The cosmological constant puzzle

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Abstract. The accelerating expansion of the Universe points to a small positive vacuum energy density and negative vacuum pressure. A strong candidate is the cosmological constant in Einstein's equations of General Relativity. Possible contributions are zero-point energies and the condensates associated with spontaneous symmetry breaking. The vacuum energy density extracted from astrophysics is 10^{56} times smaller than the value expected from quantum fields and Standard Model particle physics. Is the vacuum energy density time dependent ? We give an introduction to the cosmological constant puzzle and ideas how to solve it.

1. Introduction

Accelerating expansion of the Universe has been observed in studies of the cosmic microwave background, gravitational lensing, type 1a supernovae and ripples in the large scale distribution of galaxies. These observations point to a new "dark energy density" or negative pressure in the vacuum perceived by gravitational interactions. The simplest explanation is a small positive value for the cosmological constant in Einstein's equations of General Relativity. However, this brings with it many challenging puzzles. The cosmological constant puzzle connects the Universe on cosmological scales (the very large) with subatomic physics (the very small).

The physical world we observe today is built from spin-1/2 fermions interacting through the exchange of gauge bosons: massless spin-1 photons and gluons; massive W and Z bosons; and gravitational interactions. QED is manifest in the Coulomb phase, QCD is manifest in the confinement phase and the electroweak interaction is manifest in the Higgs phase. Further ingredients are needed to allow the formation of large-scale structures on the galactic scale and to explain the accelerating expansion of the Universe. These are the mysterious dark matter and dark energy, respectively. Current observations point to an energy budget of the Universe where just 4% is composed of atoms, 23% involves dark matter (possibly made of new elementary particles) and 73% is dark energy (the energy density of the vacuum perceived by gravitational interactions).

The vacuum energy density receives possible contributions from the zero-point energies of quantum fields and condensates associated with spontaneous symmetry breaking. The vacuum is associated with various condensates. The QCD scale associated with quark and gluon confinement is around 1 GeV, while the electroweak mass scale associated with the W and Z boson masses is around 250 GeV. These scales are many orders of magnitude less than the Planck-mass scale of around 10^{19} GeV, where gravitational interactions are supposed to be sensitive to quantum effects. The vacuum energy density associated with dark energy is characterised by a scale around 0.002 eV, typical of the range of possible light neutrino masses, and a cosmological constant, which is 56 orders of magnitude less than the value expected from the Higgs condensate with no extra new physics. Why is this vacuum "dark energy" finite, and why so small ?

The challenge presented by gravitation and the cosmological constant is fundamentally different from particle physics in that gravity couples to everything whereas other physics processes and experiments involve measuring the differences between quantities. The challenge of reconciling zero-point energies in quantum field theory with gravitation was recognised in the earliest days of these theories by Pauli [1] and remains a puzzle today [2]. The absolute value of the zero point energy of a quantum mechanical system has no physical meaning when gravitational coupling is ignored. All that is measured are changes of the zero-point energy under variations of system parameters or of external couplings.

It is commonly believed that we live in the second period of accelerating expansion of the Universe, with the first being *inflation*, where the Universe underwent a period of exponential expansion causing it to expand by at least a factor of 10^{26} in an infinitesimal time ($\sim 10^{-33}$ seconds). Are these two periods connected ? Is the vacuum energy density or cosmological constant time dependent ?

This review is directed towards particle and subatomic physicists. We emphasise conceptual details and key physics issues. Readers interested in more technical details are refered to articles [3, 4, 5, 6] (on the acceleration of the Universe) and [4, 7, 8, 9, 10, 11, 12] (for details of different dark energy models). The plan of this article is as follows. In Section 2 we discuss General Relativity including precision tests of gravitation. Section 3 deals with cosmology and the accelerating Universe. In Section 4 we discuss the cosmological constant and vacuum energies in quantum field theory. Section 5 gives an overview of recent ideas aimed at solving the "dark energy" puzzle.

2. General Relativity

We start our discussion with Einstein's theory of General Relativity to introduce the cosmological constant and motivate its connection to vacuum energy in the quantum world, joining the interface of classical gravitation and subatomic quantum physics.

General Relativity is Einstein's dynamical theory of gravity [13, 14] where gravitation is connected with the geometry of 4-dimensional spacetime. General Relativity is based on and at the same time extends Newton's theory of gravitation:

(i) Space and time form a four-dimensional manifold, called spacetime. Its geometrical properties are described by its metric $g_{\mu\nu}$. The metric is determined by gravitation, and the components are calculated from Einstein's fundamental equations. The metric may depend on the location in space and time.

(ii) The physical laws are the same in every coordinate scheme; mathematically speaking, they have to be covariant. (This condition leads to a gauge theory interpretation of gravitation.)

(iii) Locally, the gravitational field can be (almost) transformed away by choosing a coordinate system in which the metric $g_{\mu\nu}$ is flat. In such "local inertial systems" the laws of Special Relativity are valid.

These principles lead to Einstein's equations where gravitation and spacetime curvature couple to the energy-momentum tensor. Quoting John Wheeler: *"Matter tells space how to warp. And warped space tells matter how to move."* As in gauge theories of particle physics, the local symmetry properties of the theory determine the dynamics. We briefly sketch the key details of the derivation leading to the appearance of the cosmological constant or vacuum energy term.

