Fuzzy Logic Approach to Represent and Propagate Imprecision in Agri-Environmental Indicator Assessment.

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Abstract— *The indicator of groundwater contamination developed and used in agriculture is calculated from data available in the field or data estimated by an expert. The modeling of this indicator generally requires a large number of parameters whose measure is imprecise. Several information sources provide information about the same imprecise quantities which have to be combined for defining what is called an "indicator of groundwater contamination" (I_{gro}). This indicator estimates the impact of cultivation practices on the groundwater contamination.*
In this paper, we explore a possibilistic information fusion method by using the notion of maximal coherent subsets to represent the imprecisions of multisource variables of the indicator. We also calculate the bounds of this indicator, and we propagate imprecision by using an interval analysis. Finally, we present the indicator's results for pesticides applied on different crops.

Keywords— Possibility theory, fuzzy set, fusion, maximal coherent subset, interval analysis, indicator

1 Introduction

Data used in application domains are often imperfect: imprecise, uncertain, incoherent... Some data come from information sources (database, expert) and others are approximative measurements. When multiple sources –such as a database, an expert, a website, a book– deliver information about some unknown quantity, aggregating this information can be a tedious task, especially when information are incoherent. Actually, there are many approaches to specify the impact of input parameter imprecision of a model, such as statistical sampling methods, the Monte-Carlo method [1], and bayesian method. These methods require either a significant number of data and a lot of computation time, or an exact definition of the statistical properties of the input parameters. There exists also alternative approaches based on fuzzy set and possibility theory [2, 3] to express, in a non-probabilistic sense, imprecision of parameters. Each approach represents parameters imprecision in a specific way. When, we need to aggregate data from multiple sources, fuzzy approaches show more flexibility in the treatment of incoherent information. Recently, Destercke et al [4] explored a possibilistic information fusion using the notion of maximal coherent subsets to synthesize information from several incoherent data. In this paper, we will use this method to synthesize an information from several information sources to calculate variables used in indicator assessment.
An indicator measures a certain aspect of the agrosystem. It is proposed to help farmers to improve the environmental sustainability of their agricultural practices and to reduce the pollution of environment such as indicator of groundwater contamination I_{gro}. It estimates the possibility for a pesticide to reach groundwater through leaching. The modeling and the assessment of an indicator generally require a large number of parameters whose measure is imprecise such as the variable "soil depth", that indicates how thick the soil cover is. Multiple sources provide information about the imprecise quantities which compose the indicator I_{gro}, for example, pesticide characteristics such as pesticide half-life $DT50$. The calculation of I_{gro} is based on a system of decision rules using fuzzy sets [5]. Authors in [5] do not take into account the imprecision in parameters of the indicator.
In this paper, we present an original work that, at our knowledge, has not been carried out until now. We propose here an analysis of imprecision of the indicator, especially variables used and their nature. We are interested in imprecision of parameters provided by several sources. Then, we use imprecision for evaluating the amplitude and the effect of imprecision on the value of the indicator I_{gro}. Firstly, we change the input values of variables in the process of calculation. Then, we use intervals of values in place of single values of input variables in the calculation process of the indicator I_{gro}. We also explore a fusion method, using the notion of maximal coherent subset, for summarizing information about the quantity given by several sources [4]. Finally, we calculate a variation interval for the indicator I_{gro} given by the input intervals of the variables.
The paper is divided as follows: the first section presents the indicator of groundwater contamination I_{gro}, its definition, structure and data used. Theoretical preliminaries are introduced in the third section. Then, the fourth section describes the fusion method used and the way how maximal coherent subsets can be used to obtain intervals of the imprecise variables; we also explore a way to compute the lower and upper bounds of the indicator. Then, the fifth section presents the indicator of groundwater contamination, its definition and its structure. Finally, the sixth section shows the results of the indicator for four pesticides applied in different crops.

2 Application description

In this section, we define the indicator I_{gro}, its structure and data used in the process of calculation.

