Analysis of Slot-Pole Combination of Fractional-Slots PMSM for Embedded Applications

H. Dogan, *Student Member, IEEE*, F. Wurtz, *Member, IEEE*, A. Foggia, *Life Senior Member, IEEE*, L. Garbuio

Grenoble Electrical Engineering Laboratory (G2Elab) ENSE³ (Grenoble INP-UJF, CNRS UMR 5529) BP 46 – 38402 Saint Martin d'Hères Cedex, France

Abstract—During the design of permanent magnet synchronous machines (PMSM), the first step that engineers have to do, concern the choice of the topology, i.e. the slot/pole combination. Given that the performances of the machines are strongly related to this combination, the choice has to be fully justified. Fractional-slots (FS)-PMSM offers several advantages and have been focused by a lot of researches. However, depending on the topology, these machines could have major drawbacks as magnetic noises, high iron and eddy-current losses and low fault-tolerance. The aim of this paper is to give an overview of each machine considering the most common slot/pole combination in order to single out if a topology is suitable or not. A special focus is also done on the homopolar component which traduces the machine's fault tolerant capability required for embedded applications.

Keywords— *permanent magnets synchronous motor; fractional-slots; torque ripple; homopolar inductance; eddy-current losses; subharmonics.*

I. INTRODUCTION

Due to their numerous advantages, PMSM have found wide attention in the research and industrial areas. For instance, they own the highest torque density and efficiency among all types of motors. Moreover, thanks to their flux-weakening capability, interior PMSM allow wide speed operation with constant power making them suitable for traction and transportation systems [1].

The traditional designing of synchronous machines consist of topologies having an integer number of slots per pole per phase. The relation of the number of slots per pole per phase for these three-phase machines is written as

$$Q = \frac{N_s}{3 \cdot 2p} \quad (1)$$

where Q is the number of slots per pole per phase, N_s is the number of slots and p is the number of pole pair. These machines are well known and are also common in the market. However, they could have high value of torque ripple and low fault tolerant capability. In complement, during the recent years, the concentrated-windings PMSM have been the topic of interest for a lot of studies [2-5]. These types of machines have a fractional value of Q less than 1 which is traditionally

equal to 0.5. With the windings concentrated around each tooth, the physical, i.e. the electrical separation between phases is fulfilled. In addition, other configurations of concentrated-windings machines having coils joining the same phase around consecutive teeth allows obtaining magnetic separation between phases and thus a better fault tolerant capability. With this distinction, they are sometimes called “modular machines” [2]. Lastly, they are also well-matched for embedded applications [3], [4].

For FS-PMSM which the number of slot per pole per phase is higher than 1, the electrical and physical separation between the phases is not feasible. Therefore, not a lot of researches have been done on. However, these machines have the advantage of a very smooth torque and the magnetic separation is still possible. This is achieved if a short circuit on one phase doesn't impact the other phases. The meaning is that the mutual between phases have to be as close as possible to zero. Considering that the zero-sequence or homopolar inductance is equivalent to the self inductance minus the mutual, the magnetic separation will be perfect when the homopolar component is equal to the self component.

Ultimately, the purpose of this paper is to firstly study the main criteria that has to be taken into account for the right choice of the slot/pole combination in this context and secondly, to determine and evaluate the different candidate topologies in order to give some assumption to the designer. Then, the methodology for the evaluation of the FS-PMSM is given in section II and the results are presented in section III before discussion in the last part.

II. PERFORMANCE EVALUATION METHODOLOGY

For each application, there is a different specification. As a result, the selection criteria will have to be taken into account according to their respective significance. Usually, the main requirements for PMSM intended for embedded applications could be divided into three parts [5]:

- Ripple torque and radial forces
- Harmonics and subharmonics
- Zero-sequence (ZS) inductance ratio

A. Ripple torque and radial forces

There is a multitude amount of applications which requires a smooth torque in order to avoid mechanical vibrations, acoustic noises and troubles associated to the command [2], [6]. In the same way, a lot of researches have been done to diminish those undesirable effects as ripple torque and radial forces [7], [8].

