Single-scan skeletonization driven by a neighborhood-sequence distance

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Chapter One

Single-Scan Skeletonization Driven by a Neighborhood-Sequence Distance

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▬▬ ABSTRACT

Shape description is an important step in image analysis.

Skeletonization methods are widely used in image analysis as they are a powerful tool to describe a shape. Indeed, a skeleton is a one point wide line centered in the shape which keeps the shape's topology.

Commonly, at least two scans of the image are needed for the skeleton computation in the state of art methods of skeletonization. In this work, a single scan is used considering information propagation in order to compute the skeleton. This paper presents also a new single-scan skeletonization using different distances like d_4, d_8 and d_{ns}.

▬▬

Keywords: Skeleton, single-scan, Neighborhood sequence distance

1.1 INTRODUCTION

The skeletonization is an important method in the image analysis domain. This method is a powerful tool to extract some pertinent informations about a shape. The Skeletonization algorithm is the operation that identifies the skeleton of a shape. The skeleton is a shape subset that has to be one point wide, homotopic and centered in the shape. Moreover, if the distance information is preserved, the

skeleton is reversible. Skeletonization methods have been largely studied in the literature.

We are interested in the discrete domain methods because of the discretization of our space due to the pixels. In this domain there are some different methods like thinning [7, 8], distance based methods [1, 5] and other likes distance ordered thinning [12, 16] These skeletonization methods use more than two scans of the image in order to produce the skeleton.

the goal of this work is to calculate the skeleton with only one scan of the image.

The point of working with a single scan of the image is that the analysis task can be applied during the acquisition task. This amounts to saving time and storage.

In order to understand how this is possible, let us focus on the information propagation in the other methods. In method like that Thinning, the shape is peeled layer by layer until to produce the skeleton. Methods using the thinning from distance transform also peel the shape but they use the distance information like a layer.

These two methods compute the skeleton with a classical raster scan of the image. The dependance between two pixels depends of their distance in the raster scan order. That is to say if the distance map of a pixel is inferior than his neighbor, his neighbor will be peeled after the first one.

In section 1.2, necessary notions to compute a skeleton will be introduced. Section 1.3 explains the main idea of this work in order to compute the distance map and the medial axis with a single scan of the image. In section 1.4, we describe the computation of the skeleton through our reference algorithm and the particular cases are descibed. The results will be presented in section 1.5.

1.2   PRELIMINARIES

1.2.1   Neighborhood

Definition 1.1. We call *j*-neighborhood in *n*D, with $1 \leq j \leq n$, the set of vectors:

$$\mathcal{N}_j = \left\{ \overrightarrow{v} \ : \ |v_i| \leq 1 \text{ and } \sum |v_i| \leq j \right\} .$$

The point *q* is a *j*−neighbor of *p* if the vector from *p* to *q* belongs to the *j*-neighborhood ($\overrightarrow{pq} \in \mathcal{N}_j$).

The two neighborhoods of the 2D square grid, often called *cityblock* or 4-*neighborhood* for $\mathcal{N}_1$ and *chessboard* or 8-*neighborhood* for $\mathcal{N}_2$, are depicted on fig. 1.1.

Definition 1.2 (Simple point [13]). *Let X be a part of $\mathbb{Z}^2$ with a finite number of $\alpha - connected - components$ and $\overline{\alpha} - connected - components$. A point A of X is $\alpha - simple$ in X if:*

- *the set X and $X \setminus \{A\}$ have the same $\alpha - connected - component$ number;*
- *the set $\overline{X}$ and $\overline{X \setminus \{A\}}$ have the same $\overline{\alpha} - connected - component$ number;*
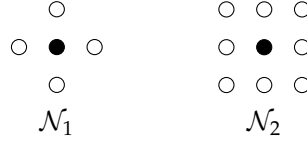
Figure 1.1.   1-neighborhood, $\mathcal{N}_1$, and 2-neighborhood, $\mathcal{N}_2$, in the 2D square grid

1.2.2   *Discrete distances*

Definition 1.3 (Discrete distance and metric). *Consider a function $d : \mathbb{Z}^n \times \mathbb{Z}^n \to \mathbb{N}$ and the following properties $\forall x, y, z \in \mathbb{Z}^n$:*