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2.1 *The risk of groundwater contamination I_{gro}*

2.1.1 *Definition and data used*

The indicator module I_{gro} reflects the potential of a pesticide to reach groundwater through leaching and to affect its potential use as a source of drinking water. I_{gro} as proposed by Van der Werf and Zimmer [5] depends on four input variables : (1) pesticide leaching potential (GUS); (2) position ; (3) leaching risk, and (4) toxicity of the pesticide for humans (based on Acceptable Daily Intake). Table 1 shows variables used to calculate I_{gro}.

Table 1: All variables for the I_{gro} and their description.

Variables	Description
Pesticide characteristics (quantitative variables)	
GUS	pesticide leaching potential
DT50	pesticide half-life
koc	organic-carbon partition
ADI	Acceptable Daily Intake
Environmental characteristics (qualitative variables)	
Leaching risk	Quantity leaching
Application characteristics (quantitative variables)	
Position	Position of application

Variable ”pesticide leaching potential” (GUS) provides an estimation of the risk of leaching of the compound. This potentiel is calculated by the formula:

$$GUS = \log(DT50) * (4 - \log(koc)) \quad (1)$$

where variable DT50 is the pesticide half-life and ”koc” is the organic-carbone partition coefficient.

Variable DT50 is the time required for the pesticide concentration under defined conditions to decrease to 50% of the amount in application. Variable ”koc” is the organic-carbon constant that describes the tendency of a pesticide to bind to soil particles. Variable ”position” is the position of application of the pesticide (on the crop, on the soil, in the soil). This position is the interception rate of active ingredient per leaf area of studied culture. It is obtained from variable ”soil cover”. Variable ”soil cover” is a value thats depends on the leaf surface of the culture. The variation of ”soil cover” over time is fixed. Then, the position depends on the date of application by the farmer. The variable ”leaching risk” depends on characteristics of the soil. In I_{gro}, the estimation of soil ”leaching risk” is given by experts on a scale between 0 (minor leaching risk) and 1 (major leaching risk). The ”position” and ”leaching risk” take values between 0 and 1. The variable ”Acceptable Daily Intake” (ADI) reflects chronic toxicity from humans. The pesticide properties (DT50, koc, and ADI) are given by several heterogeneous information sources such as Agritox (database, France), RIVM (database, Netherlands), Pesticide Manual (book, england), Dabene (expert, France), etc . . . . For each pesticide, these infomation sources give fuzzy intervals for each characteristic. Let us consider the following example: Three sources (Pesticide manuel: a book, Agritox: a database, Dabene: an expert) provide information, about the variable ”DT50” (pesticide half-life) of ”I_{gro}”, in term of two intervals Int_1 and Int_2. The information is summarized in Table 2 and represented on Figure 1. The interval Int_2 represents the most plausible values for the variable DT50.

Table 2: Example information from sources

Source	Int_1	Int_2
Dabene	[10,42]	[20,35]
Agritox	[46,53]	50
Pesticide Manual	[21,56]	[35,40]

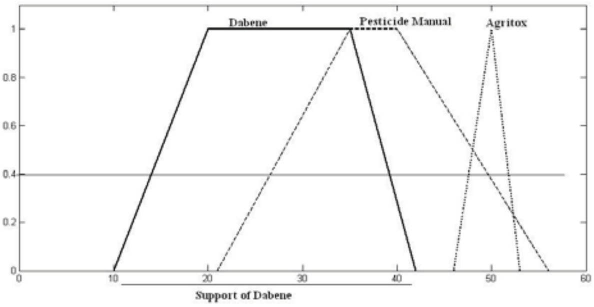

Figure 1: An example of values of DT50 given by sources, α-cut with $\alpha = 0.4$ and the support of data given by Dabene

2.1.2 *The structure of the indicator of groundwater contamination I_{gro}*

The indicator of groundwater contamination I_{gro} is calculated on a scale between 0 (maximal risk) and 10 (minimal risk). These values are calculated according to fuzzy if-then rules and to the degree of membership of the input variables to fuzzy subsets. The mechanism is explained below.