General Relativity is a gauge theory, built on invariance under local co-ordinate transformations

$$x^\mu \to \tilde{x}^\mu(x). \tag{1}$$

For a general tensor

$$S^{\mu}{}_{\nu\rho} \to S^{\mu'}{}_{\nu'\rho'} = \frac{\partial\tilde{x}^{\mu'}}{\partial x^{\mu}}\frac{\partial x^{\nu}}{\partial\tilde{x}^{\nu'}}\frac{\partial x^{\rho}}{\partial\tilde{x}^{\rho'}} S^{\mu}{}_{\nu\rho}. \tag{2}$$

If an equation between two tensors holds in one frame, it holds in all frames. To formulate General Relativity we need an invariant volume element and the gravitational equivalent of a gauge covariant derivative. To define an invariant volume element, first note that

$$d^4x \to d^4\tilde{x} \equiv \mathtt{det}\Big(\frac{\partial\tilde{x}^{\mu'}}{\partial x^{\mu}}\Big) d^4x. \tag{3}$$

If we define

$$-g \equiv \mathtt{det}(g_{\mu\nu}), \tag{4}$$

then

$$(-g) \to (-\tilde{g}) = \Big[\mathtt{det}\Big(\frac{\partial\tilde{x}^{\mu'}}{\partial x^{\mu}}\Big)\Big]^{-2}(-g) \tag{5}$$

and the invariant volume element is

$$\sqrt{-g}\ d^4x = \sqrt{-\tilde{g}}\ d^4\tilde{x}. \tag{6}$$

We next formulate the gravitational covariant derivative. Derivatives of scalar fields transform as

$$\partial_{\mu}\phi \to \partial_{\mu'}\phi = \frac{\partial x^{\mu}}{\partial\tilde{x}^{\mu'}}\partial_{\mu}\phi. \tag{7}$$

For a derivative acting on a vector field transforming as $V^{\mu} \to V^{\mu'} = \frac{\partial\tilde{x}^{\mu'}}{\partial x^{\mu}} V^{\mu}$, one finds

$$\begin{aligned} \partial_{\mu}V^{\nu} \to \partial_{\mu'}V^{\nu'} &= \Big(\frac{\partial x^{\mu}}{\partial\tilde{x}^{\mu'}}\partial_{\mu}\Big)\Big(\frac{\partial\tilde{x}^{\nu'}}{\partial x^{\nu}}V^{\nu}\Big) \\ &= \frac{\partial x^{\mu}}{\partial\tilde{x}^{\mu'}}\frac{\partial\tilde{x}^{\nu'}}{\partial x^{\nu}}(\partial_{\mu}V^{\nu}) + \frac{\partial x^{\mu}}{\partial\tilde{x}^{\mu'}}\frac{\partial^2\tilde{x}^{\nu'}}{\partial x^{\nu}\partial x^{\mu}}V^{\nu}. \end{aligned} \tag{8}$$

To avoid and cancel the second term on the right hand side of Eq.(8), we need to replace the simple derivative ∂_{μ} by the gravitational covariant derivative

$$\nabla_{\mu}V^{\nu} = \partial_{\mu}V^{\nu} + \Gamma^{\nu}_{\mu\lambda}V^{\lambda}. \tag{9}$$

Here the $\Gamma^{\nu}_{\mu\lambda}$ are called connection coefficients and satisfy the transformation rule

$$\Gamma^{\nu}_{\mu\lambda} \to \Gamma^{\nu'}_{\mu'\lambda'} = \frac{\partial x^{\mu}}{\partial\tilde{x}^{\mu'}}\frac{\partial x^{\lambda}}{\partial\tilde{x}^{\lambda'}}\frac{\partial\tilde{x}^{\nu'}}{\partial x^{\nu}}\Gamma^{\nu}_{\mu\lambda} - \frac{\partial x^{\mu}}{\partial\tilde{x}^{\mu'}}\frac{\partial x^{\lambda}}{\partial\tilde{x}^{\lambda'}}\frac{\partial^2\tilde{x}^{\nu'}}{\partial x^{\mu}\partial x^{\lambda}}. \tag{10}$$

Eq.(9) is the gravitational analogue of the gauge covariant derivatives in particle physics. The connection coefficients have a natural expression in terms of the metric and its derivatives

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2}g^{\sigma\rho}\Big(\partial_{\mu}g_{\nu\rho} + \partial_{\nu}g_{\rho\mu} - \partial_{\rho}g_{\mu\nu}\Big) \tag{11}$$

known as Christoffel symbols. For covariant derivatives of tensors with lower indices, there is an additional minus sign and change of the index which is summed over:

$\nabla_\mu V_\nu = \partial_\mu V_\nu - \Gamma^\lambda_{\mu\nu} V_\lambda$. One can check that the covariant derivative of the metric vanishes

$$\nabla_\sigma g_{\mu\nu} = \nabla_\sigma g^{\mu\nu} = 0. \tag{12}$$

Information about the curvature of a space-time manifold is encoded in the Riemann curvature tensor $R^\sigma{}_{\mu\alpha\beta}$ which is defined like the field tensor in gauge theories,

$$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) V_\rho \equiv -R^\lambda{}_{\rho\mu\nu} V_\lambda \tag{13}$$

where

$$R^\sigma{}_{\mu\alpha\beta} \equiv \partial_\alpha \Gamma^\sigma_{\mu\beta} - \partial_\beta \Gamma^\sigma_{\mu\alpha} + \Gamma^\sigma_{\alpha\lambda} \Gamma^\lambda_{\mu\beta} - \Gamma^\sigma_{\beta\lambda} \Gamma^\lambda_{\mu\alpha}. \tag{14}$$