(1) ***positive definiteness*** $d(x,y) \geq 0$ *and* $d(x,y) = 0 \Leftrightarrow x = y$,
(2) ***symmetry*** $d(x,y) = d(y,x)$,
(3) ***triangle inequality*** $d(x,z) \leq d(x,y) + d(y,z)$,
(4) ***translation invariance*** $d(x+z, y+z) = d(x,y)$.

d is called a distance *if it verifies conditions* 1 *and* 2*, a* metric *with conditions* 1 *to* 3 *and a translation-invariant metric with the all 4 conditions.*

Practically, path-based distances are distance functions that associate to each couple of points (p, q), the minimal cost of a path from p to q. A path P is a sequence of points $(p_0 = p, p_1, \ldots, p_n = q)$ such that two successive points are neighbors: $\overrightarrow{p_{r-1}p_r} \in \mathcal{N}$. The cost (or length) of a path $P = (p_0 = p, p_1, \ldots, p_n = q)$ is the number of its displacements, n.

Rosenfeld and Pfaltz proposed the first path-based distances, namely d and d^* (often denoted by d_4 and d_8), that are respectively generated by neighborhoods of types 1 and 2 [15]. Later, they defined the octagonal distance, a better approximation of the Euclidean distance, using alternated neighborhoods [14]. By allowing longer periodic sequences of neighborhoods, Das *et al.* introduced the concept of neighborhood-sequence (NS) distances that was afterwards extended to nonperiodic sequences [3, 4].

Let $B(r), r \in \mathbb{N}^*$ be a sequence of integers; each $B(r)$ represents the type of neighborhood allowed for the r$^{\text{th}}$ displacement. *i.e.,* a path $(p_0 = p, p_1, \ldots, p_n = q)$ is valid if any two successive points p_{r-1} and p_r are $B(r)$−neighbors: $\overrightarrow{p_{r-1}p_r} \in \mathcal{N}_{B(r)}$. In the 2D case, $B(r) \in \{1, 2\}$, We denote by $\mathbf{J}_B(r)$, with $1 \leq J \leq n$, the number of times the *j*-neighborhood appears in sequence B till index r:

$$\mathbf{J}_B(r) = \left| \left\{ r \; : \; 1 \leq i \leq r, B(i) = j \right\} \right| .$$

For any real number x, let $\lfloor x \rfloor$ (resp. $\lceil x \rceil$) denote the largest integer less (resp. greater) than or equal to x. A Beatty sequence with parameter α where α is a positive irrational number ($\alpha \in \mathbb{R}_+ \setminus \mathbb{Q}$), is the sequence of the positive multiples of α rounded to the largest smaller integers: $b(i) = \lfloor i\alpha \rfloor$. András and Lajos Hajdu suggested to base the neighborhood sequence on a Beatty sequence, taking $B(r) = \lfloor \alpha r \rfloor - \lfloor \alpha(r-1) \rfloor + 1$ [6]. Clearly, if $\alpha \in [0,1]$ then $\forall r \in \mathbb{N}^*, B(r) \in \{1, 2\}$

and α represents the asymptotic rate of '2s' in the sequence. Then (even for a rational α):

$$\mathbf{1}_B(r) = r - \lfloor \alpha r \rfloor = \lceil (1-\alpha)r \rceil \,,$$
$$\mathbf{2}_B(r) = \lfloor \alpha r \rfloor \,.$$

Definition 1.4 (Disk). *For a given distance d, the closed disk $D_\le$ and open disk $D_<$ of center c and radius r are the sets of points of $\mathbb{Z}^n$:*

$$D_<(c,r) = \{p \,:\, d(c,p) < r\} \,;$$
$$D_\le(c,r) = \{p \,:\, d(c,p) \le r\} \,. \tag{1.1}$$