There are three sources of torque ripple coming from the machine: a) cogging torque, b) difference between permeances of the air gap in the d- and q-axis (reluctance torque), and c) distortion of the magnetic flux density waveform in the air gap [6]. The cogging torque is related to the interaction between the permanent magnets located in the rotor with the stator teeth. Then, the slot/pole combination has direct effects on the value of this cogging torque. One way to lessen its value is to obtain the least common multiplier (LCM) between the slot number and the pole number the higher as possible. Evenly, the value of the LCM traduces the value of the first harmonic of the cogging torque. Thus, higher is the LCM, lower will be the cogging torque. Another criterion is the greatest common divider between (GCD) the slot and pole numbers. The GCD illustrates the balanced radial forces applied on the rotor. A high value of the GCD is synonymous of good balanced forces on the rotor. Likewise, a low value of the GCD indicates poor radial symmetry and unbalanced forces. Therefore, these forces could generate important magnetic noises. To avoid these negative effects, the GCD has to be an even number and also as high as possible [6].

B. Harmonics and subharmonics

As well known, the MMF distribution is never perfectly a sinus wave and harmonics waves which are asynchronous to the fundamental occur. These MMF harmonics induce losses in the iron part of the machine and also EMF harmonics in the windings. Then, one has to find the topology and the winding giving the lowest harmonic distortion. In general rules, the main winding factors as the 1st, 3rd, 5th and 7th order harmonics are chosen to decide if a topology is favorable or not. The 1st harmonic has to be as high as possible since it traduces the torque density of the machine. And the other harmonics have to be as low as possible.

However, those parameters are sufficient for PMSM having an integer number of slots per pole per phase. In the case of FS-PMSM, the MMF present in the air-gap has also harmonic components with order lower than the fundamental. Those harmonics are called subharmonics [9]. Because of their higher wavelength and their higher rotational speed compare to the main harmonic, flux lines of subharmonics penetrate deeply into the rotor. Consequently, subharmonics involve higher losses than other order harmonic [10]. It is also demonstrated that the most responsible of the eddy-current losses induced in the magnets of the rotor are the subharmonics. For the analysis involved in section III, a criterion based on the MMF harmonic distortion will indicate the negative effects implicated by the subharmonics.

In addition, considering the LCM of slot/pole combination as the first harmonic of the cogging torque, particular attention has to be given to the harmonic order of the MMF distribution that could interact with. For instance, if we reckon the topology having 12 slots and 4 poles pair, i.e. 8 poles, the LCM will be 24. Then, in order to prevent high value of ripple torque, the harmonics 24 - 4 (4 poles pair) and 24 + 4, i.e. 20th and 28th in the mechanical frame or 5th and 7th order harmonics in the electrical frame, have to be as low as possible.

Finally, MMF distribution of the windings is of great importance in determining performances of the machines. Then, the harmonic spectra could be simply obtained with the winding function [11]. It also corresponds to the MMF created in the air-gap when one phase is supplied.

C. Zero-sequence inductance

The last criterion implicated in this study concerns the ratio between the ZS inductance and the self inductance. In embedded applications, the machines have to get a high fault tolerance in order to operate even under faulty conditions. Hence, the magnetic separation is fulfilled when the ratio between the ZS inductance and the self inductance is equal to one.

[12] proposes a very simple, fast and precise method to obtain quickly the value of this ratio. The calculation is done with those assumptions:

- The machine has no magnetic leakage
- The reluctance of the magnetic material of the stator and the reluctance of the rotor are negligible compared to the non-magnetic material (air)
- The air gap is constant

The method presented is based on the calculation of the air gap energy when only one phase is fed, traducing the self inductance, and when all phases are fed with the same current. This is the zero-sequence energy.

$$\begin{cases} E_s = \frac{1}{2} L_s \cdot I^2 \\ E_0 = \frac{3}{2} L_0 \cdot I^2 \end{cases} \Rightarrow \frac{L_0}{L_s} = \frac{1}{3} \frac{E_0}{E_s} \quad (2)$$