All components of $R^\sigma{}_{\mu\alpha\beta}$ vanish if and only if the (4-dimensional) space is flat (meaning that there is a global co-ordinate system in which the metric components are everywhere constant). One can define two useful contractions: the Ricci tensor

$$R_{\alpha\beta} = R^\lambda{}_{\alpha\lambda\beta} \tag{15}$$

and Ricci scalar

$$R = g^{\mu\nu} R_{\mu\nu}. \tag{16}$$

Using symmetry properties under exchange of indices of the Riemann curvature tensor one finds the Bianchi identity

$$\nabla_{[\lambda} R_{\mu\nu]\rho\sigma} = 0 \tag{17}$$

where we anti-symmetrise over the indices λ, μ and ν. We next define the Einstein tensor

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{18}$$

Then, applying the Bianchi identity, one obtains

$$\nabla^\mu G_{\mu\nu} = 0, \tag{19}$$

so that the Einstein tensor is *covariantly conserved*.

In the presence of matter we would like to couple gravity to the matter source. We require a symmetric tensor. Einstein's guess was to take the energy momentum tensor $T_{\mu\nu}$ which is then also covariantly conserved, viz.

$$\nabla^\mu T_{\mu\nu} = 0. \tag{20}$$

One thus obtains Einstein's equations of General Relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^2} T_{\mu\nu} + \Lambda g_{\mu\nu}. \tag{21}$$

Here Λ is the cosmological constant. It is a number and is independent of the local metric $g_{\mu\nu}$. The cosmological constant term $\Lambda g_{\mu\nu}$ appears both as an integration constant in the covariant conservation of Einstein's tensor (following from Eq.(12)) and as an invariant in the gravitational action

$$S = \int d^4x \sqrt{-g} \frac{1}{16\pi G} \Big(R - \Lambda + 16\pi G\, \mathcal{L}_M \Big). \tag{22}$$

(Here $\mathcal{L}_M$ is the matter contribution.) Einstein's equations link the geometry of spacetime to the energy-momentum tensor. The metric in the equations of motion for General Relativity determines the geodesics – that is, how particles move under the influence of the gravitational field.

When placed on the right-hand side of Einstein's equation the cosmological constant term acts like a vacuum energy density $\rho_{\rm vac}$. Since the vacuum is Lorentz invariant, the vacuum expectation value of $T_{\mu\nu}$ is an invariant, symmetric tensor which is proportional to the metric tensor

$$\langle T_{\mu\nu} \rangle_{\rm vac} = -\frac{g_{\mu\nu}}{c^2} \rho_{\rm vac}. \tag{23}$$

The cosmological constant is given by

$$\Lambda = 8\pi G \rho_{\rm vac} + \Lambda_0. \tag{24}$$

Possible contributions include zero-point energies and the condensates associated with spontaneous symmetry breaking; Λ_0 is a possible counterterm. If the net vacuum energy is finite it will have gravitational effect.

Newton's theory of gravitation is recovered as the low curvature limit of General Relativity for slowly moving particles. Consider a metric which is almost Minkowski, but with a specific kind of small perturbation $ds^2 = (1 + 2\Phi/c^2)c^2 dt^2 - (1 - 2\Phi/c^2) d\vec{x}^2$ where $\Phi = -GM/r$. The 00 component of Einstein's equation is just the Poisson equation for Newtonian gravity $\nabla^2 \Phi = 4\pi G \rho$ where ρ is the matter density.

2.1. Tests of General Relativity

General Relativity has proved very successful everywhere the theory has been tested from laboratory torsion balance experiments through to large scale relativistic effects in double pulsar systems plus gravitational lensing. Ongoing and future tests include the search for gravitational waves.

At short distances, recent torsion balance experiments [15] have found that Newton's Inverse Square Law (ISL) holds down to a length scale of 56 μm. This result is especially interesting because this length is less than the dark-energy length scale $\lambda = (\hbar c/\rho_{\rm vac})^{\frac{1}{4}} \sim 85 \mu$m corresponding to energy density $\rho_{\rm vac} \sim 3.8$ keV/cm^3 (or $(0.002 \text{ eV})^4$) extracted from dark energy astrophysics experiments.

The Double Pulsar system PSR J0737-3039A/B provides a precise test of General Relativity. This is the first known binary system containing two neutron stars both emitting regular radio pulses. It is also the most relativistic binary pulsar discovered so far with higher mean orbital velocities and accelerations than those of other binary pulsars. Precision tests of General Relativity observables in the strong-field regime have verified the theory at the 0.05% level [16].

The bending of light by gravity can lead to gravitational lensing, where multiple images of the same distant astronomical object are visible in the sky. A large scale precision test of General Relativity was recently proposed based on the determination of a quantity E_G that combines measures of large-scale gravitational lensing, galaxy clustering, and the growth rate of structure. Reyes et al. [17] report $E_G = 0.39 \pm 0.06$ based on data from a sample of more than 70,000 distant galaxies, which is consistent with the value of 0.4 predicted by General Relativity.