Disks of a neighborhood-sequence distance are built by a series of Minkowski sums:

$$D_<(c,r+1) = D_<(c,r) \oplus \mathcal{N}_{B(r)} \,. \tag{1.2}$$

1.2.3   Distance map and medial axis

Definition 1.5 (Distance transform). *The distance transform DT_X of the binary image X is a function that maps each point p to its distance from the closest background point:*

$$\mathrm{DT}_X : \mathbb{Z}^n \to \mathbb{N}$$
$$\mathrm{DT}_X(p) = \min\left\{d(q,p) \,:\, q \in \mathbb{Z}^n \setminus X\right\} \,. \tag{1.3}$$

Alternatively, since all points at a distance less than $\mathrm{DT}_X(p)$ to p belong to X ($\check{D}_<(p,\mathrm{DT}_X(p)) \subset X$) and at least one point at a distance to p equal to $\mathrm{DT}_X(p)$ is not in X ($\check{D}_<(p,\mathrm{DT}_X(p)+1) \not\subset X$) then:

$$\mathrm{DT}_X(p) \ge r \iff \check{D}_<(p,r) \subset X \,, \tag{1.4}$$

where $D_<(c,r)$ and $\check{D}_<(c,r)$ are symmetric to each other with regards to their center c, $\check{D}_<(c,r) = \{p \,:\, d(p,c) < r\}$. The DT is usually defined as the distance *to* the background which is equivalent to the distance *from* the background by symmetry. The equivalence is lost with asymmetric distances, and this definition better reflects the fact that DT algorithms always propagate paths from the background points.

Definition 1.6 (Medial Axis). *We call maximal disk of the set X, a disk contained in X but not contained in any other disk also contained in X. By definition of DT_X, if $D_<(p,r)$ is maximal then necessarily $r = \mathrm{DT}_X(p)$. The medial axis MA_X of the set X maps each point to the radius of the maximal disk centered in that point if it exists, 0 otherwise:*

$$\mathrm{MA}_X(p) = \begin{cases} 0 & \text{if } \exists\, q,r \text{ s.t. } D_<(p,\mathrm{DT}_X(p)) \subsetneq D_<(q,r) \subset X \,, \\ \mathrm{DT}_X(p) & \text{otherwise} \,. \end{cases} \tag{1.5}$$

For NS distances, a simple local criterion is sufficient to extract the medial axis from the distance map:

$$\text{MA}_X(p) = \begin{cases} 0 & \text{if } \exists \, \overrightarrow{v} \in \mathcal{N}_{B(\text{DT}_X(p))}, \text{DT}_X(p + \overrightarrow{v}) > \text{DT}_X(p) \, , \\ \text{DT}_X(p) & \text{otherwise} \, . \end{cases} \quad (1.6)$$

Since the null vector is, by definition 1.1, included in neighborhoods of all types, we can also write:

$$\text{MA}_X(p) = r > 0 \iff \begin{cases} \text{DT}_X(p) \geq r \\ \forall \, \overrightarrow{v} \in \mathcal{N}_{B(r)}, \text{DT}_X(p + \overrightarrow{v}) \leq r \end{cases} \, . \quad (1.7)$$

1.3 DISTANCE MAP AND MEDIAL AXIS COMPUTATION

Distance from the background to each pixel is the primary order criterion in our skeletonization algorithm. For path-based distances, distance maps are computed by propagating path costs from the background towards the inside of the shape. This process is common to all kinds of algorithms, parallel [15], queue-based [11, 17] or sequential [9, 15]. Classical sequential algorithms require at least two scans (exactly two for simple and chamfer distances) of the image data in reverse order [15]. Wang and Bertrand [18] described a generalized distance transform using one neighborhood or two alternated neighborhoods. When the neighborhoods satisfy the forward scan condition, path costs are only propagated in directions compatible with the raster scan thus a single scan is required. A new generalized distance transform for arbitrary sequences, periodic or not, was presented in [10]. It uses a translated version of neighborhoods $\mathcal{N}_1$ and $\mathcal{N}_2$ as shown in fig. 1.2.. Furthermore, it was shown that the regular, centered, distance map can be obtained from the generalized distance map obtained with translated disks. This correcting step requires a random access to pixels in a limited band of image rows, but it follows the main raster scan data flow.