Then, with the developed expression of the energy, the inductance ratio becomes:

$$E = \frac{\Re}{2} \sum_{n=1}^N \phi^2 \Rightarrow \frac{L_0}{L_s} = \frac{1}{3} \frac{\sum_{n=1}^N \phi_0^2}{\sum_{n=1}^N \phi_s^2} \quad (3)$$

In order to calculate this inductance ratio, we need first to determine the MMF distribution, the air gap flux and then the air gap energy. Given that the air gap flux is the image of the MMF, these different steps could be easily done by graphically counting the surface of the piecewise waveforms [12] for the two cases. It also corresponds to the winding function when only one phase is fed by a current I and when all phases are fed by the same current I .

III. FS-PMSM FOR EMBEDDED APPLICATIONS

The methodology previously suggested for the analysis of the FS-PMSM is applied in this section. Once more, the aim of this study is to get an overview of the tendencies given by the topologies in the context of the most common embedded applications. Then, it may concerns the PMSM employed in transportation systems as the traction in hybrid electric vehicle (HEV) and electric vehicle (EV). Therefore, the area of the review will be constrained for the slot and pole numbers. The slot number will start from 9 to a maximum value of 60, and the pole number will be included between 2 and 10. This value is mainly due to the higher frequency of the power supply inducing higher switching losses.

Obviously, not only one winding is possible for each topology. In our context, the choice of the winding comprises a double-layer. Moreover, coils (go and return coil sides) may surround a different number of teeth depending of the choice made by the designer. The arrangement of the coils for each phase needs an additional study and several combinations could be possible. An illustration is given for the topology having 12 slots and 10 poles. The star of slots is used to represent the phasors of the main harmonic of the EMF induced in each coil side within the stator [11]. Since the EMF phasor of one coil side is always rotated of a fixed angle (depending of the number of teeth surrounded) to the other coil side, only the EMF induced in one coil side will be represented. Fig. 1 shows the star of slots of the 12/10 topology. The number given to each vector corresponds to the number of the slot containing the corresponding coil side [11]:

Fig.1 Star of slots of 12/10 topology

Then, the coil sides of phase A have to be chosen. The other two phases B and C correspond to the same choice with a phase shift of $2\pi/3$ and $4\pi/3$ respectively. The topology disposes of 12 slots meaning 12 coils (double-layer) for the machine and 4 coils per phase. Thus, 4 vectors have to be chosen for phase one. These vectors could be picked from the right side and/or from the left side of the star of slots. Consequently, there are three possibilities which are the combinations: 2–2 (2 from the right side and 2 from the left side); 3–1 and 4–0. Mostly, 2–2 combination is preferred since it offers the highest value of the main harmonic. In this paper, this type of combination will be chosen for all the topologies. Then, the slots selected for phase A and for one side of the coils could be 1-6-7-12. For the other combinations 3–1 and

4–0, the slots could be 1-6-8-7 and 1-6-8-11 respectively. A particular attention has to be done on the selection of the slots; the phasor vectors have to be the nearest from each other. Thus, for combination 2–2, the selection 1-6-5-7 (or 1-6-5-12, ...) leads to an impossible winding. Last point concerns the number of teeth N_{teeth} that have to be surrounded by the coils. In order to obtain a high torque density (fundamental component) and to reduce at the same time the undesirable harmonics, N_{teeth} is generally chosen in such a way to get:

$$120^\circ \leq N_{teeth} \cdot \frac{360 \cdot p}{N_s} \leq 180^\circ \quad (4)$$

For certain machine having a large number of slots, several values of N_{teeth} is possible. In the followings, the winding configuration chosen for each slot/pole combination is realized considering the specifications of a PMSM dedicated to an EV. The mains constraints are a high torque density, low values of EMF harmonics, high fault tolerance and low eddy-current losses.