Black holes are a prediction of General Relativity and are now well established by observation. Black holes are regions of space from which nothing, not even light, can escape: Due to a very compact mass the escape velocity exceeds the speed of light. Black holes are believed to be the end state for massive star collapse. They can be inferred by tracking the movement of a group of stars that orbit a region in space. Alternatively, when gas falls into a stellar black hole from a companion star, the gas spirals inward, heating to very high temperatures and emitting large amounts of radiation that can be detected in experiments. Supermassive black holes (SMBHs) with masses up to a few billion solar masses are believed to occupy the centre of galaxies. The Milky Way is believed to contain a supermassive black hole at its centre called Sagittarius A* (Sgr A*) with mass about 4 million solar masses. Using the Hubble telescope, SMBH candidates have been observed in about 100 galaxies. There are strong correlations (scaling relations) between the masses of these supermassive black holes and the global properties of their host galaxies (including dark matter contributions) [18]. SMBHs have profound effects on both the dynamical and structure properties of the entire galaxy in which they reside, suggesting that they play an important role in the galaxy formation process.

There are open challenges in both experimental and theoretical investigations of gravitation. Considerable experimental effort is focussed on the search for gravitational waves predicted by General Relativity [19]. Strong indirect evidence comes from observations of binary pulsars. Gravitational waves involve propagation of disturbances in space-time which can be triggered during cataclysmic events involving stars or black holes, and they could even have been generated in the very early Universe, well before any star formed, merely as a consequence of the dynamics and expansion of the Universe. In the latter case, these waves should provide a background signal of gravitational waves coming from all directions in space. The effects that are of interest are small, but experiments are gradually achieving a sensitivity that will test cosmological models. However, gravitational waves have not yet been detected directly. Direct detection is a major goal of current relativity-related research. Key experiments aimed at the search for gravitational waves are VIRGO near Pisa in Italy, LIGO (the Laser Interferometer Gravitational-Wave Observatory) in the United States and the planned LISA experiment in space which should also permit gravitational wave astronomy.

Unanswered questions remain, the most fundamental being how General Relativity can be reconciled with the laws of quantum physics to produce a complete and self-consistent theory of quantum gravity [20]. Do we need new fields and interactions, or do the key ingredients involve just the Standard Model fields plus the gravitational metric with an *asymptotic safe* ultraviolet fixed point [21] ?

While laboratory experiments, solar systems tests and cosmological observations have all been in complete agreement with General Relativity, these tests do not eliminate the possibility for the graviton to have a very small mass. A conservative bound based on galaxy clusters of size 580 kpc is $m_{\rm graviton} \leq 10^{-29}$ eV [22].

3. Cosmology and the expanding Universe

For cosmology one starts with the assumption that the Universe is homogeneous and isotropic. Then, the metric takes the general Friedmann-Lemaitre-Robertson-Walker (FLRW) form

$$ds^2 = c^2 dt^2 - a^2(t)\Big[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big]. \tag{25}$$

Here k is the three-space curvature constant ($k = 0$,+1,-1 for a spatially flat, closed or open Universe). ‡ The function $a(t)$ is known as the scale factor and tells us the relative sizes of the spatial surfaces. It represents the radius of curvature of the 3-dimensional space and describes the time-dependence of the 3-dimensional space. Here the time t, called cosmic or proper time, is the one measured by a clock at rest in these co-ordinates.

The Universe is modelled as a perfect fluid, characterised by its rest frame energy density ρ and isotropic pressure p. It has no viscosity or shear stresses. The average density ρ and pressure p have the same value everywhere, but can depend on time. We write the energy-momentum tensor $T_{\mu\nu}$ as $\texttt{diag}[\rho, p, p, p]$, viz.

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu} \tag{26}$$

where U_μ are four-velocities and $g^{\mu\nu} U_\mu U_\nu = -1$. Then Einstein's equations (21) for the metric (25) and diagonal components of the energy-momentum tensor give (setting $c = 1$):

$$\begin{aligned} \left(\frac{\dot a}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{1}{3}\Lambda \\ \frac{\ddot a}{a} &= -\frac{4\pi G}{3}(\rho + 3p) + \frac{1}{3}\Lambda. \end{aligned} \tag{27}$$

These equations are known as the Friedmann equations. The Λ cosmological constant contribution is written explictly. It is assumed to contain all contributions from the vacuum energy density.

The Hubble parameter

$$H = \frac{\dot a}{a} \tag{28}$$

measures the expansion of the Universe. The empirical value for the present era is

$$H_0 = (\dot a / a)_0 = h_0 \times 100 \text{ km/s/Mpc}, \quad h_0 = 0.70 \pm 0.01. \tag{29}$$

The ratio of the two equations in (27) defines the deceleration parameter

$$q \equiv -\frac{\ddot a a}{\dot a^2} = -\Big(1 + \frac{\dot H}{H^2}\Big). \tag{30}$$

‡ For a flat geometry the interior angles of a triangle defined by three beams of light will sum to 180°. For a closed Universe the interior angles of a triangle would sum to more than 180°. For an open Universe the angles would sum to less than 180°.

The expansion of the Universe accelerates if the second derivative of the Universe scale factor, $\ddot{a}$, is positive, which follows if the equation of state of the Universe is such that $w = p/\rho < -1/3$.

