On the Effect of Quality Overestimation in Remanufacturing

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On the Effect of Quality Overestimation in Remanufacturing

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Abstract

We study a simple reverse supply chain consisting of a remanufacturing facility and a number of independent locations where used products are returned by the end-users. At the collection locations, the returned products are graded and classified based on a list of nominal quality metrics provided by the remanufacturer. It is assumed that this classification is subject to errors; specifically, the returns condition is overestimated because of a stochastic proportion of returned units which are classified in classes corresponding to better quality than the actual. The scope of the paper is to study how these classification errors affect the optimal procurement decisions of the remanufacturer as well as the associated profit for the cases of both constant and stochastic demand in a single-period context. Moreover, in the former case we study the impact of these classification errors on profit variability. The quantification of the impact of quality overestimation provides intuition on the value of reliable classification and on the extent of the necessary investments and initiatives to improve classification accuracy.

Keywords: remanufacturing, reverse logistics, grading errors, quality of returns, supplier evaluation.

1. Introduction

The issue of information regarding the quantity and quality of used items in reverse supply chains has recently attracted a lot of attention by both practitioners and academics. The quality categorization of the returns is the first step among a series of necessary procedures before a used product can be considered suitable to satisfy demand for remanufactured products.

The present paper addresses the issue of overestimation of returned products quality and its impact on the operation and profitability of reverse supply chains. This study is motivated by the case of ReCellular, a firm that remanufactures and sells used mobile phones. ReCellular’s practice is to collect used products at a number of different locations. At each location, a primary sorting of the returns is carried out based on a list provided by ReCellular itself. Specifically, there are several quality characteristics of a returned unit that determine the quality category in which the returned item will be classified. This list is shown in Table 1, adopted by Guide and Van Wassenhove (2001).

Table 1 about here

In total, there are 6 different product classes, which have been introduced with respect to the associated cost to remanufacture a unit of a certain quality standard. Table 2 shows the remanufacturing cost for each product class as reported in Guide et al. (2003), along with the estimated acquisition prices.

Table 2 about here

After returned products reach ReCellular’s remanufacturing facility, they are sorted and graded at a grading station. Each of the new quality classes that emerge is processed in a different remanufacturing cell. Although all quality classes can be remanufactured in any of the available cells, inaccurate sorting at ReCellular facilities may also increase remanufacturing time and cost. However, the procedure at ReCellular’s grading station can be considered accurate. On the other hand, the classification of returns carried out by the suppliers is considered inaccurate, for a number of reasons, which are explained below. Therefore, despite the suppliers’ categorization of a unit in a certain class, the cost that the firm bears during the remanufacturing process can be larger than estimated. This possible underestimation

of remanufacturing cost, which is a result of quality overestimation, can be attributed to three factors:

1. The list of product characteristics which determine the specific class in which a unit falls can not be exhaustive. Considering the complexity of some products it is easily understood that there can be some characteristics which are not included in that list but can increase the remanufacturing cost.

2. Since the classification process involves, to some extent, the subjective evaluation of a number of characteristics, it is prone to errors. It is not actually reported by ReCellular that these classification errors lead to overestimation of the overall returns quality. However, since a supplier is paid more for a higher quality product, we believe that in the majority of reverse supply chains quality overestimation is rather common.

3. The remanufacturing cost for returned units with similar quality problems is not necessarily the same for all those units. The presence of a certain attribute induces a minimum remanufacturing cost because of the standard remanufacturing process. However, in some cases more effort may be required, resulting in different remanufacturing cost for units classified in the same quality class.

The present paper examines how misclassification of returned products at the collection location (supplier) affects the profitability of remanufacturing and consequently, to what extent a firm should undertake some actions in order to improve the classification accuracy. Moreover, we provide some ideas on how the accuracy of quality evaluation can be incorporated in the procurement decision process.

2. Literature review

The management of products acquisition is an important issue in the reverse supply chains context. As Guide and Jayaraman (2000) point out, one of the complicating characteristics that the remanufacturing industry has to deal with is the increased uncertainty regarding returns' quality. The motivation of the current research comes from ReCellular practices regarding quality assessment and procurement of used products. An analytical description of ReCellular practices can be found in Guide and Van Wassenhove (2001) and in Guide et al. (2003). It should be noted that in Guide et al. (2003) the procurement and selling price are considered