Ultimately, table II shows the analysis achieved for all the feasible topologies. For each topology, there are two rows. In the first row, the first four columns correspond to the winding factors F_{w1} , F_{w3} , F_{w5} and F_{w7} , i.e. 1st, 3rd, 5th and 7th order harmonics. Next column indicates the ratio of the ZS inductance L_0 with the self inductance L_s . Values of the ratio superior to 0.7 are in bold. In the second row, the first two columns characterize the LCM and the GCD of the slot and pole numbers. Finally, last case denotes the harmonic distortion of the MMF induced by the windings. Its high value leads to high eddy-current losses. The calculation takes into account the distortion of the harmonics until the $2*2p$ component order and without the components order multiple of 3. Values of the harmonic distortion superior to 100 per cent are in bold. Following as the model given below:

TABLE I: Model of the analysis for one topology

F_{w1}	F_{w3}	F_{w5}	F_{w7}	L_0/L_s
LCM	GCD	Harmonic Distortion (%)		

IV. DISCUSSION

Considering Table I, several observations can be done. For the topologies having 2 poles, the overall performances are quite good. Indeed, the 1st harmonic order is high and the other harmonics, particularly 5th and 7th are relatively reduced. Moreover, the ZS inductance ratio is always superior to 0.7, meaning a good fault tolerance, and the harmonic distortion is almost equal null. Thus, no eddy-current losses are predicted. In addition, due to the lower number of poles, the electrical frequency is also lower compared to the other topology. Then, iron losses will be also lower. The main drawbacks are that due to the long end windings, the torque density is not so high and for certain topologies there is a riskiness of ripple torque and radial forces.

TABLE II: Analysis of the FS-PMSM feasible topologies.