To calculate the indicator I_{gro}, we need to aggregate expertise knowledge and variables heterogeneous (qualitatives and quantitatives), authors in [5] are used fuzzy control [6] for achieving this two goals. For all input variables, two fuzzy subsets F (Favourable, i.e. the sets of values giving rise to acceptable environmental effect) and U (Unfavourable, i.e. the set of values giving rise to unacceptable environmental effect) are defined. Table 3 shows favourable and unfavourable limits for input variables of I_{gro}, which are extracted from literature or based on human expert knowledge.

Table 3: Favourable and unfavourable limits of input variables

Variable	Favourable limit	Unfavourable limit
DT50	1	30
GUS	1.8	2.8
ADI	1	10^{-4}
Position	1	0
Leaching risk	0	1

For each class (favourable, unfavourable, and transition interval which corresponds to values between favourable and unfavourable limits), we have defined a membership function on knowledge of experts.

The membership of values of input variables takes any value in the interval [0,1]. The value 0 represents complete non-membership and the value 1 represents complete membership. Values between 0 and 1 are used to represent partial membership. The membership function is defined in such a way that the value of an input variable either belongs fully to one of the

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two fuzzy subsets (favourable, unfavourable), or partially to both. In the latter case, the value is within a transition interval. We used membership functions that are sinus shaped in the transition interval, as they provide smoother variations of the output values than membership functions that are linear in the transition interval [5].
For example, experts classify the leaching potential GUS into three classes. They affect pesticides classified as "leacher" (i.e. GUS > 2.8 or called unfavourable subset) a membership value of 1 for the fuzzy subset U and membership value of 0 for the fuzzy subset F. Pesticides classified as "no-leacher" (GUS < 1.8 or called favourable subset) are given a membership value of 0 for the subset U and a membership value of 1 for the fuzzy subset F. The class of borderline compounds (1.8 < GUS < 2.8) falls within a transition interval where the membership value for F decreases from 1 (GUS = 1.8) to 0 (GUS= 2.8), and the membership value of U increases from 0 to 1 (see Fig. 2). The favourable (respectively unfavourable) membership function "F-Function" (respectively "U-Function") of GUS is the degree for GUS to be in the favourable subset (respectively unfavourable subset). We have also "U-Function" = 1 - "F-Function". The function of subset favourable of GUS is $0.5 + 0.5\cos(\pi(\text{GUS} - 1.8))$. If variable GUS has a value of 2.5, then "F-Function" = 0.16 and "U-Function" = 0.84.

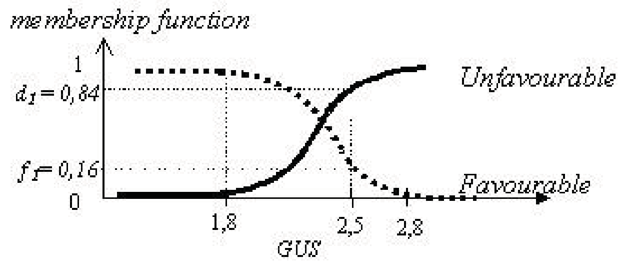

Figure 2: F-Function and U-Function function of variable GUS

The calculations are carried out according to a set of if-then rules. The experts attribute values between 0 and 10 for each rule.They consider firstly –what determines the leaching– the properties of organic matter (molecule), so they use variable GUS. If the variable GUS is favourable (GUS < 1.8), then there is no problem. The molecule does not present the characteristics of leaching. If variable GUS is unfavourable (GUS > 2.8), the experts are interested in the other variables of indicator. When a molecule is able to leach, it is able to reach the soil. Then, the experts use variable "position" of application. If the variable "position" is favourable, experts set a value to 9 for indicating the risk of leaching, but it is not too important. Then the environment is taken into account through the variable "leaching risk" (is the molecule leaching or not?, is the soil sensitive to the leaching?) and, finally they set on the variable "ADI".
Fig. 3 shows the if-then rules of I_{gro}. For example, if all input variables of I_{gro} are F then conclusion is 10. If all input variables of I_{gro} are U then conclusion is 0.

