Study and experimentation of a rotor current excited vernier reluctance machine aimed to direct-driven applications

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Abstract. The present paper deals with the study and experimentation of a rotor current excited vernier reluctance machine. First, the structure is introduced and the conditions on the teeth and the polarity numbers necessary for the smooth running of such a structure are presented. Then, a simple procedure is given, based on some simplifying hypotheses, to obtain the initial geometrical and electrical sizes of a designed prototype. To validate the operating principle of the structure, a prototype is designed and studied using 2D FEM. Finally, this prototype was constructed and simulation results were compared to those obtained from the prototype.

PACS. 84.50.+d Electric motors - 84.60.Bk Performance characteristics of energy conversion systems; figure of merit

1. Introduction

Recently, growing interest has been paid to electromechanical structures designed for used in direct drive applications. Direct drive applications is a general expression which embodies different fields. Hence, it covers structures at very high speed [1,4], structures with multiple degrees of freedom (MDoF) [5,8] as well as structures ensuring high torque, or high force, at very low speed [9-27]. Therefore, many studies have dealt with original structures or with the improvement and the optimisation of known machines for specific uses.

Applications involving high torque/force at very low speed are numerous, with chemical mixers, lifts, naval propulsion and wind energy conversion being the most known. To implement such applications, the simplest way is obviously the use of classical machines supplied at a very low frequency. However, this is neither practical nor efficient. Another solution, which has given rise to numerous studies, is to increase the pair pole number of classical machines while supplying them with standard frequency [16,17]. This possibility takes advantages of the design and control know-how used in the common electrical machines but essentially suffers from an important increase in the diameter of the structure which induces a decrease of the power/mass ratio. To limit this drawback, attempts have been made to use PM axial flux machines [18,19]. This solution is very attractive but it is extremely sensitive to the uniformity of the air gap width. Moreover, mechanical problems involving axial forces on the moving part can occur which makes the use of such structures very sensitive.

The use of the air gap reluctance variation in the electromagnetic energy conversion has been known for decades. In the field of electrical energy generation, different studies have been carried out. In the field of wind energy conversion, researches dealt with the use of the doubly fed synchronous reluctance machine to convert the wind energy into electrical energy [20-22]. This structure is very interesting for this application in that it introduces two supplementary degrees of freedom, namely the magnitude and frequency of the control winding current. The power of the control winding being significantly lower than that of the armature winding, the whole device operates in an efficient and well adapted manner. However, as the rated speed of such structures is relatively high, the use of a gearbox can not be avoided. Therefore, it is not possible to use this structure in low speed direct drive applications.

The vernier reluctance machine can constitute an appropriate alternative structure to replace either classical machines at standard speed coupled to mechanical gearboxes, or machines with a high pole pair number. Due to its inherent electromagnetic conversion principle, the vernier reluctance structure can achieve high torque at low speed while avoiding an increase in the armature pair pole number. It can then constitute a solution while remaining maintenance free. The main drawback of these machines is their very limited power factor which leads to an oversizing of the static converter intended for their supply.

To overcome this problem, solutions using a second excitation circuit have been proposed using either permanent magnets or current excitation circuits [23-26]. In both cases, the operation of these machines is similar to that of smooth rotor synchronous machines with low torque ripples. Different topologies have been studied using magnets located either in the stator or the rotor [23-26]. Utilising the difference between the pair pole numbers of the armature winding and the excitation permanent magnets, the studied structures can generate high torque (or force) with low speed while yielding very favourable performance. However, the use of PMs necessitates a degree of care and does not permit degrees of freedom to be used in the control operation. Furthermore, care must be taken during no load operation. To avoid this limitation, structures with current excitation circuits can be used [27]. As in the case of permanent magnets, the circuit can be located either in the stator or the rotor. Furthermore, it can be supplied by a DC current or by a balanced polyphase current, leading to the introduction of two supplementary degrees of freedom, which can be used in the control of the machine. Previous work [27] concerned a structure excited by a stator circuit. It has been shown that despite the benefit of the excitation circuit location, this solution induces a high saturation level which decreases the performance of the machine.

In this paper, our aim is to study a vernier reluctance machine endowed with a polyphase circuit in the rotor which

fulfils the excitation, or control winding, function **in order to validate its principle through an experimental prototype**.

In the first part, we introduce the rotor excited vernier reluctance machine and an analytical model which permits to obtain the necessary relations between the teeth and the polarities in order to perform synchronous running.

In the second part, we describe a **basic** procedure, based on an analytical approach, to design an initial prototype of an excited vernier reluctance machine.

