability. This front acts as a wave-maker and imposes its frequency on the entire system. The front frequency equals the (real) local absolute frequency $\omega_0^{ca} = \omega_0(z^{ca})$ prevailing at its location. At $\mathrm{Re} = 300$, this frequency selection criterion yields a prediction of $\mathrm{St} = \omega_0^{ca}/2\pi \simeq 0.17$, in reasonable agreement with the observed $\mathrm{St} = 0.14$. Bearing in mind that the present flow is rather far from parallel, one may consider that this establishes the validity of the proposed underlying frequency selection mechanism.

*LMFA, École centrale de Lyon, 36 avenue Guy-de-Collongue, F-69134 Écully.
[1] Pier et al. *Physica D*, **148**, 49 (2001).

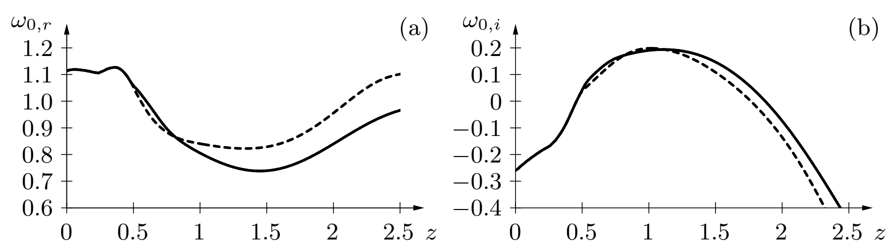

Figure 1: (a) Real and (b) imaginary parts of local absolute frequency $\omega_0(z)$ computed for the two most unstable modes of the non-axisymmetric basic wake flow at $\mathrm{Re} = 300$ for a sphere of radius 0.5 with center at $z = 0$.