One can solve the Friedmann equations (27) for possible flat Universes ($k = 0$).

- For a matter dominated Universe, the equation of state has $p = 0$.

$$\begin{aligned} a(t) &\propto t^{\frac{2}{3}} \\ \rho_{\rm matter} &\propto a^{-3} \end{aligned} \tag{31}$$

- For a radiation dominated Universe (with just photons) the equation of state has $p = \frac{1}{3}\rho$, which corresponds to a traceless energy-momentum tensor. (Photons are massless so there is no finite mass scale.)

$$\begin{aligned} a(t) &\propto t^{\frac{1}{2}} \\ \rho_{\rm radiation} &\propto a^{-4} \end{aligned} \tag{32}$$

- For a vacuum dominated Universe the equation of state is $p = -\rho$ (or $T_{\mu\nu} \propto g_{\mu\nu}$). Here

$$\begin{aligned} &a(t) \propto e^{Ht} \\ &\rho_{\rm vac} = \texttt{constant} = \frac{\Lambda}{8\pi G} \end{aligned} \tag{33}$$

  with Hubble constant

$$H = \sqrt{\frac{8\pi G \rho_{\rm vac}}{3}} = \texttt{constant}. \tag{34}$$

To understand these results, the expansion of the Universe is the same in all three spatial dimensions. A given volume expands as a^3. When the volume grows, the density of matter inside it dilutes and the energy density $\rho_{\rm matter}$ drops inversely to the volume, $\sim a^{-3}$. For photons, $\rho_{\rm radiation}$ drops faster because the wavelength gets stretched and the frequency of radiation falls by an additional factor of $1/a$. The cosmological constant is a property of the vacuum and does not dilute when the Universe expands. If the flat Universe has a finite cosmological constant term it will eventually come to dominate the expansion as matter and radiation are sufficiently diluted. The result is an eternally accelerating Universe. Going back in time with fixed cosmological constant, we should eventually come to a radiation dominated phase (associated with the Big Bang). Given the very different time dependences of the various terms, it is interesting that $\rho_{\rm vac} \sim 2\rho_{\rm matter}$ today. This is known as the Coincidence Problem.

3.1. The energy density content of the Universe

The overall composition of the Universe can be conveniently described by the density parameter, Ω, which is defined as the average energy density of the Universe divided by the critical density $\rho_{\rm crit}$ needed for a spatially flat Universe. From the first Friedmann equation in Eq.(27) we find

$$\Omega \equiv \frac{\rho}{\rho_{\rm crit}} = \frac{8\pi G \rho}{3H_0^2} = 1 + \frac{k}{\dot{a}^2} \tag{35}$$

where

$$\rho_{\rm crit} = \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-29} h_0^2 \ {\rm gcm}^{-3} \tag{36}$$

and h_0 is given in Eq.(29). In general Ω will change with time unless it is equal to one. An open Universe has $\Omega < 1$ and a closed Universe has $\Omega > 1$. The energy density ρ receives contributions from vacuum, radiation and matter contributions

$$\rho = \rho_{\rm vac} + \rho_{\rm radiation} + \rho_{\rm matter}. \tag{37}$$

For these three contributions we define $\Omega_i = \rho_i/\rho_{\rm crit}$; $\Omega_k \equiv 1 - \Omega = -k/\dot{a}^2$ for the curvature contribution. For a flat Universe ($k = 0$) the deceleration parameter is

$$q = \frac{1}{2}(\Omega_m + 2\Omega_\gamma + \{1 + 3w\}\Omega_\Lambda) \tag{38}$$

where $w = p/\rho$ is the equation of state for the "dark energy" contribution.

Historically, the cosmological constant was introduced by Einstein as a mechanism to obtain a stable solution of the gravitational field equation that would lead to a static Universe, the idea being to use Λ to balance gravitational attraction. For a closed Universe ($k = +1$), the Friedmann equations, Eqs. (27), admit the mathematical solution $\{a = 1/\sqrt{\Lambda}, \dot{a} = 0, \rho_{\rm matter} = 2\rho_{\rm vac}\}$. This solution is, however, unstable against small perturbations away from $\rho_{\rm matter} = 2\rho_{\rm vac}$. If $\rho_{\rm matter} = (2+\epsilon)\rho_{\rm vac}$ with ϵ positive, the Universe starts contracting and ϵ keeps increasing until matter dominates the collapse. The opposite happens if ϵ starts slightly negative: in that case the Universe expands faster and faster diluting $\rho_{\rm matter}$. Thus, local inhomogeneities would lead ultimately to either the runaway expansion or contraction of the Universe. The static Universe of Einstein is unstable. This result and Hubble's discovery of the redshift and expanding Universe then led to Einstein throwing away the cosmological constant. It returned to phenomenology in 1990 with the suggestion that observations of large scale structure imply a spatially flat cosmology with finite cosmological constant and $\Omega_\Lambda < 0.8$ [23], and then with the discovery in 1998 of the accelerating expansion of the Universe [24].

3.2. Dark Energy and the Λ-CDM Model

Supernovae, the Cosmic Microwave Background, Baryon Acoustic Oscillations and gravitational lensing provide complementary constraints on the fractional energy densities of matter, vacuum energy, and the dark energy equation of state. There is good agreement between all measurements [5, 6].