1.3.1 *Asymmetric Distance Transform*

Let d be a 2D NS distance with sequence $B(r) = \lfloor \alpha r \rfloor$. The main criterion of the thinning order of a shape X is the distance from the background obtained by the distance transform with distance d: DT_X. Instead of directly computing DT_X (this would require at least three scans of the image), we deduce it from the distance transform DT'_X obtained with another (generalized because asymmetric) distance d'. d' is defined with the same sequence B as d but with translated neighborhoods.

For each neighborhood $\mathcal{N}_j, j \in \{1, 2\}$, we apply a translation vector $\overrightarrow{t_j}$ such that the translated neighborhood $\mathcal{N}'_j = \mathcal{N}_j \oplus \{\overrightarrow{t_j}\}$ is in forward scan condition. In a translation preserved scan order, $\overrightarrow{t_j}$ translates the first visited point of $\mathcal{N}_j$ to the
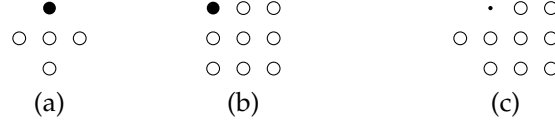
Figure 1.2.   Neighborhoods used for the translated NS-distance transform. (a) and (b) are respectively the type 1 and 2 translated neighborhoods, $\mathcal{N}_1'$ and $\mathcal{N}_2'$. (c) is the whole set of non-null displacements, $\mathcal{N}'^* = (\mathcal{N}_1' \cup \mathcal{N}_2') \setminus \{O\}$.

origin. Assuming a 2D standard raster scan order:

$$\overrightarrow{t_1} = (0,1) \qquad\qquad \overrightarrow{t_2} = (1,1) \,. \tag{1.8}$$

The translated neighborhoods $\mathcal{N}_1'$ and $\mathcal{N}_2'$ obtained with $\overrightarrow{t_1} = (0,1)$ and $\overrightarrow{t_2} = (1,1)$ are depicted in fig. 1.2.a and fig. 1.2.b.

$\quad$ DT_X' can computed in a single raster scan using [10]:

$$\mathrm{DT}_X'(p) = \begin{cases} 0 & \text{if } p \notin X \\ \min\left\{ \mathrm{LUT}_{\overrightarrow{v}}(\mathrm{DT}_X'(p - \overrightarrow{v})), \overrightarrow{v} \in \mathcal{N}'^* \right\} & otherwise \end{cases} \tag{1.9}$$

where $\mathrm{LUT}_{\overrightarrow{v}}(r)$ is the final cost of a path of initial cost r after an extra displacement $\overrightarrow{v}$:

$$\mathrm{LUT}_{\overrightarrow{v}}(r) = \begin{cases} \mathrm{LUT}_1(r) = \left\lfloor \dfrac{\lceil (1-\alpha)r \rceil}{1-\alpha} \right\rfloor + 1 & \text{if } \overrightarrow{v} \in \mathcal{N}_1' \text{ and } \overrightarrow{v} \notin \mathcal{N}_2' \\[4mm] \mathrm{LUT}_2(r) = \left\lceil \dfrac{\lfloor \alpha r \rfloor + 1}{\alpha} \right\rceil & \text{if } \overrightarrow{v} \notin \mathcal{N}_1' \text{ and } \overrightarrow{v} \in \mathcal{N}_2' \\[4mm] \mathrm{LUT}_{12}(r) = r + 1 & \text{if } \overrightarrow{v} \in \mathcal{N}_1' \text{ and } \overrightarrow{v} \in \mathcal{N}_2' \end{cases}$$

1.3.2   *Equivalence with the Centered Distance Transform*

Disks generated by neighborhoods $\mathcal{N}_1'$, $\mathcal{N}_2'$ and the sequence $B(r)$ are such that:

$$D_\leq'(p,r) = D_\leq(p + \overrightarrow{t}(r), r) \qquad \check{D}_\leq'(p,r) = \check{D}_\leq(p - \overrightarrow{t}(r), r) \tag{1.10}$$

where the translation vector $\overrightarrow{t}(r)$ is:

$$\begin{aligned} \overrightarrow{t}(r) &= \overrightarrow{t}(r-1) + \overrightarrow{t_{B(r)}} \\ &= \mathbf{1}_B(r)\overrightarrow{t_1} + \mathbf{2}_B(r)\overrightarrow{t_2} \\ &= (\mathbf{2}_B(r), \mathbf{1}_B(r) + \mathbf{2}_B(r)) \\ &= (\mathbf{2}_B(r), r) \,. \end{aligned}$$