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3 decision variables which influence both return and demand rates. A similar issue is
4 examined and in Vadde et al. (2007). In the current paper these two parameters, as
5 well as returns quantity and demand, are considered exogenous to the system, since
6 our main interest is to study the effect of classification errors on procurement and
7 production decisions as well as on the system profitability.
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12 The problem of procurement and production planning at ReCellular is also
13 examined in Jayaraman (2006). Jayaraman (2006) formulates the problem as a
14 mathematical programming model, considering however that the classification of
15 returns into different quality classes is accurate.
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19 The issue of timely classification of returns when multiple product classes
20 exist has been addressed in a number of recent papers. In Guide et al. (2006) the value
21 of time is explored: since the sorting of returns accelerates remanufacturing
22 procedures it increases the value salvaged by the used products. An implementation of
23 such practice in a real situation is included in Guide et al. (2005).
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27 Ferrer (2003) studies the procurement and remanufacturing decisions in a
28 single-period context under a number of different scenarios regarding the timing of
29 information concerning the quality of returns. Aras et al. (2004) study a similar
30 problem in a multi-period context. There is a recent stream of studies, in which the
31 value of returns' quality information is evaluated. In most of these papers returns'
32 quality is modeled as a random variable that follows a continuous probability density
33 function. Each of the possible infinite degrees of quality condition is related to a
34 different value of remanufacturing cost. In this context falls the work of Robotis et al.
35 (2005), Galbreth and Blackburn (2006) and Ketzenberg et al. (2006).
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39 The issue of grading errors during the sorting procedure of the returns has not
40 been addressed extensively in the existing literature. Two contributions that fall in this
41 context are those of Souza et al. (2002) and Zikopoulos and Tagaras (2008). Both
42 papers study the effect of grading errors on the value of advanced yield information,
43 by simulation and analytical methods, respectively.
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47 The contribution of the present paper is that a different aspect of grading errors
48 is explored. Specifically, we study the effect of quality overestimation in a reverse
49 supply chain, in order to quantify the loss that the remanufacturer suffers because of
50 supplier classification errors. Thus, it is possible to evaluate to what extent it would be
51 advisable for a remanufacturing firm to invest in the improvement of the classification
52 system and to work on the supplier reliability issue. Moreover, we examine the effect
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of the variability in the returns classification on the expected profit of the remanufacturer.

In Section 3 we introduce the problem structure, the basic assumptions and the notation used throughout the paper. In section 4 the formulation and analysis of the problem are presented, for both deterministic and stochastic demand for remanufactured products. Section 5 includes a brief numerical example and, finally, Section 6 summarizes the paper and presents some extensions of the current work.

3. Problem description

We study a reverse supply chain consisting of a remanufacturer, R , and N independent suppliers of returned items. The suppliers collect the products from end-users and grade them according to a list provided by the remanufacturer. This grading procedure results in the categorization of returns in M distinct quality classes. Each quality class, $j = 1, 2, \dots, M$, is characterized by different acquisition price, α_j , and remanufacturing cost, r_j . The suppliers inform R about the number of units per class that they have available for sale and R decides from which suppliers to order and the amount of returns in each class (s_{ij}). The class with the best quality is class $j = 1$ which has the higher value of acquisition cost, $\max_{j=1, \dots, M} \alpha_j = \alpha_1$.

Upon receipt, returns are inspected at the grading station so that their actual condition is revealed. We assume that at this stage, the sorting procedure is error-free. For each product of a certain class, there is a constant remanufacturing cost. Obviously, the better the condition of a unit, the lower the remanufacturing cost is: $\min_{j=1, \dots, M} r_j = r_1$. The remanufacturing cost of each unit is defined by the actual condition of a unit rather than by the supplier's classification. After classification, returns are remanufactured in order to reach a condition that makes them suitable to satisfy demand for remanufactured products, in a single-period context. Demand for remanufactured products is assumed known. This assumption is relaxed later permitting stochastic demand with known probability density function, $f(\cdot)$, and distribution function, $F(\cdot)$. After remanufacturing, all units are in the same condition regardless of their quality at the time that they were returned by end-users.

We assume that the suppliers tend to overestimate the quality of the returns. Specifically, we assume that in a lot of returns that supplier i has classified as

category j there is only a fraction of products $p_{ij} (\leq 1)$ that are indeed category j returns. The remaining products of the lot are category $j + 1$ returns. That is, quality overestimation is at most by one class. Each fraction p_{ij} is treated as a random variable following a known probability density function $g_{ij}(\cdot)$ and distribution function $G_{ij}(\cdot)$, apart from the fraction of products classified in category M , p_{iM} , for which we assume that they are always classified correctly ($p_{iM} = 1$). The available quantity of items in each category is:

$$q_{i1} = s_{i1}p_{i1}, \text{ and,}$$
$$q_{ij} = s_{ij}p_{ij} + s_{i(j-1)}(1 - p_{i(j-1)}), j = 2, \dots, M.$$

Since p_{ij} is a random variable, q_{ij} is also a random variable. After the remanufacturing process all available units can be used to satisfy demand for remanufactured units. The selling price, v , is the same for all units. When the remanufactured quantity does not match exactly demand the remanufacturer faces costs, related to shortage, b , or excess inventory, h , respectively. The cost of excess inventory is positive when the firm has to pay for the disposal of the unsold units, or negative if these units can be sold at a salvage value. Table 3 summarizes the notation used throughout the paper. The reverse supply chain for the problem examined is shown in Figure 1.

| Table 3 about here |
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| Figure 1 about here |
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A technical issue that affects the plausibility of the analysis that follows is the method employed for the evaluation of p_{ij} . A commonly used approach in similar situations is to follow a Bayesian update type procedure. Specifically, an initial estimation can be based on historical data. Each time that a specific supplier is used, the actual quality outcome is recorded and consequently the information is updated. The use of distribution of the Beta family is advisable for three reasons:

1. Distributions of the Beta family are usually used to model fractions of a quantity with a certain property.
2. The resulting distribution from the Bayesian update procedure belongs also in the Beta distributions family.
3. By appropriate selection of the distribution parameters a large number of different characteristics can be captured.

It should be mentioned that although the product has a limited sales period, this does not imply that the firm's relationship with its suppliers is not long-term. An issue that remains unanswered is the case of a new supplier for which no historical data is available. In this case the firm could use a rather rough estimation based on the experience with similar suppliers.