For each rule, we obtain the truth value by applying the minimum operator on the set of membership of the rule. For the first rule, if all input variables are F, the memberships of

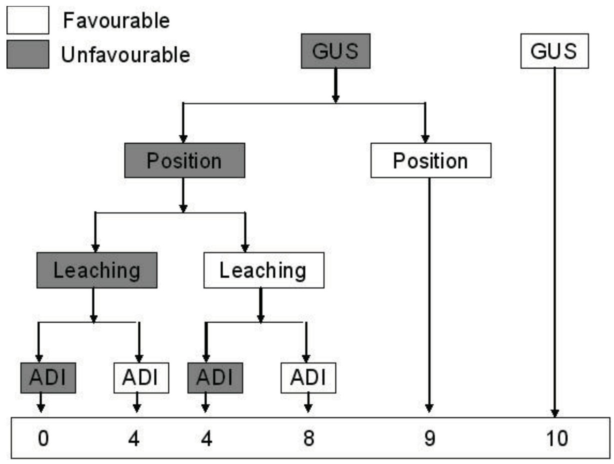

Figure 3: If-then rules of the indicator I_{gro} (I_{gro} has $2^4 = 16$ rules)

four input variables are f_i for each variable, then the truth value is $\omega_1 = \min_{i=1,\ldots,4}(f_i)$. The final score of the indicator is the average of rule conclusions weighted by their values of truth

$$I_{gro} = \frac{\sum_{i=1}^{16} \omega_i s_i}{\sum_{i=1}^{16} \omega_i}$$

3 Preliminaries

3.1 Problem statement

In this paper, we consider the indicator of groundwater contamination I_{gro}, which depends on parameters: GUS, DT50, "koc", ADI, "leaching risk" and "position". It is calculated by Van der Werf and others in [5] as $I_{gro} = \frac{\sum_{i=1}^{16} \omega_i s_i}{\sum_{i=1}^{16} \omega_i}$, where ω_i depends of variables of I_{gro}, and $s_i \in [0, 10]$.
We are interested here in multisource variable DT50, koc, ADI. Assume that the set of n information sources provide n fuzzy intervals about these variables (see Table 2). We have to search the most plausible values to calculate the indicator. We use here the support of fuzzy interval (α-cut with $\alpha = 0$ i.e. the interval Int_1 in Table 2). We use the notion of maximal coherent subset into a possibilistic fusion to synthesize a final result for multisource variables. These results combined with values of others variables will be used to compute the bounds (lower and upper) of the indicator I_{gro}.
In this paper, $\forall x \in \mathbb{R}$, x^- and x^+ represent the lower and upper bounds of interval where x can vary.

3.2 Possibility theory, fuzzy sets

In this section, we briefly summarize basic concepts of possibility theory, which are needed for understanding this paper. Fuzzy sets were firstly introduced by Zadeh [2] as a possible way to handle uncertainty in processing incertain or imprecise data, and to control expert knowledge. This theory allows the notion of graduation to express whether an element belongs to a set (For more details, see[2, 3, 7]).

3.2.1 Possibility theory

Possibility theory was introduced in 1978, in connection with fuzzy set theory, to allow reasoning to be carried out on imprecise or vague knowledge, making it possible to deal with uncertainties on knowledge [3, 8]. The basic tool of possibility

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theory is possibility distribution. A possibility distribution is equivalent to the definition of a normalized fuzzy membership function. We are interested in this theory, in this paper, because we need to synthesize information of a variables which come from various sources. When we want to synthetize the value of a variable (i.e. approximate the value of the variable depending a set of a different sources (This process is called ”information fusion” (see here after). In such a situation, possibility theory can be used to handle this kind of problems. This theory presents many choices of fusion operators in different contexts.

3.3 Information fusion

Definition 1 *Information fusion consists of merging, or exploiting conjointly, several sources of information for answering questions of interest and make proper decisions.*

A large set of information fusion operators has been designed in the possibility theory framework [3]. They can be split into three subsets, according to the behavior of operators:

- Conjunctive type operators: it is the equivalent of a set intersection. It makes the assumption that all sources are reliable (i.e. all sources give information with high confidence), and usually results in very precise information. If there is an incoherence between variable values (i.e. detected by domain expert), then the result of the conjunction becomes poorly reliable, or even empty.
- Disjunctive type operators: it is the equivalent to a set union. It makes the assumption that at least one source is reliable. The result of a disjunctive operator can be considered as very reliable.
- Trade-off type operators: they are compromise operators between conjunctive and disjunctive operators. They are typically used when sources are partially conflicting (i.e. in contradiction). As its name indicates, such an operator tries to make a trade-off between disjunction and conjunction for achieving a good balance between informativeness and reliability.