The last part is devoted to the study of a designed prototype using a numerical model based on the 2D FEM. Results at no load are given and the similarity with smooth synchronous machine operation is shown. These results are then compared to measurements obtained using the constructed prototype. Other numerical results at load are also presented and discussed.

2. The Vernier reluctance machine

The basic generic structure of a vernier machine is shown in figure 1. There are N_s teeth in the stator and N_r teeth in the rotor. The three phase distributed armature winding, of pole pair number p_a , is located in the stator. The excitation circuit is made up of another three phase distributed winding, of $2p_e$ poles, located in the rotor.

Fig. 1. The basic structure of a doubly slotted variable reluctance machine.

To determine the relation between N_s , N_r , p_a and p_e which allows synchronous operating of the vernier machine, we use an energetic model based on the expression of the magnetic energy in the structure under the hypothesis of a **very high permeability** of the iron. Even if this hypothesis is very strong in the case of vernier machines, the model in question, which is not intended to quantify the performance of the machine, is sufficient to determine the required relation.

Let θ_s (resp. θ_r) be the position of a given point M in the air gap with respect to the stator (resp. rotor) frame. θ_s and θ_r are then linked by the following relation:

$$\theta_s - \theta_r = \theta \quad (1)$$

where θ is the angle between the rotor and the stator frames.

The magnetic coenergy in the vernier machine, which is stored solely in the air gap, can be expressed as follows:

$$W_{em} = \frac{1}{2} \int_0^{2\pi} (V_a + V_e)^2 \cdot P(\theta_s, \theta) \cdot d\theta_s \quad (2)$$

where V_a and V_e are the magnetomotive forces (m.m.f) due to the armature and excitation circuits respectively and P is the air gap permeance function.

The m.m.f. due to a three phase armature winding, supplied by balanced currents of magnitude I_{amax} and pulsation ω_a , can be expressed by:

$$V_a = \sum_{i=0}^{\infty} \left[\frac{3}{\pi} \frac{n_a I_{a \max}}{(2i+1)} (-1)^i k_{aB}^i \right] \cos(\omega_a t - (2i+1) p_a \theta_s) \quad (3)$$

where k_{aB}^i denotes the winding coefficient of the circuit and n_a is the conductor number per slot. In the same manner, the three phase excitation m.m.f. can also be expressed as:

$$V_e = \sum_{i=0}^{\infty} \left[\frac{3}{\pi} \frac{n_e I_{e \max}}{(2i+1)} (-1)^i k_{eB}^i \right] \cos(\omega_e t - (2i+1) p_e \theta_r) \quad (4-a)$$

$$V_e = \sum_{i=0}^{\infty} \left[\frac{3}{\pi} \frac{n_e I_{e \max}}{(2i+1)} (-1)^i k_{eB}^i \right] \cos(\omega_e t - (2i+1) p_e (\theta_s - \theta)) \quad (4-b)$$

where k_{eB}^i and n_e are respectively the excitation winding coefficient and the conductor number per slot. $I_{e \max}$ and ω_e represent respectively the magnitude and the pulsation of the excitation balanced currents.

The air gap permeance function is given by the following relation [28]:

$$P(\theta_s, \theta) = P_0 + \sum_{j=1}^{\infty} P_{js} \cos(j N_s \theta_s) + \sum_{m=1}^{\infty} P_{mr} \cos(m N_r (\theta_s - \theta)) \quad (5)$$

$$+ \frac{1}{2} \sum_{j=1}^{\infty} \sum_{m=1}^{\infty} P_{js-mr} \cos[j N_s \theta_s + \varepsilon m N_r (\theta_s - \theta)]$$

ε is a coefficient whose value is equal to 1 or -1 and P_0 , P_{js} , P_{mr} and P_{js-mr} are coefficients whose values depend on geometric characteristics (see appendix).

The expression of the coenergy is then obtained by inserting (3), (4) and (5) into relation (2). This yields a complex development involving the different harmonics. In order to simplify this development, initially only the first terms of (3), (4) and (5) are considered. Therefore, the air gap electromagnetic coenergy can be rewritten as follows:

$$W_{em} = \frac{1}{2} \int_0^{2\pi} \left[V_{a\max} \cos(\omega_a t - p_a \theta_s) + V_{e\max} \cos(p_e (\theta_s - \theta)) \right]^2 \\ \left[P_0 + P_{1s} \cos(N_s \theta_s) + P_{1r} \cos(N_r (\theta_s - \theta)) + \frac{1}{2} P_{1s-1r} \cos((N_s + N_r) \theta_s - N_r \theta) \right. \\ \left. + \frac{1}{2} P_{1s-1r} \cos((N_s - N_r) \theta_s + N_r \theta) \right] d\theta_s \quad (6)$$