A key assumption of the model examined is that product misclassification is associated with the overestimation of product quality by one class; all classification errors result in classifying a quality $j+1$ return in j quality class. Although this assumption can be considered rather strict from a theoretical point of view, we argue that it is reasonable for practical situations. On the one hand, since suppliers are paid according to the quality of the collected products it is unlikely that a unit would be classified in an inferior quality class. On the other hand, it is not that probable that a unit's quality would be overestimated so dramatically in order to classify a $j + 1$ return in a class with superior quality than class j quality. For example, consider that returns are classified in 4 classes according to the presence of dirt, scratches, cracks or breaks, respectively. The required remanufacturing effort for correcting a certain problem may not be deterministic but it could be associated with a probability distribution. Figure 2 shows an example of associating a family of truncated normal probability distributions functions with the respective remanufacturing cost, shown on the horizontal axis.

Figure 2 about here

The four vertical lines in Figure 2 separate the remanufacturing cost to five regions (region 1 corresponds to the lowest remanufacturing cost). Obviously, the length of each quality class does not match exactly the length of a certain cost region. Consider for example the returned units that have only some scratches. Although they are all classified in class 2, a proportion of them would induce remanufacturing cost comparable to the cost of the majority of class 3 units. Moreover, there is also a proportion of class 2 units with remanufacturing cost in the fourth cost region. Assuming that each cost region extends for a width equal to the value of one standard deviation, σ , of the truncated normal distribution it would be easy to estimate the probability that a unit requires larger remanufacturing costs: there is a 68% probability of correct classification, 27% probability of quality overestimation by one class, 4% probability of quality overestimation by two classes, 0.2% probability of

quality overestimation by three classes, etc. Section 4 that follows introduces the mathematical model for the problem of returns' misclassification and comments on some of its properties.

4. Model formulation and analysis

The analysis that follows is presented in two sections, which differ in the nature of the single-period demand: firstly, the case of known demand is presented, followed by the analysis when demand is considered stochastic. In both cases, we derive the conditions for the determination of the optimal procurement policy. For the case of known demand the impact of the variance of p on the variance of the total profit is also examined.

4.1. Optimal procurement policy when demand is known

Since demand is known and equal to D , the objective function of the remanufacturing business depends on the relation between $\sum_{i=1}^N \sum_{j=1}^M s_{ij}$ and D . The objective function is given by:

$$E(\pi) = v \min \left(\sum_{i=1}^N \sum_{j=1}^M s_{ij}, D \right) - \sum_{i=1}^N \sum_{j=1}^M r_j E(q_{ij}) - \sum_{i=1}^N \left(\sum_{j=1}^M a_j s_{ij} \right) - h \left(\sum_{i=1}^N \sum_{j=1}^M s_{ij} - D \right)^+ - b \left(D - \sum_{i=1}^N \sum_{j=1}^M s_{ij} \right)^+ \tag{1}$$

where $(x)^+ = \max(0, x)$, and the terms in order that they appear stand for the sales revenue, the remanufacturing cost, the acquisition cost, the holding cost and the shortage cost, respectively. The first order partial derivatives of (1) with respect to each of the s_{ij} are given by

$$\frac{\partial E(\pi)}{\partial s_{ij}} = -r_j E(p_{ij}) - r_{j+1} [1 - E(p_{ij})] - a_j - h < 0 \text{ if } \sum_{i=1}^N \sum_{j=1}^M s_{ij} > D$$
$$\frac{\partial E(\pi)}{\partial s_{ij}} = v - r_j E(p_{ij}) - r_{j+1} [1 - E(p_{ij})] - a_j + b \text{ if } \sum_{i=1}^N \sum_{j=1}^M s_{ij} \leq D$$

whereas, for every second order partial (direct or mixed) derivative holds:

$$\frac{\partial^2 E(\pi)}{\partial s_{ij}^2} = \frac{\partial^2 E(\pi)}{\partial s_{ij} \partial s_{kl}} = 0,$$

and therefore, the optimal ordering policy can be evaluated using solely the first order conditions. It is obvious that $\sum_{i=1}^N \sum_{j=1}^M s_{ij} > D$ can never be optimal. Therefore, the optimal solution is such that $\sum_{i=1}^N \sum_{j=1}^M s_{ij} \leq D$, and the Karush - Kuhn - Tucker (K-K-T) conditions are written as follows:

$$I. \frac{\partial E(\pi)}{\partial s_{ij}} - \mu_{ij} - \mu_D \leq 0$$

$$II. s_{ij} \left[\frac{\partial E(\pi)}{\partial s_{ij}} - \mu_{ij} - \mu_D \right] = 0$$

$$III. s_{ij} - S_{ij} \leq 0$$

$$IV. \mu_{ij} (s_{ij} - S_{ij}) = 0$$

$$V. \sum_{i=1}^N \sum_{j=1}^M s_{ij} - D \leq 0$$

$$VI. \mu_D \left[\sum_{i=1}^N \sum_{j=1}^M s_{ij} - D \right] = 0,$$

where μ_{ij} , μ_D , are non-negative constants.