$\quad$ DT_X' has equivalence with values of DT_X:

$$\begin{aligned} \mathrm{DT}_X(p) \geq r &\iff \check{D}_\leq(p, r-1) \subseteq X \\ &\iff \check{D}'(p + \overrightarrow{t}(r-1), r-1) \subseteq X \\ &\iff \mathrm{DT}_X'(p + \overrightarrow{t}(r-1)) \geq r \,. \end{aligned} \tag{1.11}$$

Consequently:

$$\text{DT}_X(p' - \overrightarrow{t}(r-1)) = r \iff \text{DT}_X(p' - \overrightarrow{t}(r-1)) \geq r \text{ and } \text{DT}_X(p' - \overrightarrow{t}(r-1)) < r+1$$
$$\iff \text{DT}'_X(p' + \overrightarrow{t_{B(r)}}) \leq r \leq \text{DT}'_X(p'). \tag{1.12}$$

Knowing $\text{DT}'_X(p')$ and $\text{DT}'_X(p' + \overrightarrow{t})$ for a given vector $\overrightarrow{t}$, we can deduce $\text{DT}_X(p' - \overrightarrow{t}(r-1))$ for all values of r between $\text{DT}'_X(p' + \overrightarrow{t})$ and $\text{DT}'_X(p')$ for which $\overrightarrow{t} = \overrightarrow{t_{B(r)}}$.

In [10], an algorithm is given to fully recover the centered distance transform from the asymmetric one. However, for the distance ordered skeletonization process, values of the centered distance map DT are not required to be stored as an image. Instead, we compute these values from DT', on the fly.

1.3.3 *Medial Axis*

Let MA'_X be the medial axis of X with distance d'. Similarly to (1.7):

$$\text{MA}'_X(p) = r > 0 \iff \begin{cases} \text{DT}'_X(p) \geq r \\ \forall \overrightarrow{v} \in \mathcal{N}'_{B(r)}, \text{DT}'_X(p + \overrightarrow{v}) \leq r \end{cases}$$

using (1.11) and by definition of $\mathcal{N}'_j$:

$$\text{MA}'_X(p) = r > 0 \iff \begin{cases} \text{DT}_X(p - \overrightarrow{t}(r-1)) \geq r \\ \forall \overrightarrow{v} \in \mathcal{N}_{B(r)}, \text{DT}_X(p + \overrightarrow{v} - \overrightarrow{t}(r-1)) \leq r \end{cases}$$
$$\iff \text{MA}_X(p - \overrightarrow{t}(r-1)) = r > 0$$

Like all Minkowski-based distances, the medial axis of the generalized distance d', MA'_X, can be computed by a local maximum criterion. MA'_X is equivalent, up to a translation of each maximal disk, to the medial axis of the regular distance d, MA_X.

1.4   ONE PASS SKELETON COMPUTATION

This section presents a single-scan skeletonization algorithm. We start from a multi-scan reversible distance-ordered algorithm where the thinning order is guided by the distance to the background. From this "reference" algorithm, we derive a single-scan skeletonization algorithm that, despite the fact that the thinning order is modified, preserves the dependencies of pixel removals.

1.4.1 *Reference algorithm (rlk)*

Like DOHT [12] or Svensson's work [16], we use a distance-controlled thinning method. The reference algorithm proceeds by thinning pixels, layer by layer, according to their distance to the background. Medial axis points are not deleted because it allows the reconstruction of a shape from its skeleton.