From the electromagnetic coenergy, we can express the electromagnetic torque under the following form:

$$T_e = \frac{\partial W_{em}}{\partial \theta} \Big|_{at\ constant\ current} \quad (7)$$

Thus, to yield an electromagnetic energy conversion, the coenergy must obviously be nonzero and must also depend on the rotor position θ . The development of expression (6) yields two different parts. Only one, W_{em-int} , represents the interaction between both magnetic fields through the final term of the air gap permeance. Its expression is given below:

$$W_{em-int} = A \\ \int_0^{2\pi} \left\{ [\cos((\omega_a + \omega_e)t - (p_a + p_e - N_s - N_r)\theta_s + (p_e - N_r)\theta)] \right. \\ + [\cos((\omega_a - \omega_e)t - (p_a - p_e - N_s - N_r)\theta_s - (p_e + N_r)\theta)] \\ + [\cos((\omega_a - \omega_e)t - (p_a - p_e + N_s + N_r)\theta_s + (-p_e + N_r)\theta)]_s \\ + [\cos((\omega_a + \omega_e)t - (p_a + p_e - N_s + N_r)\theta_s + (p_e + N_r)\theta)] \\ + [\cos((\omega_a + \omega_e)t - (p_a + p_e + N_s - N_r)\theta_s + (p_e - N_r)\theta)] \\ + [\cos((\omega_a - \omega_e)t - (p_a - p_e - N_s + N_r)\theta_s + (-p_e + N_r)\theta)] \\ \left. + [\cos((\omega_a - \omega_e)t - (p_a - p_e + N_s - N_r)\theta_s - (p_e + N_r)\theta)] \right\} d\theta_s \quad (8)$$

where $A = \frac{1}{8} V_{e\max} V_{a\max} P_{1s-1r}$.

Expression (8) is non zero, and thus a function of the mechanical displacement θ , if the terms under the integral are independent of θ_s . Only one term of the seven given in expression (8) fulfils this function if the following relation is respected [29, 30]:

$$|N_s + \varepsilon N_r| = |p_a + \varepsilon p_e| \quad (9)$$

This strict condition which links the teeth and polarity numbers allows the term independent of θ_s to yield an electromagnetic energy conversion. Thus, according to relation (7), W_{em-int} generates an electromagnetic torque whose expression is given by:

$$T_e = (N_r - p_e) \frac{\pi}{4} V_{a\max} \cdot V_{e\max} \cdot P_{1s-1r} \cdot \sin[(\omega_a + \varepsilon \omega_e)t + \varepsilon (N_r - p_e)(\theta - \theta_0)] \quad (10)$$

where θ_0 is a constant angle. This torque is constant when the rotor rotates at the following angular speed:

$$\Omega = \frac{\omega_a + \varepsilon \omega_e}{\varepsilon (N_r - p_e)} \quad (11)$$

which constitutes the synchronous speed of the structure.

In addition to the interaction between both magnetic fields, respecting condition (9) also allows the electromagnetic energy conversion of other parts of W_{em} . The most significant are the additional torque contributions due to the modulation of either the armature or the excitation m.m.f. by the air gap reluctance, which constitute the main electromagnetic conversion in the case of non excited vernier reluctance machines. However, even if they constitute secondary contributions with regard to the one arising from W_{em-int} , these contributions have to be avoided because they are not constants for the specific speed given in (11) and thus induce torque ripples.

Consequently, in order for the machine to operate using only interaction between the excitation and armature fields, besides condition (9), other secondary conditions must also be respected [31], notably:

$$|N_s + \varepsilon N_r| \neq 2p_e \neq 2p_a \quad (12-a)$$

$$|N_s + \varepsilon N_r| \neq p_e \neq p_a \quad (12-b)$$

The excited vernier machine thus operates in a manner similar to that of a smooth rotor synchronous machine with low torque ripples. Only the mutual inductances between both windings are sinusoidal with respect to the rotor position, while all the other inductances are constant.

3. Design of a prototype

In this part, we give a basic procedure to design a given prototype in terms of right polarity topology and electromagnetic design. The purpose of this procedure is only to reach reasonable sizes of a prototype in order to experimentally validate the operating principle of the proposed structure. Thus, the thermal part is not treated in depth and is limited to the use of conventional thermal constraints for sizing conductors. Obviously, to test the performance of the structure, a more detailed design, coupled with an optimization procedure would be necessary. This is not the aim of this paper which is limited to study the electromagnetic feasibility of the designed prototype.