If for some combinations of i and j holds $\frac{\partial E(\pi)}{\partial s_{ij}} < 0$, condition (II) yields $s_{ij} =$

0. Define R as the state space such that $ij \in R$ if and only if $\frac{\partial E(\pi)}{\partial s_{ij}} > 0$. If $\sum_{ij \in R} S_{ij} \leq D$,

then the optimal decision is to procure all available quantities from classes $ij \in R$.

However, if $\sum_{ij \in R} S_{ij} > D$, there should be a number of $ij \in R^*$ ($R^* \subseteq R$) combinations,

for which $s_{ij} < S_{ij}$. For these ij combinations, by (IV) $\mu_{ij} = 0$. Since $\frac{\partial E(\pi)}{\partial s_{ij}} > 0$ for

every $ij \in R^*$ by (I) $\mu_D > 0$, and consequently by (VI), $\sum_{ij \in R^*} s_{ij} = D$. The optimal

procurement quantity from each ij combination is defined as follows:

for every ij combination for which $\frac{\partial E(\pi)}{\partial s_{ij}} > \mu_D$ by (I) $\mu_{ij} > 0$ and by (IV) $s_{ij} = S_{ij}$.

for every ij combination for which $\frac{\partial E(\pi)}{\partial s_{ij}} < \mu_D$ by (II) $s_{ij} = 0$, and

for combination ij for which holds $\frac{\partial E(\pi)}{\partial s_{ij}} = \mu_D$ the optimal $s_{ij} \in [0, S_{ij}]$ so that condition (VI) holds.

Based on the previous analysis, there is a simple procedure for determining the optimal ordering policy. The first step is to evaluate the value of $v + b - \{r_j E(p_{ij}) + r_{j+1} [1 - E(p_{ij})]\} - a_j$.

For all ij such that: $v + b - \{r_j E(p_{ij}) + r_{j+1} [1 - E(p_{ij})]\} - a_j > 0$, the sum of available quantities, $\sum_{ij \in R} S_{ij}$ is computed. If $\sum_{ij \in R} S_{ij} \leq D$, then the optimal solution is to order all available quantities from supplier i and category j , that is all units for which : $v + b - \{r_j E(p_{ij}) + r_{j+1} [1 - E(p_{ij})]\} - a_j > 0$

Otherwise, all s_{ij} are classified in a descending order according to the value of the expression:

$$v + b - \{r_j E(p_{ij}) + r_{j+1} [1 - E(p_{ij})]\} - a_j. \quad (2)$$

For the first ij in the aforementioned classification for which $\sum_{ij \in R^*} S_{ij} \geq D$, the optimal procurement quantity consists of all available quantities from the previous ij and s_{IJ} such that

$$\sum_{ij \in R^* - \{IJ\}} S_{ij} + s_{IJ} = D.$$

Since the values of v and b are constant across all suppliers and product classes, we rewrite (2) without taking into account these parameters. The selection of suppliers and product classes can be based on the expression:

$$r_j E(p_{ij}) + r_{j+1} [1 - E(p_{ij})] + a_j, \quad (3)$$

by selecting those supplier-class combinations that yield small values of (3). Therefore, not only the remanufacturing and acquisition costs are important, but also the classification error at the supplier. Expression (2) stands for the expected profit

from the procurement, remanufacturing and sale of one returned unit that supplier i has identified as a quality j return, while expression (3) stands for the respective variable cost per unit.

Since (3) yields the variable cost of each supplier - class combination, it can be used for the estimation of the deviation of the actual value of the variable cost from the one that the remanufacturer uses for the decision process. It is easy to show that the difference of these two values is given by:

$$(r_j - r_{j+1})[E(p_{ij}^e) - E(p_{ij}^a)]. \quad (4)$$

where $E(p_{ij}^e), E(p_{ij}^a)$ are the estimated and actual, respectively expected values of p_{ij} .

Setting $E(p_{ij}^e) = 1$ in (4) yields the difference in the product evaluation if the effect of misclassification is ignored. As can be observed by (4), the difference in product evaluation depends on the value of overestimation and the difference in the remanufacturing cost of the two classes involved in the misclassification phenomenon. On the other hand, the remanufacturer's cost related to the quality overestimation is associated with the difference in the acquisition cost of these two classes, namely:

$$(\alpha_j - \alpha_{j+1})[1 - E(p_{ij}^a)]. \quad (5)$$

Since (5) yields the savings in variable costs because of the improvement in the classification accuracy it can help in the estimation of a premium that can be established for encouraging suppliers to improve their classification accuracy. Figure 3 shows graphically the savings per product class by the implementation of an error-free classification procedure (i.e. $p_{ij} = 1$) for all possible values of $E(p_{ij}^a)$ at a certain supplier j , using the ReCellular's α_j and r_j values (Table 2).

Figure 3 about here

An interesting special case of the problem examined here, is the case in which the cost of procuring and remanufacturing of a returned unit is the same independently of the unit class, namely:

$$r_j + \alpha_j = C, \text{ for } j = 1, 2, \dots, M,$$

and, moreover, the probability of quality overestimation is identical for each product class at a certain supplier, namely:

$$p_{ij} = p_i, \text{ for } j = 1, 2, \dots, M-1.$$

It can be shown that in this case, (3) simplifies to:

$$(a_j - a_{j+1})[1 - E(p_i)] + C. \tag{6}$$

Thus, the procurement from a specific supplier can be based on the relationship of the acquisition cost between sequential returns classes. When considering procurement from different suppliers the criterion for procurement of a certain class, i , from supplier k over class j from supplier m is reduced to:

$$(a_i - a_{i+1})[1 - E(p_k)] < (a_j - a_{j+1})[1 - E(p_m)]. \tag{7}$$

That is the decision is affected by the supplier classification accuracy as well as the relationship between acquisition costs.