4 Methodology

4.1 Fusion based on maximal coherent subsets to assess imprecision of multi-source variables

Information sources which supply the information about imprecise variables (DT50, koc, ADI) are heterogeneous (book, expert, database). They come from different places (France, USA, Netherlands), then the type of soil, weather and laboratory conditions are not similar. Furthermore, we ignore the reliability of sources (they do not supply a degree of confidence with the value). They provide data in term of a fuzzy interval (or fuzzy number). Also, no information about conflict and relations between sources is available. Then, we can not use a conjunctive rule (respectively disjunctive and compromise rule). We need a fusion method which take into account all information given by sources (i.e. without discarding any source). The notion of maximal coherent subset is a natural way for achieving this goal. The fusion method using the notion of maximal coherent subset consists firstly inn applying a conjunctive operator into each non-conflicting subset of sources, then a disjunctive operator between those partial results. With such a method, as much precision as possible is gained when we do not ignore any sources [9, 10]. We will explain in details how this approach applies to support of fuzzy intervals of variables of I_{gro} (see the section 4.3).

4.2 Computing maximal coherent subsets of intervals

In this section, we describe how we can obtain values of multisource variable using the fusion method based on maximal coherent subset of intervals, which are give by information sources. Let us consider n intervals, $I_i = [a_i, b_i], (i = 1, \ldots, n)$. This method allows to find every maximal subsets of sources. A maximal subset is obtained when $\bigcap_{i \in [n]} I_i \neq \emptyset$, where $[n]$ represents the set of 2^n subsets of the set $\{1, \ldots, n\}$. Then, we apply the union of these partial results (i.e. $\bigcup_j \bigcap_{i \in [n]} I_i$).

Algorithm 1: Maximal coherent subset of intervals

Input: n intervals
Output: List of m maximal coherent subsets I_j
List=$\emptyset$;$j = 1$;$I_j = \emptyset$;
Order in an increasing order
$\{a_i, i = 1, \ldots, n\} \cup \{b_i, i = 1, \ldots, n\}$;
Rename them $\{c_i, i = 1, \ldots, 2n\}$with $type(i) = a$ if $c_i = a_k$ and $type(i) = b$ if $c_i = b_k$;
for $i = 1$ to $2n - 1$ **do**
    if $type(i) = a$ **then**
        Add source k to I_j t.q. $c_i = a_k$;
        if $type(i + 1) = b$**then**
            Add k to List (I_j);
            $j = j + 1$;
    else
        Remove source k from I_j such as $c_i = b_k$;

Algorithm 1, that finds maximal coherent subsets, was introduced by Dubois and others in [11]. The algorithm 1 is linear in the number of intervals, and thus computationally efficient. The algorithm is based on an increasing sorting of the interval end-points into a sequence $(c_i)_{i=1,\ldots,2n}$. Each time and only then, an element c_i of type upper bound which is an upper bound of an interval, followed by an element c_{i+1} of type lower bound wich is a lower bound of an interval meet, a maximally coherent set is obtained.

4.3 Example of application

We consider the variable DT50 (pesticide half-life) provided by three information sources (see Table 2): a French expert (Dabene), a French database (Agritox), and an England book (Pesticide Manual).
Using algorithm 1 on the level $\alpha = 0$ (i.e. the support Int_1 of fuzzy intervals in Table 2), we find two maximal coherent subsets $K_1 = \{\text{Dabene, Pesticide Manual}\}$ and $K_2 = \{\text{Agritox, Pesticide Manual}\}$. After applying the maximal coherent subset method, the result of the variable DT50 is: (Dabene $\cap$ Pesticide Manual) $\cup$ (Agritox $\cap$ Pesticide Manual) = [21,42] $\cup$ [46,53].