Hence, to basically design such a rotor current excited vernier machine, we have to firstly choose the right combination of N_s , N_r , p_a and p_e . Thus, using the desired synchronous speed of the prototype, the frequency of the stator armature coils and an upper limit on the frequency of the excitation coils, equation (11) gives values for $(N_r - p_e)$

Subsequently, using relations (10) and (12) while respecting the constraints which link N_s to q_a and N_r to q_e , and the slot numbers per pole and per phase (i.e. $N_s = 6 p_a q_a$ and $N_r = 6 p_e q_e$), we obtain many solutions for N_s , N_r , p_a , p_e , q_a and q_e .

Adding the fact that all these variables are integers and the difference between the stator and rotor teeth must be as low as possible in order to obtain a smooth sinusoidal air gap permeance, we deduce the values of the teeth and pole numbers.

An initial mechanical dimensioning can then be obtained from a classical value of the potential electromagnetic pressure on the surface of the rotor (1 to 2 N/cm²). Using this approach, we obtain the standard value of $D_r^2 L_a$ (D_r is the outer rotor diameter and L_a is the active length) which characterizes the active volume of a machine.

To calculate D_r and L_a separately, we have to fix one of the variables. In the case of classical structures, this is achieved by taking account of constraints on the inertial moment or on the peripheral speed. For the structure studied, no such constraint is imposed. We can thus use the criterion which involves taking a ratio of the active length on the polar step (L_a/τ) of 1.5. This criterion leads to a decrease in the length of the end windings [32].

To finish the design of the prototype, we have to compute the m.m.f of the armature and excitation circuits. This can be obtained from the desired values of e.m.f and torque while respecting the maximal linear current density in order to ensure the machine remains cooled. The latter criterion is also used to determine the depth of the rotor and stator slots.

The e.m.f induced in an armature winding, at no load, is linked to the flux linkage ϕ_s by the relation $e_s = -d\phi_s/dt$. In the case of the studied machine, the pair pole numbers between the armature and excitation circuits are not the same. Thus, the relations commonly used in the classical machines are not valid. Based on this fact, we can express ϕ_s using the following relation:

$$\phi_s = \int_0^{2\pi} V_e(\theta_s, t) F_a(\theta_s) P(\theta_s, \theta) d\theta_s \quad (13)$$

where F_a is the winding distribution of the armature coil, whose expression is:

$$F_a(\theta_s) = \sum_{i=1}^{\infty} \left(\frac{2}{\pi} \right) \frac{k_{aB}^i}{i} n_a \cos \left(i p_a \theta_s + \frac{k 2\pi}{3} \right) \quad (14)$$

where $k = 0, 1$ or 2 and denotes the armature phase.

Substituting the first harmonics of (4), (5) and (14) into (13) leads to the following expression of the e.m.f. induced in an armature phase:

$$E_s = \frac{1}{2} \omega_a P_{ls-lr} k_{aB}^1 n_a V_e \cos(\omega_a t - \alpha) \quad (15-a)$$

Then, by expressing P_{ls-lr} as $P_{ls-lr} = \frac{2}{\pi^2} \mu_0 D_r L_a$ (see appendix), we yield the following expression for the no load e.m.f.

$$E_s = \omega_a \frac{3}{\pi^3} \frac{\mu_0 D_r L_a}{e} k_{aB}^1 n_a k_{eB}^1 n_e I_{emax} \cos(\omega_a t - \alpha) \quad (15-b)$$

Conversely, equation (10) gives the maximal value of the electromagnetic torque at synchronous speed:

$$T_e = (N_r - p_e) \frac{\pi}{4} V_{amax} \cdot V_{emax} \cdot P_{ls_lr} \quad (16-a)$$

Which becomes, when substituting V_{amax} , V_{emax} and P_{ls-lr} by their expressions, as:

$$T_e = (N_r - p_e) \frac{9}{2\pi^3} \frac{\mu_0 D_r \cdot L_a}{e} k_{aB}^1 n_a I_{a\max} \cdot k_{eB}^1 n_e I_{e\max} \quad (16-b)$$

It is clear that knowledge of the expression of the couple would lead to that of the e.m.f. This can also be verified through expression (15) and (16).

Using these two expressions, combined with the equalization of both equivalent m.m.f. to optimize the energy conversion, we then get the values of the magnetomotive forces from the desired magnitude of the e.m.f. and torque. Thus, by choosing the magnitude of both armature and excitation currents, we can determine the conductor numbers.

