Experimental Test of Universality of the Anderson Transition
Matthias Lopez, Jean-François Clément, Pascal Szriftgiser, Jean Claude Garreau, Dominique Delande

To cite this version:

HAL Id: hal-00613108
https://hal.archives-ouvertes.fr/hal-00613108
Submitted on 2 Aug 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Experimental Test of Universality of the Anderson Transition

Matthias Lopez, Jean-François Clément, Pascal Szriftgiser, Jean Claude Garreau, and Dominique Delande

1Laboratoire de Physique des Lasers, Atomes et Molécules, Université Lille 1 Sciences et Technologies, CNRS; F-59655 Villeneuve d’Ascq Cedex, France
2Laboratoire Kastler-Brossel, UPMC-Paris 6, ENS, CNRS; 4 Place Jussieu, F-75005 Paris, France

(Dated: August 2, 2011)

We experimentally test the universality of the Anderson three dimensional metal-insulator transition. Nine sets of parameters controlling the microscopic details of this second order phase transition have been tested. The corresponding critical exponents are independent (within 2σ) of these microscopic details, and the average value 1.63 ± 0.05 is in very good agreement with the numerically predicted value, ν = 1.58.

PACS numbers: 72.15.Rn, 03.75.-b, 05.45.Mt, 64.70.Tg

In the presence of a disordered potential, the classical diffusive transport of a particle can be inhibited by quantum interference among the various paths where the particle is multiply scattered by disorder, a puzzling phenomenon known as Anderson localization [1]. The dimensionality of the system plays a major role, which can be understood qualitatively from the scaling theory of localization [2]. In dimension d = 3 and above, there is a delocalized-localized (or metal-insulator in solid-state physics language) transition — known as the Anderson transition. An energy “mobility edge” Ec, which is a decreasing function of disorder, separates localized motion at low energy from diffusive motion at high energy. On the localized side, the localization length ξ diverges algebraically ξ(E) ∝ (Ec − E)−ν, with ν the critical exponent of the transition. On the diffusive side, the diffusion constant vanishes like D(E) ∝ (Ec − E)σ with, according to the scaling theory, σ = (d − 2)ν [2, 3]. A key prediction of the scaling theory is that the critical exponents are universal, that is, they do not depend on the microscopic details of the system, such as the correlation functions of the disorder, the dispersion relation of the particles, etc. Numerical experiments on simple models [4–6] such as the tight-binding Anderson model, have confirmed this universality, with a non-trivial value of the critical exponent around ν = 1.58 for spinless time-reversal invariant 3-dimensional (3D) systems [7]. However, there is a huge lack of experimental results. In this letter, we present accurate measurements of the critical exponent ν in 3D, and, by varying the various experimental parameters, test whether its value is universal.

Anderson localization is due to interference between long multiple scattering paths and is thus very sensitive to any mechanism destroying the phase coherence of the wavefunction, making its experimental observation and characterization very difficult [8]. In the context of electronic transport in disordered samples, electron-electron interaction is sufficiently important to partly invalidate the one-body Anderson scenario, leading to a critical exponent close to unity [9]. In a slightly different context, Anderson localization of acoustic [10] and electromagnetic [11–13] waves has been experimentally observed. There, absorption is a limitation, and the critical exponent ν ≈ 0.5 estimated in [13] does not agree with the numerical simulation result 1.58.

Cold atomic gases are conceptually simple systems which can be exposed to well controlled disorder. For sufficiently cold samples, the phase coherence time of the wavefunction describing the center of mass motion can be kept sufficiently long for atomic interferometry experiments to be routinely performed. Atom-atom interaction can be reduced to a minimum, either by using dilute cold gases, or by using a Feshbach resonance [14]. Direct observation of Anderson localization of atomic matter waves in a one-dimensional disordered [15] or quasi-periodic [16] potential has been reported, the disordered potential being created by the effective interaction with a detuned laser beam with a random spatial profile (speckle). Very recently, observations of the Anderson localization in a 3D atomic gases have been claimed [17]. However, the interpretation of both results involves heuristic assumptions, and a better understanding is needed before the observation of a mobility edge can be asserted.