N _s /2p	2					4					6					8					10									
9	0.945	0.58	0.14	0.06	0.79	0.945	0.58	0.14	0.06	1.01						0.945	0.58	0.14	0.06	0.92	0.945	0.58	0.14	0.06	0.92					
	36	1		3		36	1		33							72	1		148	90	1		210							
	0.933	0.5	0.07	0.07	0.92	0.866	0	0.87	0.87	1.33						0.75	0	0.75	0.75	1	0.933	0.5	0.07	0.07	1					
12	12	2		0		12	4		0							24	4		119	60	2		110							
	0.951	0.62	0.17	0.11	1.28	0.951	0.62	0.17	0.11	1.26						0.951	0.62	0.17	0.11	1.15	0.76	0	0.76	0.76	0.97					
15	60	1		0		60	1		24							120	1		60	30	5		174							
18	0.945	0.58	0.14	0.06	0.77	0.931	0.5	0.09	0.02	1.06						0.866	0	0.87	0.87	1	0.931	0.5	0.09	0.02	0.67	0.945	0.58	0.14	0.06	1.33
	36	2		0		36	2		24							18	6		263	72	2		65	90	2		44			
21	0.953	0.63	0.18	0.12	1	0.953	0.63	0.18	0.12	1.11						0.932	0.5	0.08	0	0.35	0.953	0.63	0.18	0.12	0.83					
	84	1		0		84	1		14							168	1		59	210	1		63							
24	0.95	0.6	0.16	0.1	0.76	0.933	0.5	0.07	0.07	0.92						1	1	1	1	0.33	0.925	0.46	0.05	0.04	0.5					
	24	2		0		24	4		0		24	8		0		120	2		39											
27	0.941	0.55	0.12	0.05	0.89	0.941	0.55	0.12	0.05	0.98	0.83	0	0.19	0.15	0.95	0.941	0.55	0.12	0.05	0.26	0.941	0.55	0.12	0.05	0.35					
	108	1		0		108	1		12		54	3		405	216	1		45	270	1		66								
30	0.951	0.62	0.17	0.11	0.74	0.946	0.59	0.15	0.08	1.12						0.946	0.59	0.15	0.08	0.36	1	1	1	1	0.33					
	60	2		0		60	2		14							120	2		43	30	5		0							
33	0.946	0.58	0.15	0.08	1.1	0.928	0.48	0.07	0.01	1.15						0.933	0.5	0.07	0.07	0.9	0.954	0.63	0.19	0.13	0.39	0.945	0.58	0.15	0.08	0.28
	132	1		0		132	1		9		264	1		29	330	1		52												
36	0.952	0.62	0.18	0.12	0.73	0.945	0.58	0.14	0.06	0.77	0.933	0.5	0.07	0.07	0.9	0.931	0.5	0.09	0.02	0.18	0.942	0.56	0.13	0.05	0.33					
	36	2		0		36	4		0		36	6		186	72	4		29	180	2		31								
39	0.948	0.6	0.16	0.09	0.8	0.948	0.6	0.16	0.09	0.8						0.954	0.63	0.19	0.13	0.38	0.954	0.63	0.19	0.13	0.37					
	156	1		0		156	1		8							312	1		23	390	1		37							
42	0.945	0.58	0.14	0.07	0.82	0.929	0.49	0.08	0	1.29						0.83	0	0.17	0.13	0.89	0.951	0.61	0.17	0.11	0.24	0.953	0.63	0.18	0.12	0.42
	84	2		0		84	2		11		90	3		198	168	2		41	210	2		21								
45	0.95	0.61	0.17	0.1	0.71	0.941	0.55	0.12	0.05	0.72	0.83	0	0.17	0.13	0.89	0.941	0.55	0.12	0.14	0.17	0.931	0.5	0.09	0.02	0.27					
	180	1		0		180	1		7		90	3		198	360	1		26	90	5		41								
48	0.947	0.59	0.15	0.09	0.72	0.95	0.6	0.16	0.1	0.76						0.966	0.71	0.26	0.26	0.2	0.953	0.63	0.05	0.04	0.32					
	48	2		0		48	4		0							48	8		0	240	2		16							
51	0.951	0.61	0.17	0.11	0.87	0.933	0.51	0.09	0.01	0.88						0.945	0.58	0.14	0.06	0.72	0.951	0.61	0.17	0.11	0.18	0.955	0.64	0.19	0.13	0.31
	204	1		0		204	1		6		54	6		210	408	1		23	510	1		25								
54	0.949	0.6	0.16	0.1	0.72	0.939	0.55	0.12	0.04	1.18	0.945	0.58	0.14	0.06	0.72	0.952	0.62	0.18	0.12	0.22	0.949	0.6	0.16	0.1	0.23					
	108	2		0		108	2		7		54	6		210	216	2		23	270	2		18								
57	0.952	0.62	0.18	0.11	0.71	0.946	0.58	0.15	0.08	0.73						0.955	0.64	0.19	0.13	0.24	0.952	0.62	0.18	0.11	0.2					
	228	1		0		228	1		5							456	1		16	570	1		29							
60	0.95	0.61	0.17	0.1	0.72	0.936	0.52	0.1	0.01	0.73						0.946	0.59	0.15	0.08	0.24	0.966	0.71	0.26	0.26	0.2					
	60	2		0		60	4		0		120	4		14	60	5		0												

Considering the topologies with 4 poles, the performances obtained look like the machines with 2 poles. The main variances concern the riskiness of ripple torque and radial forces which is lower, the ZS inductance ratio which is slightly higher and the harmonic distortion which reveal the presence of eddy-current losses for certain machines. Also, the electrical frequency is multiplied by two and the end winding are a little shorter.

With the topologies having 6 poles, it is still possible to obtain a good ZS inductance ratio. However, the harmonic distortion is very high and high eddy-current losses can be predicted.

For, the last topologies with 8 and 10 poles, the performance is more mixed. First of all, for almost topologies, the ripple torque is very low meaning a smooth torque. Though, there is a risk of radial forces for certain combinations. Besides, the main harmonic is high and a high torque density is expected. The other EMF harmonics are also diminished. In view of the ZS inductance ratio, the values are in general much lower than the first columns topologies. Conversely, for certain topologies with a small number of slots, the ZS inductance ratio is high. Unfortunately, many of them present a high harmonic distortion and eddy-current losses. Finally, the electrical frequency is higher and higher iron losses could be expected.