The dimensioning approach proposed above only enables us to find the initial values of the design. To refine the design by taking account, among other factors, of the saturation effects, numerical analysis has to be carried out in order to quantify the performance of the virtual prototype with regard to the expected analytical results.

4. Study of a prototype

4.1 The designed prototype

According to the design procedure presented, we looked at a prototype with two three phase windings, located in the stator and rotor. This prototype is designed for a rated speed of 50 rpm and a rated torque of 1000Nm. The frequency of the armature supply is fixed at $f_o=50\text{Hz}$ and that of the excitation circuits is chosen to be the lowest possible, i.e. $f_e=4.16\text{Hz}$. The choice of the armature variable frequency is motivated by the fact that the stator windings can be connected directly to the network. The control would then be achieved through the frequency and magnitude of the excitation currents. Previous work related to a stator excited vernier reluctance machine showed that this could be possible [27].

In table I, we summarize the main features of the designed prototype.

TABLE I
THE CHARACTERISTICS OF THE PROTOTYPE

$V_a=220V$	$I_a=12A$	$f_a=50\text{ Hz}$
$V_e=220V$	$I_e=2.5A$	$f_e=4.16\text{Hz}$
$N_s=60$	$p_a=5$	$q_a=2$
$N_r=66$	$p_e=11$	$q_e=1$
$D_s=360\text{mm}$	$L_r=260\text{mm}$	$e_{min}=0.4\text{mm}$

Hence, in this prototype, relation (3) is confirmed by:

$$N_r - N_s = p_e - p_d \quad (17)$$

and the synchronous rotation speed Ω is given by:

$$\Omega = \frac{\omega_a - \omega_e}{N_r - p_e} \quad (18)$$

In figure 2, a zoom of the cutting plane of the designed prototype is presented. We can see the arrangement of both armature and excitation windings. 2 slots per pole and per phase are chosen for the armature windings, whereas; due to the high pair pole number, only one slot per pole and per phase is chosen for the excitation circuit. The rotor slots are relatively deep in order to include the necessary conductors

Fig 2: A cutting plane view of the prototype designed

Furthermore, to be able to study accurately the magnetic behaviour of the machine, we added flux sensors located in the stator. They are made up of coils surrounding stator teeth.

The designed prototype was constructed. In figure 3a, we show the rotor of the structure. Fig. 3b shows the stator with a zoom where we can see the flux sensors (Fig. 3c).

Fig 3: The rotor and stator of the constructed prototype

To study the operating of the prototype, an experimental bench was constructed. We can see, in figure 4, the constructed prototype which is mechanically coupled to a DC machine through a mechanical gearbox. According to the operating of the prototype, the DC machine can run as a speed controlled motor supplied through a chopper or operate as a generator supplying a resistive load.

Fig 4: The experimental set up

4.2 Study at no load

First, the designed prototype was characterised using 2D-FEM. Figure 5 shows the field distribution, at no load, of a cross section of the prototype when the excitation circuit is supplied.

We can clearly notice the effect of the air gap variable reluctance. Indeed, given that the excitation circuit is made up of 11 pair poles, in the case of a machine with a constant air-gap, the field distribution should be made up of 11 poles of the same magnitude. However, in Figure 5, we can see that the magnitude of the poles is different along the diameter, this being due to the air gap variable reluctance.

Fig 5: Field distribution in a cross section of the studied machine

Figure 6 presents the waveforms of the teeth flux calculated by FEM and given by the flux sensor of the constructed prototype when the rotor is moved at the rated speed while the excitation circuit is supplied by the rated DC value. As the sensors are located in the stator, we can notice the effect of the rotor slots. The magnitude of the signal is different due to the accuracy of the $B(H)$ curve used to take into account the non linear behaviour of the magnetic material in the FEM. Nevertheless, the frequencies of the harmonics are relatively well quantified by the numerical modelling. It may be notice

that the waveforms presented are related to a tooth flux. The slot harmonics are not noticeable on the phase flux as it contains the flux of different teeth which are displaced amongst themselves.

Fig 6: Teeth flux versus time rated speed and DC excitation supply

At no load and the rated speed (50rpm), figures 7a and 7b show the emf induced in an armature phase of the machine with regard to time when the excitation circuit was powered with a DC current (fig. 7a) and a balanced three phase current with the rated frequency (fig. 7b).