4.4 Computation method to the fuzzy weighted average

The objective of this section is to compute the bounds of the indicator I_{gro}. The function $f(s_1, s_2, \ldots, s_n, \omega_1, \ldots, \omega_n) = \frac{\sum_{i=1}^{n} \omega_i s_i}{\sum_{i=1}^{n} \omega_i}$, where ω_i is a fuzzy interval and $s_i \in \mathbb{R}$, is called

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fuzzy weighted average by Dong and Wong [12]. Authors in [12] proposed an algorithm to compute the fuzzy weighted average. Their algorithm is based on the α-cut representation of fuzzy sets and combinatorial interval analysis. Subsequently, Liou and Wang [13] suggested an improved fuzzy weighted average algorithm to simplify the computational process. Afterwards, Lee and Park [14] have proposed an even more efficient algorithm for fuzzy weighted average by reducing the number of arithmetic operations to $O(n \log n)$. The main idea of Lee and Park in [14] is to sort the s_i variables such as for all $i < j$, we have $s_i \leq s_j$. In this order, we can find a rank k with $1 < k < n$ such as for all $i \leq k$, the function f is decreasing with respect to ω_i, and for all $i > k$, the function f is increasing with respect to ω_i. Then, we can compute the lower and the upper bounds of the fuzzy weighted average in accordance with [15],which are bounds of I$_{gro}$, as:

$$f_k^- = \frac{\sum_{j=1}^{k}(\omega_j)^+.s_j + \sum_{j=k+1}^{n}(\omega_j)^-.s_j}{\sum_{j=1}^{k}(\omega_j)^+ + \sum_{j=k+1}^{n}(\omega_j)^-} \quad (2)$$

$$f_k^+ = \frac{\sum_{j=1}^{k}(\omega_j)^-.s_j + \sum_{j=k+1}^{n}(\omega_j)^+.s_j}{\sum_{j=1}^{k}(\omega_j)^- + \sum_{j=i+1}^{n}(\omega_k)^+} \quad (3)$$

Where ω_i^- and ω_i^+ are respectively the lower and upper bounds of ω_i. The equation (2) (respectively (3)) represent the lower (upper) bound of the level k and $k = 1, \ldots, n$. Then, we determinate the fuzzy interval of indicator such as introduced by Fortin and others in [15]:

$$[\min_{k=1,\ldots,n}(f_k^-), \max_{k=1,\ldots,n}(f_k^+)] \quad (4)$$

5 Results

In this application when computing I_{gro}, about fifteen information sources are used such as ARS (USA), Agritox (France), Pesticide manual (England)... These sources provide information based on a fuzzy interval or on a fuzzy number. In this paper, we use the support of fuzzy intervals given by the sources. The decision rules do not change. For each variable, we replace the input value by an input interval. We propagate the imprecision into computation indicator to calculate a fuzzy interval of indicator. For a pesticide application, an indicator of groundwater contamination score is calculated.
The application considers the indicator of groundwater contamination I$_{gro}$ in several sites with different crops. The positions of application of pesticides is shown in Table 4. Its value is calculated as: "position $= \frac{\text{soil coverd by the crop}}{100}$". The "soil covered" by the crop is estimated by the user of the systems [16]. The variable "leaching risk" set the value of 0.9 [16].

Table 4: Values of the variables of application condition

Pesticide name	Position	soil covered
Chloridazone	0.9	90
Glyphosate	0.75	75
Nicosulfuron	0.1	10
Isoproturon	0.1	10

Pesticide properties (DT50, koc, and ADI) given by several information source are calculated by fusion based on maximal coherent subsets. Table 5 shows the results of pesticide characteristics. The values shown in Table 5 are convexified to simplify the calculation. By using the interval analysis [17] and the equation 1, we compute the fuzzy interval of variable GUS, such as:

$$\text{GUS}^- = \log(\text{DT50}^-) \times (4 - \log(\text{koc}^+)) \quad (5)$$

$$\text{GUS}^+ = \log(\text{DT50}^+) \times (4 - \log(\text{koc}^-)) \quad (6)$$

where ([DT50$^-$,DT50$^+$]) and ([koc$^-$,koc$^+$]) are fuzzy intervals of variables DT50 and Koc.