4.1.2. The impact of the variance of p

In this section we study the impact of the variance of the suppliers grading errors on remanufacturer's profitability. In other words, we examine how the variance of p affects the variance of π . The evaluation of the variance of π is important because of difficulties in the estimation of the actual percentage of misclassified units. The poor estimation of p can be expressed by selecting a distribution of p with a large variance. For example, an initial estimate for the distribution $g(p)$ could be the uniform distribution with mean 0.5. Monitoring of the suppliers history and continuous updates of the distribution parameters result in better understanding of the procedure and therefore in more accurate estimation of $E(p)$ and in different posterior distribution, $g'(p)$, characterized by lower variance. Consequently, the quantification of the effect of the variance of p on π offers some information about the extent to which a firm should invest in order to minimize the variance of p .

There are two factors that are responsible for the non-zero variance of p . The first one is related to the fact that suppliers' errors are not the same at every replenishment instant. The second factor is related to the estimation of the value of p by the remanufacturing company. For example, when there is not enough information about the actual distribution of the errors of suppliers, the remanufacturer should use the distribution parameters that give a greater value of variance. As more information becomes available, this estimation can be updated and eventually the variance will be minimized. The case with zero variance can be considered as a rather optimistic bound for the variance of p .

For clarity of illustration, we present the analysis in two parts. Firstly, we deal with the case that the optimal procurement policy calls for procuring only one class of returns from a single supplier. This is the simpler case which provides some intuition in order to proceed to the general case in which more than one of the suppliers and quality classes are used.

Procurement of a single quality class from a single supplier

Assume that the optimal policy is to procure only s_{ij} . In such case the optimal profit for the remanufacturer is:

$$\begin{aligned}\pi &= v s_{ij} - r_j s_{ij} p_{ij} - r_{j+1} s_{ij} (1 - p_{ij}) - a_j s_{ij} - b (D - s_{ij})^+ \\ &= \left[(v - r_{j+1} - a_j) + (r_{j+1} - r_j) p_{ij} \right] s_{ij} - b (D - s_{ij})^+.\end{aligned}\quad (8)$$

Since π is a linear function of p_{ij} , the variance of the profit when the optimal policy is implemented is given by:

$$Var(\pi) = (r_{j+1} - r_j)^2 (s_{ij})^2 Var(p_{ij}), \quad (9)$$

while for the standard deviation holds:

$$StDev(\pi) = (r_{j+1} - r_j) s_{ij} StDev(p_{ij}).$$

The general case

When more than one of the suppliers and quality classes are used the optimal profit is given by:

$$\pi = v \sum_{i=1}^N \sum_{j=1}^M s_{ij} - \sum_{i=1}^N \sum_{j=1}^M r_j s_{ij} p_{ij} - \sum_{i=1}^N \left(\sum_{j=1}^M a_j s_{ij} \right) - b \left(D - \sum_{i=1}^N \sum_{j=1}^M s_{ij} \right)^+, \quad (10)$$

where some $s_{ij} = 0$, if certain combinations of quality class and supplier are not used. The optimal profit is a linear combination of the random variables p_{ij} , therefore the variance of the optimal profit can be computed by:

$$\begin{aligned}Var(\pi) &= \sum_{i=1}^N \sum_{j=1}^M \left[(r_{j+1} - r_j)^2 Var(p_{ij}) s_{ij}^2 \right] \\ &\quad + \sum_{i=1}^N \sum_{\substack{k=1 \\ k \neq i}}^N \sum_{j=1}^M \sum_{\substack{m=1 \\ m \neq j}}^M \left[(r_{j+1} - r_j) (r_{m+1} - r_m) Cov(p_{ij}, p_{km}) s_{ij} s_{km} \right].\end{aligned}\quad (11)$$

It is natural to assume that the covariance of the percentage of misclassified units can take non-zero values either between different product classes for the same supplier or for the same product class at different suppliers. For the former case, non-zero covariance can be attributed to the fact that at each supplier the classification process is carried out following a certain procedure which results in a certain tendency to overestimate the true returns quality. The covariance between products of the same class coming from different suppliers can be related to the nature of the product. For example consider that a certain product type has a particular defect that can not be easily detected. If this defect is such that it changes the classification of the returned unit, we expect to have non-zero correlation between the suppliers, and more specifically this correlation is expected to be positive.

In accordance with the former remarks, we can set in (11):

$$Cov(p_{ij}, p_{km}) = 0, \tag{12}$$

$$Cov(p_{ij}, p_{kj}) \neq 0, \text{ and} \tag{13}$$

$$Cov(p_{ij}, p_{im}) \neq 0. \tag{14}$$

It is interesting to note that if (13) and (14) indicate the presence of positive correlation both across different return's classes at a certain supplier and across different suppliers for a certain quality class, this results in an increase of the overall profit variability. Another interesting conclusion, derived by the previous analysis and the form of (11), is that the variance of remanufacturer's profit increases in $(r_{j+1} - r_j)$. Therefore, the introduction of new quality classes of returned units may decrease the overall profit variability, given that a suitable low-error sorting procedure can be implemented.