Fig 7: The emf in an armature phase at $N=50\text{rpm}$ with a DC excitation (a) and a three phase excitation current of frequency $f_e=4.16\text{Hz}$ (b)

In both cases, the waveforms are quite sinusoidal. Only the frequencies are different ($f = 45.83\text{Hz}$ for a DC excitation and $f = 50\text{Hz}$ for the three phase excitation).

The internal characteristic $E(I_{exc})$ of the prototype was determined both in the case of DC and AC three phase excitation currents. In the case of a DC excitation current, the control windings are connected as shown in figure 8, while they are star connected for a balanced three phase excitation.

Fig 8: The control winding connection in the case of a DC excitation current.

In figure 9, we present a comparison between the calculated and measured values of the induced emf with respect to the magnitude of the DC excitation current. This comparison is identical to the case of a three phase excitation. The 2D model obviously overestimates the emf magnitude. This is due to the accuracy of the $B(H)$ curve approximation and the real minimal value of the prototype air gap.

Fig 9: The induced emf with respect to the DC excitation current magnitude or the rms three phase excitation current.

Furthermore, the active length of the machine is limited, inducing leakage flux which is not taken into account in a 2D-FEM analysis. However, the waveforms are close and correspond to the specific characteristic of current excited vernier reluctance machines, i.e. the induced emf reaches a maximal value for an excitation current I_{excmax} and subsequently decreases [27, 33]. This can be explained by considering the saturation effects. From I_{excmax} , the teeth are saturated and the equivalent air gap becomes larger. Therefore, the higher the current, the greater the saturation effect and the larger the equivalent air gap. This leads to a decrease of the average value of the permeance and also to the decrease of the magnitude of the coupling terms, which leads to a decrease in the e.m.f. magnitude.

Other calculations and measurements were carried out in order to identify the inductances of the prototype. As pointed out above, the operation of this structure can be considered as similar to that of a smooth rotor synchronous machine with an electromagnetic gearbox. Therefore, both armature and excitation winding inductances are independent of the rotor position. Thus, no electromagnetic conversion arises directly from the variation of the air gap reluctance.

In figure 10, we present the waveforms of the calculated mutual inductances, with respect to the rotor position. As expected, they constitute a three phase system. Given that the neutral of the excitation circuit was not accessible, we could not measure these mutual inductances. In the case of the self and mutual inductances of the armature windings, their invariance with respect to the rotor position was verified both by numerical calculations and measurements [27].

Fig 10: The mutual inductances (H) between armature and excitation circuits calculated by FEM

In table II, we give a comparison between the values of the different inductances and the rated emfs obtained by the analytical approach, the fem analysis and the measurements.

The analytical values are obtained from the design approach while the fem values are calculated from the linkage and mutual fluxes when a phase winding is supplied by a DC current. In the case of measurements, the self inductances are obtained from impedance calculations for a low current magnitude and the mutual inductances are deduced from the induced emf when the rotor is moved at the rated speed and a phase winding is supplied by a DC current.

The analytical values are overestimated as the non linearity of the magnetic material is not taken into account in this approach. Moreover, as introduced above, given that the neutral of the excitation circuit is not accessible, we only measured the difference between the self and mutual inductances of the excitation circuit ($L_e - M_e$) of the machine.

TABLE II
COMPARISON BETWEEN ANALYTICAL, FEM AND EXPERIMENT VALUES

	Analytical	Fem	Experiment
L_a	0.18 H	0.138 H	0.125 H
M_a	-0.09H	-0.05H	-0.068H
L_e	7.1 H	5.2 H	
M_e	-3.5H	-1.5H	

$L_e - M_e$	$10.6H$	$6.7H$	$6.4H$
M_{aemax}	$0.36 H$	$0.24 H$	$0.22H$
E_{max}	$250 V$	$210 V$	$182V$

The values obtained by FEM and those measured are relatively close. The inductances related to the control winding have high values due to the high number of conductors.

4.3 Study at load

The studies at load are carried out using the numerical modelling. Thus, calculations for both excitation cases were performed. The goal of these studies is to quantify the torque ripples at load and to determine the waveform of the electromagnetic torque with respect to the load angle.

For both excitation cases, the simulations are carried out, at synchronous speed, when the armature circuit is supplied with the rated voltages at the rated frequency. Then, the torque is determined using Maxwell's stress tensor.