Depending on the application, the specifications will differ and some topologies may be preferred. If the constraint on the torque density is not severe and low eddy-current losses are desired, topologies with 2 or 4 poles will be favored. On the other side, if a high torque density and a smooth torque are required, machines 15/8, 18/10 [5] and 21/10 could be

interesting. Finally, if the rotation speed is not so elevated as in HEV or EV, machine 12/10 is ideal for a perfect magnetic isolation and also good overall performances.

CONCLUSION

This paper has been concerned with the investigation of favorable slot/pole combination intended for embedded applications. The first part was concentrated on the assessment of FS-PMSM with the main criteria that has to be taken into account for this type of applications. Then, the second part demonstrated the contribution and the usefulness of this analysis allowing designers of machines to get an additional sense on FS-PMSM. Finally, it has been found that even if the electrical separation is not accomplished for certain topologies, magnetic separation and at the same time good performances can be achieved.

ACKNOWLEDGMENTS

The authors would like to thanks the French automotive cluster *Mov'eo*, for supporting the project of this work, and the industrial partners *Valeo* and *Leroy-Somer*.

REFERENCES

- [1] M. Barcaro and *al.*, "Performance evaluation of an integrated starter alternator using an interior permanent magnet machine," *IET Electr. Power Appl.*, vol.4, Iss. 1, pp.539-546, 2010.
- [2] F. Magnussen and H. Lendenmann, "Parasitic Effects in PM Machines With Concentrated Windings", *IEEE Trans. On Ind. Appl.*, Vol. 43, No. 5, Sept./Oct. 2007
- [3] J. T. Chen and Z. Q. Zhu, "Winding Configurations and Optimal Stator and Rotor Pole Combination of Flux-Switching PM Brushless AC Machines", *IEEE Trans. On Energy Conv.*, vol. 25, No. 2, June 2010

- [4] M. T. Abolhassani and H. A. Toliyat, "Fault Tolerant Permanent Magnet Motor Drives for Electric Vehicles", *IEEE Int. Electric Machines and Drives Conference*, IEMDC'09, Miami, USA, May 2009
- [5] H. Dogan, F. Wurtz, A. Foggia and L. Garbuio., "Performance Analysis and Comparison of three IPMSM With High Homopolar Inductance for Electric Vehicle Applications", *IEEE Power Electronics and Applications Conference*, EPE'11, Birmingham, Aug./Sept. 2011
- [6] J. A. Güemes et al., "Comparative Study of PMSM with Integer-slot and Fractional-slot Windings", *IEEE Int. Conf. On Electrical Machines*, ICEM'10, Rome, September 2010
- [7] T. Sun and al., "Effect of Pole and Slot Combination on Noise and Vibration in Permanent Magnet Synchronous Motor", *IEEE Trans. On Magnetics*, Vol. 47, No. 5, May 2011
- [8] Z. P. Xia, Z. Q. Zhu and al., "Comparison of Radial Vibration Forces in 10-pole/12-slot Surface-Mounted and Interior PM Brushless AC Machines", *IEEE Int. Conf. on Electrical Machines*, ICEM'10, Rome, Sept. 2010
- [9] N. Bianchi and al., "A General Approach to Determine the Rotor Losses in Three-Phase Fractional-Slot PM Machine", *IEEE Int. Conf. On Electric Machines and Drives*, IEMDC'07, Antalya, May 2007
- [10] Y. Liu, "Critical Wavelength of Eddy-currents and its Influence on Harmonic Losses in Solid Poles", *IEEE Int. Conf. on Electrical Machines*, ICEM'10, Rome, Sept. 2010
- [11] N. Bianchi and al., "Design considerations for fractional-slot winding configurations of synchronous machines", *IEEE Trans. On Ind. Appl.*, vol. 42, No. 4, Jul./Aug. 2006
- [12] L. De-Sousa and H. Dogan, "Method of Evaluating the Zero-Sequence Inductance Ratio for Electrical Machines", *IEEE Power Electronics and Applications Conference*, EPE'11, Birmingham, Aug./Sept. 2011