4.2 Stochastic demand

Having defined the optimal procurement policy in the case of known demand, we proceed to the case that some uncertainty on the demand side exists. The objective function in this case is given by:

$$E(\pi) = v \left(\int_0^{\sum_{i=1}^N \sum_{j=1}^M s_{ij}} x f(x) dx + \int_{\sum_{i=1}^N \sum_{j=1}^M s_{ij}}^{\infty} \sum_{i=1}^N \sum_{j=1}^M s_{ij} f(x) dx \right)$$

$$\begin{aligned}
& -\sum_{i=1}^N \sum_{j=1}^M r_j E(q_{ij}) - \sum_{i=1}^N \left(\sum_{j=1}^M a_j s_{ij} \right) \\
& -h \int_0^{\sum_{i=1}^N \sum_{j=1}^M s_{ij}} \left(\sum_{i=1}^N \sum_{j=1}^M s_{ij} - x \right) f(x) dx - b \int_{\sum_{i=1}^N \sum_{j=1}^M s_{ij}}^{\infty} \left(x - \sum_{i=1}^N \sum_{j=1}^M s_{ij} \right) f(x) dx.
\end{aligned}$$

Taking derivatives with respect to the procurement quantities, s_{ij} , the first and second order partial derivatives are obtained, as follows:

$$\frac{\partial E(\pi)}{\partial s_{ij}} = v + b - (v + b + h) F \left(\sum_{i=1}^N \sum_{j=1}^M s_{ij} \right) - \left\{ r_j E(p_{ij}) + r_{j+1} [1 - E(p_{ij})] \right\} - a_j$$

$$\frac{\partial^2 E(\pi)}{\partial s_{ij}^2} = \frac{\partial^2 E(\pi)}{\partial s_{ij} \partial s_{kl}} = -(v + b + h) f \left(\sum_{i=1}^N \sum_{j=1}^M s_{ij} \right) < 0.$$

The objective function has to be maximized subject to the $M \cdot N$ constraints of the form:

$$s_{ij} \leq S_{ij}.$$

The K-K-T conditions are similar to the ones presented for the known demand case. Examining the K-K-T conditions it is easy to conclude that the optimal solution will involve procurement of all the available quantity for a number of i and j combinations, plus a quantity that lies between 0 and S_{ij} for a specific class and supplier combination and no procurement from all other classes. As the value of the expression

$$\frac{v + b - \left\{ r_j E(p_{ij}) + r_{j+1} [1 - E(p_{ij})] \right\} - a_j}{(v + b + h)} \quad (15)$$

decreases it becomes less probable that the specific combination of class and supplier will enter the optimal solution. Since v , b and h are constant across all supplier - quality class combinations, we conclude that the evaluation of each combination can be based on condition (3). Thus, although the nature of demand complicates the decision procedure, it does not alter the criterion for the supplier - product class classification.

It is interesting to note that since both (2) and (15) define the relevant profitability of a certain combination of supplier and quality class, it is the difference

of the values of (2) and (15) for sequential classes that impacts the overall profitability and the importance of error-free classification. This feature is further explored in Section 5 via a numerical example.

5. Numerical illustration and discussion

In this section we present a couple of numerical examples in order to illustrate the optimization procedure proposed in the previous sections and to provide further intuition on some aspects of the problem studied. For the first of these examples we use some general parameter values while for the second we use real data that corresponds to ReCellular actual acquisition and remanufacturing costs. The numerical analysis is carried out for a reverse supply chain consisting of a remanufacturer and $N = 2$ suppliers. We assume that there are $M = 3$ different quality classes of returns characterized by different acquisition prices and remanufacturing costs.

In the first numerical example we use two different sets for these costs, as shown in Table 4, differing in the relevant profitability of the first two quality classes.

Table 4 about here

In the first case the sum of procurement and remanufacturing costs for class 1 returns is equal to \$21, while for the class 2 returns is \$52. The second value set assumes a \$36 total cost for quality class 1 and \$37 for class 2. However, the values selection is such that the cost of procuring and remanufacturing one class 1 and one class 2 units is equal for these two sets (i.e. \$73). The procurement and remanufacturing costs for class 3 returns are equal in both cases, while the selling price exceeds by approximately 1.64% the sum of acquisition and remanufacturing costs of class 3 ($v = 62$). The demand for remanufactured products is assumed to be known and equal to 100 units. The quantity per quality class collected at each supplier is set equal to 25 units. Since there are 3 quality classes at the 2 available suppliers, there are in total 150 units available for the remanufacturer to purchase. It is assumed that the probability of misclassification at a certain supplier is constant across the first two quality classes, $p_{ij} = p_i$, $i = 1, 2$ and $j = 1, 2$, while $p_{i3} = 1$, $i = 1, 2$. Furthermore, it is assumed that p_I follows a Beta distribution with parameters $a_I = 10$ and $b_I = 2$, which yield $E(p_I) = a_I / (a_I + b_I) = 10/12$. The probability of misclassification at the

second supplier is examined in two levels, low and high misclassification probability. In the former case the parameters of the Beta distribution are $a_2 = 8$ and $b_2 = 4$ while in the latter case they are $a_2 = 4$ and $b_2 = 8$, resulting in $E(p_2) = 8/12$ and $E(p_2) = 4/12$, respectively. The two different values of classification accuracy are examined comparatively to the case of error-free classification at the second supplier, i.e. $E(p_2) = 1$.