Let p and q be two points, r_p and r_q be the distance map value to the point p and q respectively ($DT_x(p)$ and $DT_x(q)$). A lexicographic order (here called *rlk*) is defined such that :

$$p \prec^{rlk} q \iff \begin{cases} r_p < r_q \\ r_p = r_q \text{ and } p \prec q \end{cases} \tag{1.13}$$

Algorithm 1: The *rlk* algorithm

Data: A binary image X, DT_X, MA_X
Result: The skeleton Sk_X
for $r \leftarrow 1$ **to** $\max_{p \in X}\{DT_X(p)\}$ **do**
    for $l \leftarrow 0$ **to** L **do**
        p: point (k, l)
        for $k \leftarrow 0$ **to** K **do**
            if $DT_X(p) = r$ **and** $MA_X(p) \neq r$ **and** p *is simple* **then**
                $Sk_X(p) \leftarrow 0$;
            end
        end
    end
end

1.4.2   Single-scan algorithm ($l'k'r$)

Now the goal is to switch the order of the loops to produce a skeletonization algorithm in one scan. The order of the loops for the reference algorithm was radii, lines and columns (*rlk* order scan).

Thanks to the translation of the center of the disks, it is possible to change the order of the loops. Now the order is translated lines (l'), translated columns (k') and raddi ($l'k'r$ order).

The following equivalences appear :

Let $l'_p = l_p + r_p - 1$, $l'_q = l_q + r_q - 1$ and $k'_p = k_p$, $k'_q = k_q$ for the distance d_4 and $k'_p = k_p + r_p - 1$, $k'_q = k_q + r_q - 1$ for the distance d_8.

$$p \prec^{l'k'r} q \iff \begin{cases} l'_p < l'_q \iff p \prec^{rlk} q \\ l'_p = l'_q \text{ and } \begin{cases} k'_p < k'_q \\ k'_p = k'_q \text{ and } r_p < r_q \iff p \prec^{rlk} q \end{cases} \end{cases} \tag{1.14}$$

Algorithm 2: The $l'k'r$ algorithm

Data: A $K \times L$ binary image X, DT'_X, MA'_X

Result: The skeleton Sk_X

for $l' \leftarrow 0$ *to* L **do**

    for $k' \leftarrow 0$ *to* K **do**

        p': point (k', l')

        for $r \leftarrow \max\left\{1, \min\left\{DT'_X(p' + \overrightarrow{t_1}), DT'_X(p' + \overrightarrow{t_2})\right\}\right\}$ *to* $DT'_X(p')$ **do**

            $p \leftarrow p' - \overrightarrow{t}\,(r-1)$

            if $r \geq DT'_X(p' + \overrightarrow{t_{B(r)}})$ **then**

                if $MA'_X(p') \neq r$ **and** p *is simple* **then**

                    $Sk_X(p) \leftarrow 0$

                    if *Next open disk built with* N_8 **and** $k' \geq 0$ **and**

                    $MA_X[l][k-1] = d$ **and** $MA'_X \neq r$ **and** *simple* **then**

                        $Sk_X(k-1, l) \leftarrow 0$

                    end

                end

            end

        end

    end

end

1.4.3   Particular cases

We note two particulars cases in our algorithm. The first one is a simplicity partic-ular case, due to our special order scan. Figure 1.3. shows this particular case.

In this example the deletion causes a change in the topology of the shape if you remove the point p (creating a hole). In an other algorithm, the point p was removed after his right neighbor. Due to our special order scan, p is seen before its right neighbor and is not (for the moment) simple.

The solution is to re-evaluate the simplicity of the previous point when the current point is deleted. That is to say, in the example figure 1.3., the deletion of the point q lead the simplicity evaluation of the point p.
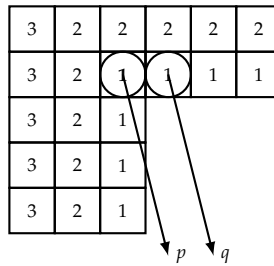
Figure 1.3.   Special case in one scan algorithms

The second particular case is due to the translation of the center of the disk d_4. Figure 1.4. shows the particular case.
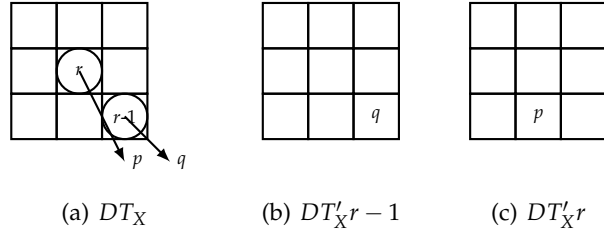

(a) DT_X (b) $DT'_X r - 1$ (c) $DT'_X r$

Figure 1.4. $p \prec^{l'k'r} q \iff l'_p = l'_q$ et $k'_p < k'_q$.