Table 5: Results of pesticide characteristics

Pesticide name	GUS	DT50	koc	ADI
Chloridazone	[2.53,3.19]	[21,46]	120	0.025
Glyphosate	[0.73,2.97]	[18,47]	[167,2640]	0.3
Nicosulfuron	[2.78,5.39]	[15,43]	[5,43]	0.4
Isoproturon	[2.68,3.03]	[22,28]	[80,99.99]	0.4

Afterwards, we compute the membership intervals of input variables of indicator. For each input variable that varies in $[a, b]$ and has a membership function f, we compute the membership interval by using the function:

$$f([a,b]) = \begin{cases} [f(a), f(b)] & \text{if } f \text{ is increasing} \\ [f(b), f(a)] & \text{if } f \text{ is decreasing} \end{cases} \quad (7)$$

Then, we calculate the truth intervals of the sixteen decision rules of I$_{gro}$. These truth intervals can be computed by minimum extension applied on intervals such as $\min([a,b],[c,d]) = [\min(a,c), \min(b,d)]$. Then, we use the equations (2) and (3) to compute respectively the lower and upper bounds of indicator for the sixteen α-cuts established by the fuzzy input membership intervals. Finally, we obtain the final interval of indicator by the equation (4).
The Table 6 shows the results of I$_{gro}$ for different pesticides. The fuzzy interval in Table 6 represent the exact interval when the indicator is varying. It represents also the support, of the fuzzy interval of the indicator I$_{gro}$, that can obtain from all fuzzy intervals of input variables.

Table 6: Results of indicator

Pesticide name	Interval of I$_{gro}$
Chloridazone	[4.3,9.82]
Glyphosate	[0,10]
Nicosulfuron	[4.42, 4.45]
Isoproturon	[4.42,5.27]

The results presented in Table 6 show a lot of imprecision in some cases. We take example of the organic matter "glyphosate": the indicator obtained by application of glyphosate on the soil varies between 0 and 10. This imprecision is due to the variable GUS –which varies in [0.73,2.97]– that reach limits fixed by experts in Table 3 (i.e (lower limit) $<$ (favourable limit), and (upper limit) $>$ (unfavourable limit)). We note that the indicator varies in [0,10], when we have one of the input variables has a lower (respectively upper) limit exceeds favourable (respectively unfavourable) limit. In the

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calculation of only one value of indicator I_{gro} [5], the role of the variable GUS is more important than the role of others variables. In this paper, all variables have the same influence in the final result.

6 Discussion and conclusions

In this paper, we introduced the calculus of the indicator of groundwater contamination I_{gro}. We try to control the imprecisions of input variables by using a possibilistic fusion method based on the maximal coherent subsets. Then, we use the notion of interval analysis to propagate the imprecision in the process of calculus of indicator. A fusion method for merging fuzzy subsets, based on the notion of maximal coherent subsets, is proposed to represent the imprecision of input parameters of the indicator of groundwater contamination. This notion appears as a very natural way to conciliate two objectives, gaining information and considering all the conflicts between sources. The method of fusion is simple: it can be applied without any additional information, and its computational complexity remains affordable. The way is summarizes information is conceptually attractive: maximal coherent subsets are the best we can do in the presence of conflict.
We have also described a way to calculate the bounds of indicator. The method of fuzzy interval arithmetic used in this paper is also simple. The support of the fuzzy number of intervals is calculated. Also, we can calculate the membership function of indicators by computing several values of α-cut of input variables. This method allows to obtain exact fuzzy profiles of indicator with reducing time complexity as in the sampling case. It still remains to validate the fusion method using the notion of maximal coherent subset in contrast with other fusion rules. We have to calculate the fuzzy interval for all α-cut to obtain the distribution of I_{gro}. We plane to use the methodology described in this paper (representing by fuzzy intervals the imprecision and propagating this imprecision by interval analysis) to calculate other indicators such as indicator of pesticide in environment which has a process of calculation similar to process of I_{gro}.

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