The optimal procurement decisions are obtained using the conditions provided in Section 4. In all cases the optimal policy is to procure all available class 1 and 2 units. That is, the remanufacturing facility pays the acquisition cost of 50 class 1 units and 50 class 2 units. However, after the sorting and grading procedure the true quality condition of the returns is revealed. For example, for $E(p_1) = 10/12$ and $E(p_2) = 8/12$ there are on average 37.5 class 1, 50 class 2 and 12.5 class 3 units available. Naturally, the remanufacturing cost depends on the actual quality condition of the remanufacturing quantity. Table 5 shows the expected profit for the optimal policy as well as the percentage profit reduction due to classification errors at the second supplier.

Table 5 about here

When classification at the second supplier is accurate the values of expected profit for the two cost sets are comparable. However, decreasing the accuracy of the second supplier has more pronounced effect when the difference in the attractiveness of class 1 units over class 2 units increases.

What Table 5 basically illustrates is that the contemporary presence of poor classification accuracy and large difference in the attractiveness of sequential quality classes results in noticeable profit reduction. When only one of these factors is present, firms may not have enough motivation for establishing more reliable sorting procedures. The results of the brief numerical example can be represented schematically, as shown in Figure 4. Specifically, for two levels of classification accuracy, low and high, and two levels of sequential classes' profitability difference, the four regions formed call for different action by the remanufacturer. The region that has the widest impact on the remanufacturer's profit is the top-right-hand one, which is characterized by low accuracy and high profitability difference.

Figure 4 about here

An interesting conclusion is that the remanufacturer can in some extent reduce the profit losses because of low classification accuracy by changing the acquisition prices offered to suppliers; an acquisition policy with rather small differences between sequential quality classes can result in greater robustness of the economical outcome of remanufacturing activities.

We proceed by presenting another numerical example, using real data for the acquisition and remanufacturing costs, α_j and r_j , respectively, corresponding to the ReCellular case. Guide and Van Wassenhove (2001) report the fee that ReCellular pays to suppliers for each quality class for 4 different phone models. By repeating the numerical analysis for the first three quality classes of these 4 sets as well as for the α_j values shown in Table 2 (referred to as model 5 in Table 6), we reach to the results presented in Table 6. The r_j ($j = 1, 2, 3$) values are set equal to Table 2 values which correspond to ReCellular actual remanufacturing cost. Since for the remainder parameter values we were unable to obtain real data, the classification and demand characteristics are identical to those used in the previous numerical example. As before, v exceeds $\alpha_3 + r_3$ by approximately 1.64%. Moreover, the returns quantity is set equal to 25 units per quality class and supplier, resulting into 150 available returned units.

Table 6 about here

Table 6 reveals that the impact of quality overestimation is substantial. The relatively lower values of the expected profit of models 2 and 5 observed in Table 6 are due to the rather low profit margins of quality class 1 units of these models. Although in these cases there is a large percentage profit reduction as classification accuracy deteriorates, this phenomenon can be attributed, to a certain extent, to scale differences. However, the absolute differences of the expected profit under different classification accuracy are comparable, indicating limited interaction of profit margins with the classification accuracy effect.

6. Extensions and conclusion

The current paper addresses the issue of returns quality overestimation by the suppliers and examines the effect of such misclassification on the remanufacturer's profitability. An interesting conclusion by the analysis is that the economical impact

of poor classification is primarily affected by the extent of quality overestimation and the difference of acquisition and remanufacturing costs of sequential quality classes at the same or different suppliers.

Since the present study deals with the assessment of misclassification cost it can help in the evaluation of alternative methods that aim at the reduction of classification errors, given that they are important (i.e., the firm operates in the top-right-hand region of Figure 4). The determination of corrective actions depends on a number of problem parameters, as for example, the relative market position of the suppliers and the remanufacturer. A major remanufacturer can impose some financial penalties according to the classification accuracy. On the other hand, a remanufacturer that cooperates with a major supplier could not take such strict actions. In both cases though, the quantification of the losses associated with misclassification is useful, for the penalty evaluation in the former case and as a negotiation tool, in the latter. In every case, if misclassification is due to malicious supplier intention, monitoring of classification accuracy is basic in order to ensure that there will not be a continuous reduction of classification accuracy. The study of alternative initiatives to the suppliers in order to improve this accuracy is an interesting area.

Another alternative, apart of the provision of initiatives, is the implementation of more accurate sorting methods. When misclassification involving a certain pair of sequential quality classes is highly costly the firm can implement a certain sorting method that ensures that the probability of misclassification in these classes is minimized or even vanished. For example consider the case of ReCellular, which uses a combination of objective and subjective criteria for returns classification, as shown in Table 1. Since a quality class 3 item differs from a quality class 4 item in that the latter fails to power up, the probability of misclassification between these two classes vanishes (i.e. $p_{i3} = 1$). Similar exact tests may be available or can be created for the discrimination of classes with high cost differences.