In figure 11a, we show the electromagnetic torque calculated when the excitation circuit is supplied with the rated DC current. *As the mechanical equation is not taken into account, the initial simulation conditions are the following: The excitation windings are supplied by their rated currents and the rotor is moving at its rated speed. The rated armature voltages are then applied at the stator windings. This is obviously not a physical phenomenon which explains the numerical transient on the torque waveform. Once the transition is completed, the obtained temporal torque waveform corresponds to the steady state operating with a stator voltage supply.* Fig. 11b presents a zoom on the calculated torque steady state waveform.

Fig. 11. Electromagnetic torque with a DC excitation

We can notice that the torque ripples are relatively low with regard to the rated value.

The same operating point was studied when the control winding is supplied by a three phase excitation with the rated magnitude and frequency. Figures 12 give the results obtained.

As in the previous case, Fig. 12a shows also a numerical transient and the steady state related to the motor operating. On the zoom of the electromagnetic torque waveform at steady state (Fig. 12b), we can see that the torque ripples are slightly higher than in the case of the DC excitation with much more harmonics due to the excitation frequency. However, their magnitudes remain very low related to the rated value.

Fig. 12. Electromagnetic torque with a three phase excitation

Hence, in both excitation cases, the electromagnetic torque at steady state is relatively smooth.

In order to determine the performances of the prototype and to quantify the maximal electromagnetic power it can generate, we performed several calculations, at synchronous speed, when the excitation and armature windings were powered with the rated currents. As the mechanical equation is not taken into account in the 2D-FEM software, we carried out the calculations while varying the shift phase δ of the armature currents. Therefore, the outlined characteristic $T(\delta)$, corresponds to the waveform of the electromagnetic torque with regard to the load angle. Furthermore, it allows us to determine the maximal generated torque.

Fig. 13. Electromagnetic torque with respect to angle δ

In figure 13, we can see the waveform $T(\delta)$ obtained for the designed prototype. It is similar to that of a classical smooth rotor synchronous machine and shows that the prototype can generate about 800Nm, as opposed to the 1000Nm predicted by the design specifications. This can be explained by the fact that the design procedure is based on an analytical approach which does not take into account the saturation effect. Therefore, in the case of such a machine, an iterative procedure has to be used in order to adapt the geometric and electric parameters to obtain the desired mechanical characteristics

4.4 Discussion

The obtained simulation results, whether at no load with a comparison to the measures, or at load, show that the studied structure has a similar operation to that of a smooth rotor synchronous machine. Both self and mutual inductances related to each of the armature and excitation windings are constant with regard to the rotor position. Only the mutual inductances between both windings are sinusoidal function of the rotor position and constitute a balanced three phase system.

The different values of the inductances show that the basic design procedure yields overestimated results with respects to the experiment. This is also the case of numerical simulations but to a lesser extent. However, for the teeth flux (Fig. 6), as well as the no load e.m.f. (Fig. 7) and the internal characteristic (Fig. 8), the simulated waveforms are close to the experimental ones and correspond to the expected assumptions of the analytical approach.

The simulation results related to the operating at load also show similarity with the operation of a smooth rotor synchronous machine. Either with a DC or balanced three phase excitation currents, the electromagnetic torque is relatively smooth even if the air gap permeance is highly variable.

The mass of the constructed prototype is of 272kg. This value is very high because of the need for rigid flanges to maintain a constant small air gap of 0.4 mm. Thus, the maximal torque/mass ratio would be of 3Nm/kg. Moreover, the maximal power factor obtained from the different simulation of the motor operating study does not exceed 0.6. These performances are, obviously, not very high and are far from the ones which can be expected using a permanent magnet structure. However, the design of the studied prototype was not performed to yield good performance but only to validate the operating principle of the proposed structure through an experimental prototype, which has been achieved.

5. Conclusion

In this paper, a prototype of a rotor current excited vernier reluctance machine was studied. This structure can yield high torque at low speed and operates in a manner similar to that of a smooth rotor synchronous machine with limited torque ripples. Furthermore, the two degrees of freedom (the magnitude and frequency of the excitation circuit) could be useful when implementing the control.

In the first step, the excited vernier reluctance machine was introduced and the combination between the teeth and the polarities necessary to yield a continuous energy conversion was presented using an analytical model based on the electromagnetic energy in the air gap of the structure. Given the specific nature of the structure, the procedure used to design classical machines can not be directly implemented. A procedure, based on the previous model, was thus given in order to design a prototype. This procedure can only yield initial values for the dimensions of this type of prototype as it does not take into account the saturation effect and the thermal design is very basic. It has obviously to be refined in the case of the design of an optimised prototype.