All data are consistent with the phenomenological Λ-CDM model, where Λ denotes the cosmological constant and CDM denotes cold dark matter. The model uses the FLRW metric and the Friedmann equations (27) to describe the observable Universe for all times after the inflationary epoch. The Universe is nearly spatially flat with an expansion rate $H = 70.4^{+1.3}_{-1.4}$ km/s/Mpc and with structures grown out of a primordial linear spectrum of nearly Gaussian and scale invariant energy density perturbations. With 6 parameters the model explains the existence and structure of the cosmic microwave background, the large scale structure of galaxy clusters and the distribution of light elements (hydrogen, helium, lithium, oxygen) plus the accelerating expansion of the Universe observed in the light from distant galaxies and supernovae. It is the simplest model that is in general agreement with observed phenomena.

One finds [25] consistent, overlapping confidence level contours, that in combination indicate a concordance cosmology with $\Omega_m = 0.273 \pm 0.014$, $\Omega_\Lambda = 0.728^{+0.015}_{-0.016}$ at 68% confidence level (CL) for a flat $\Lambda-$CDM cosmology. One obtains $w = -0.980 \pm 0.053$ for a flat Universe with a constant (time independent) dark energy equation of state. When the curvature Ω_k was allowed to vary, the best fit yielded $w = -0.999^{+0.057}_{-0.056}$ and $\Omega_k = -0.0057^{+0.0067}_{-0.0068}$. Combining all these measurements plus also constraints from gravitational lensing, and fitting to the time dependent equation of state

$$w = w_0 + w_1(1 - a) \tag{39}$$

gives $w_0 = -0.93 \pm 0.012$ and $w_1 = -0.38^{+0.66}_{-0.65}$ at 68% CL.

3.3. Observation

From ground-based studies of supernovae to satellite probes of the cosmic microwave background (CMB), experimental evidence points to the accelerating expansion of the Universe, which was first reported in 1998 [24]. This acceleration has occurred in recent cosmic history, dating back 5 billion years corresponding to redshifts of about $z \leq 1$. This is where the acceleration becomes a deceleration due to the lesser impact of $\rho_{\rm vac}$ at earlier times.

Supernova studies of the accelerating Universe use Type Ia Supernovae. These events are caused by runaway thermonuclear explosions following accretion onto a carbon/oxygen white dwarf star. They have almost uniform brightness making them "standard candles" which can be used for the precise measurement of astronomical distances. Light from these sources is fainter than expected for a given expansion velocity, indicating that the supernovae are farther away than predicted with just normal matter densities indicating that the expansion of the Universe is accelerating.

Measurements of the temperature fluctuations in the CMB provide independent support for the theory of an accelerating Universe. At very early times the temperature was high enough to ionize the material that filled the Universe: the Universe consisted of a plasma of nuclei, electrons and photons, and the number density of free electrons was so high that the mean free path for the Thomson scattering of photons was extremely short. As the Universe expanded, it cooled, and the mean photon energy diminished. Eventually, at a temperature of about 3000 K, the photon energies became too low to keep the Universe ionized. At this time, known as recombination, the primordial plasma coalesced into neutral atoms, and the mean free path of the photons increased to roughly the size of the observable Universe. This radiation has since travelled essentially unhindered through the Universe, and provides a snapshot of the Universe when it was only 380,000 years old.

The photons we see today in the cosmic microwave background are a true photograph of the Universe at that time, now 13.7 billion years later. The radiation has cooled to microwave frequencies and is observed as the cosmic microwave background, the thermal afterglow of the Big Bang. To a very good approximation, the temperature of the CMB is uniform across the whole sky; moreover, it is the most perfect black-body spectrum known, with a mean temperature about 3K as measured by the COsmic Background Explorer (COBE) satellite in 1992. The discovery of the CMB, together with the black-body nature of its frequency spectrum, was of fundamental importance to cosmology because it validated the idea of a hot Big Bang – the Universe was hot and dense in the past and has since cooled by expansion. Equally important is the fact that the CMB has slight variations of one part in 100,000 in its temperature. The most accurate measurement of these fluctuations is by the Wilkinson Microwave Anisotropy Probe (WMAP). The temperature anisotropies reflect the primordial inhomogeneities in the underlying density field that provided the seeds for cosmological structure formation: galaxies, stars, planets and life. The temperature variations are commonly plotted as a function of the multipole moment, – that is, the angular size of the "hot" and "cold" spots. Since the cosmological constant became important only recently in the history of the Universe its main effect is to change the distance to the last scattering surface, which determines the angular size of the CMB anisotropies. Studying the CMB thus yields precise information about the geometry of the Universe – for a detailed review, see Ref. [26].

Additional evidence for accelerated expansion comes from measured ripples in the distribution of galaxies that were imprinted in acoustic oscillations of the plasma when matter and radiation decoupled as protons and electrons combined to form hydrogen atoms, 380,000 years after the Big Bang. These are the "baryonic acoustic oscillations" (BAOs). Yet another indication comes from the investigation of weak-lensing. These probes, together with the Hubble constant, indicate a large scale flat Universe geometry.