Some other interesting conclusions include the following:

- Demand uncertainty does not alter the suppliers' evaluation criterion.
- Introducing quality classes with values of the remanufacturing cost that lay between the costs of existing classes reduces the misclassification effect on remanufacturer's profit, as long as the respective sorting procedure ensures small classification errors.

- Careful selection of acquisition prices may decrease the impact of misclassification errors on the total profitability.

An interesting extension to the problem studied in the current paper would be the examination of the case in which the suppliers offer the possibility of acquiring the total quantity collected or nothing at all. The model formulation in such case would be the same, apart from the inclusion of a number of binary variables which would indicate if a certain supplier is used. Specifically, if we denote:

$$S_i = \sum_{j=1}^M S_{ij}$$
$$A_i = \sum_{j=1}^M a_j S_{ij}$$
$$R_i = r_M s_{iM} + \sum_{j=1}^{M-1} \{r_j S_{ij} E(p_{ij}) + r_{j+1} S_{ij} [1 - E(p_{ij})]\}$$

The objective function is given by:

$$E(\pi) = \nu \min \left[\sum_{i=1}^N Q_i y_i, D \right] - \sum_{i=1}^N R_i y_i - \sum_{i=1}^N A_i y_i - h \left(\sum_{i=1}^N S_i y_i - D \right)^+ - b \left(D - \sum_{i=1}^N S_i y_i \right)^+,$$

where $y_i, i = 1, 2, \dots, N$ are binary decision variables.

A similar approach can be found in Lebreton and Tuma (2006) where the case of car and truck tires is examined.

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Table 1. Nominal quality metrics for grading mobile telephones (adopted by Guide and van Wassenhove, 2001).

Nominal quality level	Description
1	Passes a functional and electrical test and with minimal cleaning, produces a nice looking phone. Phones show no water or physical damage and have a good antenna, display, lens keypad, and a working primary charging system.
2	Passes a functional and electrical test and with cleaning and buffing can produce a phone that is acceptable in appearance. Phones show no water or physical damage and have a good antenna, display, lens keypad, and a working primary charging system. Keypad and plastic may show fading or uniform discoloration.
3	Passes a functional and electrical test but will require some amount of rework to make this phone usable. Phone does not have a good display and working keypad. Phones may have some physical damage and may show signs of wear that will require the replacement of some parts.
4	Phones may have the same physical appearance as a grade 1 phone, but the lens may have some scratches. These phones failed the test or did not power up. Many of these phones are repairable and should produce a good yield with some technical knowledge.
5	Phones have the same physical appearance as a grade 2 phone, but failed the test or did not power up. Many of these phones can be repaired and should produce a good yield with some technical knowledge.
6	Phones have failed the test or did not power up. Phones have some damage but can be used for parts salvage.

Table 2. Remanufacturing cost and acquisition prices in \$ (Guide et al. 2003).

Class (j)	Acquisition price (a_j)	Remanufacturing cost (r_j)
1	26.8	5
2	17.3	20
3	9.8	30
4	6.3	35
5	4.0	40
6	2.0	45

Table 3. Notation.

j	returned units categories, $j = 1, 2, \dots, M$ (1 best quality)
i	supplier index, $i = 1, 2, \dots, N$
a_j	acquisition cost per unit classified in category j
r_j	remanufacturing cost per unit of category j
v	sales revenue
p_{ij}	fraction of returns classified by supplier i as j returns that are indeed j returns (a random variable)
$g_{ij}(\cdot)$	probability density function of p_{ij}
D	demand for remanufactured units (constant or random)
s_{ij}	quantity of returns classified as class j ordered from supplier i
S_{ij}	quantity of returns classified as class j collected at supplier i
S_i	total quantity collected at supplier i
q_{ij}	actual quantity of class j returns received by supplier i
h	cost of unsold remanufactured unit
b	shortage cost per unit of demand that is not satisfied

Table 4. Parameters (α_j, r_j) for the two cost sets.

j		1	2	3
set 1	α_j	19	14	13
	r_j	2	38	48
set 2	α_j	15	14	13
	r_j	21	23	48

Table 5. Optimal expected profit $E(\pi)$ and percentage reduction.

$E(p_2)$	cost set	$E(\pi)$	%
1	1	2454.17	
	2	2493.75	
8/12	1	2070.83	15.6
	2	2268.75	9.0
4/12	1	1687.50	31.2
	2	2043.75	18.0

Table 6. Parameter values, expected profit $E(\pi)$ and percentage reduction for ReCellular acquisition policy.

Model	a_1	a_2	a_3	v	$E[\pi E(p_2)=1]$	% reduction of $E(\pi)$ for	
						$E(p_2)=8/12$	$E(p_2)=4/12$
1	33.0	31.0	29.0	60.0	1497.92	13.9	27.8
2	85.0	75.0	65.0	96.5	368.75	39.5	73.4
3	110.0	105.0	100.0	132.0	1147.92	18.1	32.7
4	40.0	35.0	30.0	61.0	1047.92	19.9	35.8
5	26.8	17.3	9.8	40.5	542.92	34.5	57.6

Fig. 1. Schematic representation of the reverse supply chain.

Fig. 2. Example of a family of truncated normal distributions describing the relationship between quality class and remanufacturing cost.

Fig. 3. Remanufacturer savings by error-free classification.

Fig. 4. Magnitude of the effect of classification error probability and difference of sequential classes' profitability on remanufacturer's expected profit.