Using the presented procedure, a prototype was designed and studied using 2D-FEM software. Furthermore, the prototype was constructed and tested at no load operation. The results show that:

- 1- The analytical design procedure overestimates the performance by about 25% due to the fact that the non linearity of the magnetic material is not taken into account.
- 2- The FEM analysis leads to relatively accurate knowledge of the prototype performance. A 3D approach, coupled with a better B(H) curve approximation taking into account the mechanical constraints, would improve the accuracy of the results.
- 3- The variable reluctance of the air gap constitutes an essential parameter in the electromagnetic conversion of this structure. Therefore, the width of the air-gap and the geometry of the teeth of both armatures are very important. Their realization requires a high precision and care in order to obtain the expected performances.
- 4- The studied structure yields constant self inductances and even if the air gap reluctance is variable, the electromagnetic conversion arises solely due to the interaction between the excitation and armature fields.
- 5- Regardless of the excitation supply chosen, the structure presents very low torque ripples.

The studied machine can then be used in applications involving high torque at low speed, while also ensuring smooth running.

Future work will deal with other experimental tests in generator and motor operations of the prototype. Moreover, control strategies which take advantage of the two supplementary degrees of freedom will be investigated.

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Appendix

The air gap permeance can be written under the following form :

$$P(\theta_s, \theta) = P_0 + \sum_{j=1}^{\infty} P_{js} \cos(j N_s \theta_s) + \sum_{m=1}^{\infty} P_{mr} \cos(m N_r (\theta_s - \theta)) \quad (\text{A.1})$$

$$+ \frac{1}{2} \sum_{j=1}^{\infty} \sum_{m=1}^{\infty} P_{js-mr} \cos[j N_s \theta_s + \varepsilon m N_r (\theta_s - \theta)]$$

Where

$$P_0 = \mu_0 L_a R_r E_0 \quad (\text{A.2})$$

$$P_{js} = \mu_0 L_a R_r E_s e_j \quad (\text{A.3})$$

$$P_{mr} = \mu_0 L_a R_r E_r e_m \quad (\text{A.4})$$

$$P_{js-mr} = \mu_0 L_a R_r E_{s-r} e_j e_m \quad (\text{A.5})$$

R_r is the outer rotor radius, L_a is the active length and

$$E_0 = \frac{1}{e_{\min} + p_r + p_s} \left[1 + \frac{p_s}{e_{\min} + p_r} r_{ds} + \frac{p_r}{e_{\min} + p_s} r_{dr} + \frac{p_r \cdot p_s (p_s + p_r + 2e_{\min})}{(e_{\min} + p_s)(e_{\min} + p_r)e_{\min}} r_{ds} r_{dr} \right] \quad (\text{A.6})$$

$$E_s = \frac{p_s}{(e_{\min} + p_r + p_s)(e_{\min} + p_r)} \frac{2}{\pi} \left[1 + \frac{p_r (p_s + p_r + 2e_{\min})}{(e_{\min} + p_s)e_{\min}} r_{dr} \right] \quad (\text{A.7})$$

$$E_r = \frac{p_r}{(e_{\min} + p_r + p_s)(e_{\min} + p_s)} \frac{2}{\pi} \left[1 + \frac{p_s (p_s + p_r + 2e_{\min})}{(e_{\min} + p_r)e_{\min}} r_{ds} \right] \quad (\text{A.8})$$

$$E_{s-r} = \frac{p_r \cdot p_s (p_s + p_r + 2e_{\min})}{(e_{\min} + p_r + p_s)(e_{\min} + p_s)(e_{\min} + p_r)e_{\min}} \frac{4}{\pi^2} \quad (\text{A.9})$$

$$e_j = \frac{1}{j} \sin(j r_{ds} \pi) \quad (\text{A.10})$$

$$e_m = \frac{1}{m} \sin(m r_{dr} \pi) \quad (\text{A.11})$$

$$r_{ds} = \frac{l_{ds}}{l_{ds} + l_{es}} = \frac{l_{ds}}{\lambda_s} \quad \text{with} \quad \lambda_s = \frac{2\pi}{N_s} \quad (\text{A.12})$$

$$r_{dr} = \frac{l_{dr}}{l_{dr} + l_{er}} = \frac{l_{dr}}{\lambda_r} \quad \text{with} \quad \lambda_r = \frac{2\pi}{N_r} \quad (\text{A.13})$$

Where p_r and p_s represent the rotor and stator slot depths respectively, $e_{\min}$ is the minimal airgap thickness and l_{ds} and l_{es} (resp. l_{dr} and l_{er}) are the curvilinear length of the stator teeth and slot respectively (resp. the curvilinear length of the rotor teeth and slot).