In this special case, the point p is computed before the point q (because $l'_p = l'_q$ et $k'_p < k'_q$) whereas it is not the case in the reference algorithm because $p \prec^{rlk} q \iff DT_X(p) < DT_X(q)$.

So if distance in top-left corner and left neighbor (untranslated) is bigger than r behave like this point hasn't been thinned yet.

The simplicity function determines if a point is simple or not. In order to compute the simplicity of a point, we subtract from the count of 1-connected background pixels, the number of corners of $\mathcal{N}_2$. The point is simple if the result equals 1. Figure 1.5. shows the patterns used for the simplicity function.
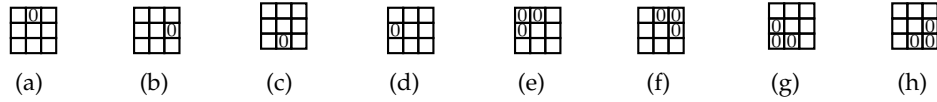

(a)   (b)   (c)   (d)   (e)   (f)   (g)   (h)

Figure 1.5.   (a)+(b)+(c)+(d)-(e)-(f)-(g)-(h) = 1 if the point is simple.

1.5   RESULTS

Table 1.1 presents the execution time of the six algorithms. The reference algorithms has been evaluated with three different distances, along with their single-scan versions. Note that, for all three distances, the single-scan algorithm is faster than the reference algorithm.

In figure 1.6. is shown an example of an original shape. The figure 1.7. shows the images resulting from the algorithms. For each one, the one scan algorithm produced the same, pixel per pixel, skeleton as the reference algorithm.

Figure 1.6.   Original shape.

Table 1.1   Execution time results

Image	number of pixel	distance used	time in second	
			ref	one scan
main	287523 pixels	d_4	0.204439	0.024109
		d_8	0.407720	0.022979
		d_{ns}	0.168092	0.030744
Bone	93632 pixels	d_4	0.021276	0.007416
		d_8	0.040200	0.007630
		d_{ns}	0.018037	0.009585
Corail	3027888 pixels	d_4	0.499546	0.149939
		d_8	0.903974	0.146726
		d_{ns}	0.386084	0.207187
anguille	5018112 pixels	d_4	1.996670	0.129146
		d_8	5.095797	0.135774
		d_{ns}	1.735280	0.211365

(a) with distance d_4 (b) with distance d_8 (c) with distance d_{ns}
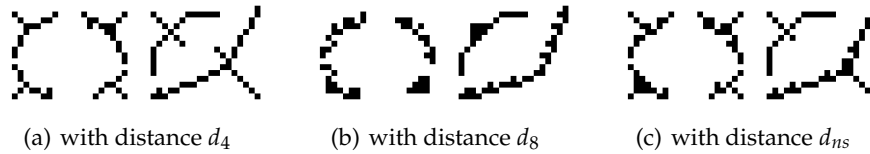
Figure 1.7.   Image results

Applications that continuously acquire an image could definitely take advantage of the proposed approach while they typically produce unbounded images. In previous works [2], this method extracted the skeleton from a radiography of trabecular bone.

1.6   CONCLUSION

This paper presented an algorithm which computes a distance map, a medial axis and a skeleton with one scan of the image. The algorithm is able to compute a skeleton using any neighborhood-sequence distance. Examples were shown for d_4 (cityblock), d_8 (chessboard) and the octagonal distance d_{oct}.

The proposed algorithm is up to 10-15 times faster than the reference algorithm. Remark that best performance improvements are obtained with large images.

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