This difficulty prompted the use of a slightly different system, where the disordered potential is replaced by classical chaotic dynamics. In the kicked rotor, a paradigmatic system of quantum chaos, quantum mechanical interference tend to suppress the classical chaotic diffusive dynamics, and to induce a phenomenon originally called dynamical localization, later discovered to be an analog of 1D Anderson localization in momentum space, by mapping the kicked rotor onto a quasirandom 1D Anderson model [18]. The experimental realization of the kicked rotor with laser-cooled atoms interacting with a pulsed standing wave allowed the first experimental observation of Anderson localization in 1D with atomic matter waves [19]. In order to observe the Anderson transition, however, an analog of the 3D Anderson model is needed [20]. Here, we focus on the so-called “quasi-periodic kicked rotor” in which three incommensurate frequencies are used to generate the 3D character [21], and shown in [22] to be
strictly equivalent to an anisotropic 3D Anderson model, a fact further confirmed by a low-energy effective field theory [23]. Meticulous numerical experiments [6] have shown the universality of the critical exponent for this system, $\nu = 1.58 \pm 0.02$, in excellent agreement with values found in the literature for the Anderson model [4].

An experiment based on this system has characterized the Anderson metal-insulator transition [24], with the first experimental determination of the critical exponent $\nu = 1.4 \pm 0.3$. The experimental setup, described in detail in [22], consists of laser-cooled cesium atoms interacting with a pulsed far-detuned standing wave (wavenumber $k_L = 7.4 \times 10^6$ m$^{-1}$ and maximum one-way intensity 180 mW). The amplitude of the kicks is modulated with two frequencies $\omega_2$ and $\omega_3$. The Hamiltonian reads:

$$H = \frac{p^2}{2} + K \cos x \left[ 1 + \varepsilon \cos (\omega_2 t) \cos (\omega_3 t) \right] \sum_{n=0}^{N-1} \delta (t - n),$$

where time is measured in units of the kicking period $T_1$, space in units of $(2k_L)^{-1}$, momentum in units of $2\hbar k_L / \hbar$, with $k = 4\hbar k_L^2 T_1 / M$ (M is the atom mass) playing the role of an effective Planck constant $(x, p) = i\hbar k$ and $K$ is the average kick amplitude. The kicks are short enough (duration $\tau = 0.8 \mu s$) compared to the atom dynamics to be considered as Dirac delta functions. If $\omega_2, \omega_3, \pi$ and $k$ are incommensurate, this 1D quasiperiodic kicked rotor has a 3D Anderson metal-insulator transition, displaying localization in momentum space. Compared to [24] and [22], the signal to noise and the stability of the experiment have been greatly improved. Atomic momentum is measured by Raman stimulated transitions. The previous Raman frequency generation setup used direct current modulation of a master laser diode to drive the Raman slave lasers [25]. This system has been replaced by a fibered phase modulator driven at 9.2 GHz. Moreover, 3 (of 4) master diode lasers working with an external cavity setup have been replaced by distributed feedback lasers, extending the experiment stability from a few hours to several days. These improvements lead to much better experimental signals, making the determination of the critical exponent much more accurate and reliable, opening the possibility to test its universality.

A Sisyphus-boosted magneto-optical trap prepares an initial thermal state of FWHM $4 \times 2h k_L$, much narrower than the final (localized or diffusive) momentum distribution, and – by time-reversal symmetry – $(p(t))$ remains zero at all time $t$. We can directly monitor the dynamics rather than rely on “bulk” quantities such as the conductance, itself related to the diffusion constant. A good quantity characterizing the dynamics is $(p^2(t))$. For practical and historical reasons, the atomic momentum $P$ is measured in units of two recoil momenta:

$$p = \frac{P}{\hbar k_L}.$$
tract a truly diverging of the fit being typically slightly smaller than 1. χ and the inhomogeneity related to the Gaussian intensity because the laser beams are not perfectly horizontal, and the inhomogeneity related to the Gaussian intensity profile of the standing wave contribute to the cutoff. We have increased the phase coherence time up to 200-300 kick periods, in agreement with theoretical calculations [22]. In order to reduce uncontrolled systematic effects, we chose to use the same time interval - from 30 to 150 kicks - for the analysis of all experimental data. The finite duration is also responsible for systematic effects: It tends to slightly underestimate Kc, but does not seem to shift significantly the critical exponent. Using a different interval, 30-120 kicks, produces ν values not differing by more than 0.05.

A typical ξ(K) curve, Fig. 2, displays a clear “divergence” near the critical point (increase by more than one order of magnitude, much better than in [24]). It is itself fitted to extract the position of the critical point Kc and the critical exponent ν, using the following formula:

\[ 1/\xi(K) = \alpha|K - K_c|^{\nu} + \beta \]  

where β is a cut-off parameter taking into account the various limitations discussed above. As seen in Fig. 2 the fit is excellent. The fitting parameter ν depends on the range of K where the fit is performed. A too small range produces a large uncertainty in ν while the quality of the fit deteriorates for a too large range. To avoid any bias, we have fitted all data sets in the interval [0.8Kc, 1.2Kc], for which the \( \chi^2 \) per degree of freedom is of the order of 1. The uncertainties are calculated using a bootstrap method starting from the raw experimental data and their error bars, ending to the determination of ν through the construction of the scaling function. For data sets presented, no statistically significant anomaly has been detected. The statistical uncertainty on Kc is very small, at most few 10^{-2}, but the finite duration of
the experiment is responsible for a systematic shift towards low $K$ (see above).

In order to test the universality of the critical exponent, we have chosen a “reference” set of parameters, noted A in table I, which has the same parameters used in [24]. We have then modified the $\omega_2$ and $\omega_3$ frequencies for set B. We have also modified the path in the $(K, \varepsilon)$ plane, either by changing $K$ only (set C) or $\varepsilon$ only (set D). In these two situations, the crossing of the critical regime is slower, making the accuracy on $\nu$ significantly worse. We have further modified the kicking period $T_1$, which affects the effective Planck’s constant $\hbar$. Several smaller $k$ values have been used in sets E, F and G. One larger value of $k$ was used in sets H and I the difference between the two sets being the duration of the laser pulses. The fact that very close values of $K_c$ and the same critical exponent are obtained is a strong indication that our experimental system is well described by the model Hamiltonian, Eq. (1).

The final results are given in table I and plotted with their error bars (one standard deviation) in Fig. 3. They unambiguously demonstrate the universality of the localized-diffusive transition in the quasi-periodic kicked rotor. Moreover, all numerical values are compatible (within 2 standard deviations) with the best numerical determinations of $\nu = 1.58 \pm 0.02$, both for the kicked rotor and the Anderson model. They all markedly differ from the value 1 predicted by the self-consistent theory of localization [29]. The later theory is an attempt to justify the scaling properties using a microscopic approach: It is qualitatively correct and gives simple physical pictures. For example, it has been used to successfully predict the momentum distribution at the critical point [26]. However, it lacks a key ingredient: At criticality, the wavefunctions display very large fluctuations which can be characterized by a multifractal spectrum [7, 30, 31]. Huge fluctuations are known to affect critical exponents of thermodynamic phase transitions, it is thus no surprise that they also affect the Anderson transition. While quantum phase transitions are usually considered for the ground state of the system, it must be emphasized that the Anderson transition deals with excited states in the vicinity of the mobility edge, displaying much richer properties. Especially, ultra-cold atomic gases open the way to experimental studies of the interplay of disorder, interference and interactions.

We thank G. Lemarié for useful discussions, and R. Holliger for help with the experiments.

* URL: www.phlan.univ-lille1.fr/aftr/cq