3.4. Energy scales associated with the cosmological constant

The presence of a nonvanishing cosmological constant provides a way of ascribing an intrinsic, observer-independent, size parameter to our Universe. Consider a flat Universe ($k = 0$). The spacetime of a distant future dominated by dark energy and exponential expansion

$$ds^2 = d\hat{t}^2 - e^{2H_\infty \hat{t}}\Big(d\hat{r}^2 + \hat{r}^2(d\theta^2 + \sin^2\theta d\phi^2)\Big) \tag{40}$$

is equivalent to static de Sitter space characterised by a horizon radius R_∞

$$ds^2 = \Big(1 - \frac{r^2}{R_\infty^2}\Big)dt^2 - \Big(1 - \frac{r^2}{R_\infty^2}\Big)^{-1} dr^2 - r^2\Big(d\theta^2 + \sin^2\theta d\phi^2\Big) \tag{41}$$

if we make the co-ordinate transformations

$$\begin{aligned} r &= \hat{r}e^{H_\infty \hat{t}} \\ t &= \hat{t} - \frac{1}{2H_\infty}\ln(1 - H_\infty^2 r^2). \end{aligned} \tag{42}$$

The horizon radius is determined in terms of the asymptotic value of the Hubble constant, $H_\infty = \lim_{t\to\infty} H(t) = \sqrt{\Omega_\Lambda} H_0$, which in turn is determined by gravitational dynamics of the vacuum energy density

$$\frac{1}{R_\infty^2} = H_\infty^2 = \frac{8\pi G}{3}\rho_{\rm vac} = \frac{\Lambda}{3}. \qquad (43)$$

There are two mass scales associated with the cosmological constant Λ. The first is the energy scale μ

$$\rho_{\rm vac} = \mu^4, \quad \mu \sim 0.002 \text{ eV} \qquad (44)$$

corresponding to the submillimeter length scale 85 μm. The second is the curvature scale

$$m^2 \sim \frac{\Lambda}{3} \sim \frac{1}{R^2}, \quad m \sim 10^{-33} \text{ eV} \qquad (45)$$

for the present Hubble length $R \sim 10^{26}$m.

4. Vacuum energy and the cosmological constant

The cosmological constant

$$\Lambda = 8\pi G \rho_{\rm vac} + \Lambda_0 \qquad (46)$$

tells us about the energy density of the vacuum perceived by gravitational interactions. Possible contributions are zero-point energies and vacuum condensates associated with spontaneous symmetry breaking and any potential in the vacuum; Λ_0 is a possible counterterm.

In quantum field theory the energy of the vacuum is badly divergent, being the sum of zero-point energies for an infinite number of oscillators, one for each normal mode, or degree of freedom of the quantum fields [27]. Before interactions, the vacuum (or zero-point) energy is

$$\rho_{\rm vac} = \frac{1}{2}\sum\{\hbar\omega\} = \frac{1}{2}\hbar \sum_{particles} g_i \int_0^{k_{max}} \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m^2} \sim \sum_i \frac{g_i k_{max}^4}{16\pi^2}. \qquad (47)$$

Here the $\frac{1}{2}\{\hbar\omega\}$ denote the eigenvalues of the free Hamiltonian; $\omega = \sqrt{k^2+m^2}$ where k is the wavenumber of the mode and m is the mass of the particle. The plus or minus sign applies to bosons or fermions, respectively; $g_i = (-1)^{2j}(2j+1)$ is the degeneracy factor for a particle of spin j, with $g_i > 0$ for bosons and $g_i < 0$ for fermions. The minus sign follows from the Pauli exclusion principle and the anti-commutator relations for fermions [27]. The vacuum energy density $\rho_{\rm vac}$ is quartically divergent in k_{max}. The question next arises, what is k_{max} ?

4.1. Quantum field theory and fine tuning

The energy-scale associated with the electroweak symmetry breaking caused by the electroweak symmetry breaking Higgs sector is $\Lambda_{\rm ew} \sim 250$ GeV (corresponding to $\Lambda_{\rm ew} = 2^{-1/4} G_F^{-1/2} = 246$ GeV). Substituting $k_{max} \sim \Lambda_{\rm ew}$ into Eq. (47) with no additional physics gives a cosmological constant

$$\Lambda_{\rm vac} \sim 8\pi G \Lambda_{\rm ew}^4 \tag{48}$$

or

$$\rho_{\rm vac} = \frac{1}{2} \sum \hbar\omega \sim (250 \text{ GeV})^4. \tag{49}$$

This number is 56 orders of magnitude larger than the observed value

$$\rho_\Lambda \sim (0.002 \text{ eV})^4. \tag{50}$$

This is the fine tuning puzzle: What dilutes this particle physics number to the physical value measured in large scale astrophysics and cosmology ? Also, summing over just the Standard Model fields in Eq. (47) gives a negative overall sign whereas the value of ρ_Λ extracted from cosmology is positive. We would have a very hard time to understand why the Universe is so well behaved if there were just the Standard Model with its Standard Higgs. The Planck scale

$$M_{\rm Pl} = \sqrt{\frac{\hbar c}{G}} = 1.3 \times 10^{19} m_{\rm proton} = 1.2 \times 10^{19} \text{ GeV} \tag{51}$$

is the scale where we expect classical gravity to break down and quantum gravity effects to become important. If we trust particle physics and quantum field theory up to this energy in Eq.(47), then we obtain a value for $\rho_{\rm vac}$ which is 10^{120} times too big.

In Quantum Field Theory (QFT) the divergence in Eq. (47) is easy to remove. Simply subtract an infinite constant from the Hamiltonian to cancel $\frac{1}{2}\sum \hbar\omega$. This can be done because absolute energies in QFT are not measurable observables. Before coupling to gravity, in QFT only energy differences have physical meaning. Subtracting out the infinite vacuum energy is equivalent to rewriting the energy-momentum operator so that all positive frequency parts of the field operators always stand to the right of negative frequency parts (taking into account the commutation or anti-commutation relations for the bosons and fermions). This procedure is known as *normal ordering.* The only effect of normal ordering here is to remove the infinite zero-point energy from the theory and to define the zero of energy as the energy of the vacuum state Φ_0. The free field zero-point energy can be cancelled by a counterterm. However comparable contributions reappear when interactions are introduced: the vacuum energy is related to the sum of all vacuum-to-vacuum Feynman diagrams. A counterterm can be introduced to cancel these contributions to any order in perturbation theory. §

§ Note that boson and fermions contributions to Eq.(47) enter with opposite signs. In a gedanken world with exact supersymmetry the boson and fermion contributions to the vacuum energy would cancel against each other. However, particle physics data tells that supersymmetry (if present in nature) must be strongly broken below the TeV scale. If we include quantum fields up to the Planck mass scale and supersymmetry beyond the TeV scale, then the mass scale $k_{\rm max}$ is the TeV scale.

Do the zero-point energies of quantum fields contribute to the cosmological constant and the energy density of the vacuum perceived by gravitation ? A related problem in nanoscopic physics is the Casimir effect in QED. There is a force between two mirrors in vacuum as a consequence of the radiation pressure of vacuum fluctuations. The Casimir force (per unit area) between parallel conducting plates separated by distance d

$$\mathcal{F} = \frac{\hbar c \pi^2}{240 d^4} \tag{52}$$

has now been measured to about 1% precision [2]. It is commonly understood as a difference in the zero-point or vacuum energy that is induced by putting boundary conditions on the fields. Absolute values of the vacuum energies do not enter, just differences. The Casimir force can also be calculated without reference to vacuum fluctuations [28], and like all other observable effects in QED, it vanishes as the fine structure constant, α, goes to zero. (The formula (52) is asymptotic, exact in the limit $\alpha \to \infty$.)

Even if one can argue away quantum zero-point contributions to the vacuum energy, the problem of spontaneous symmetry breaking remains: condensates that carry energy appear at many energy scales in the Standard Model. If there is a potential in the vacuum it will, in general, correspond to some finite vacuum energy. Why should the sum of many big numbers (plus any possible gravitational counterterm) add up to a very small number ?

Vacuum or zero-point energies are, in themselves, not Lorentz covariant without a corresponding vacuum pressure. The contribution of vacuum condensates is manifest in a covariant form via the trace anomaly. In quantum field theory, conformal symmetry is broken by the renormalisation group dependence, which introduces a momentum scale into otherwise massless theories. This, in turn, induces counterterms into the renormalisation of the energy momentum tensor so that the physical conserved operator has non-vanishing trace, even for massless fermion and boson fields. For QCD the trace anomaly gives

$$T^{\mu}_{\ \mu} = (1 + \gamma_m) \sum_q m_q \bar{q} q + \frac{\beta(\alpha_s)}{4\alpha_s} G_{\mu\nu} G^{\mu\nu}. \tag{53}$$

Here m_q is the running quark mass, γ_m is the mass anomalous dimension and $\beta(\alpha_s)$ is the QCD beta function. The corresponding classical operator has vanishing trace for massless quarks. Taking the vacuum expectation value of the trace in Eq. (53) will yield a finite cosmological constant term in Einstein's equations.

Condensates form at different times in the early Universe, suggesting some time dependence to $\rho_{\rm vac}$. For example, the QCD condensates formed at times of order 10^{-5}s as the temperature T decreased below the confinement-deconfinement temperature $T_{\rm dec} \simeq 200$ MeV.

5. Seeking a possible explanation

5.1. Neutrinos

It is very interesting that the dark energy or cosmological constant scale in Eq.(44) is of the same order of magnitude that we expect for the light neutrino mass, viz. 0.002 eV [29]:

$$\mu_\Lambda \sim m_\nu \sim \Lambda_{\rm ew}^2/M \tag{54}$$

where $M \sim 3 \times 10^{16}$ GeV is logarithmically close to the Planck mass $M_{\rm Pl}$ and typical of the scale that appears in Grand Unified Theories. It is also interesting that the gauge bosons in the Standard Model which have a mass through the Higgs mechanism are also the gauge bosons which couple to the neutrino. The non-perturbative structure of chiral gauge theories is not well understood. Changing the external parameters of the theory can change the phase of the ground state. For example, QED in 3+1 dimensions with exactly massless electrons is believed to dynamically generate a photon mass [30]. In the Schwinger Model for 1+1 dimensional QED on a circle, setting the electron mass to zero shifts the theory from a confining to a Higgs phase [31]. What happens to the structure of non-perturbative propagators and vacuum energies when we decouple chiral degrees of freedom ?

5.2. Dark energy, scalar fields, time dependence

It is commonly believed that the Universe went through an initial period of accelerated expansion called inflation [32, 33, 34]. Inflation or rapid exponential expansion of space in the very early Universe is required to explain the horizon and flatness problems. The horizon problem is that the CMB is isotropic to better than 10^{-4}. Without inflation, one should ask why the Big Bang was homogeneous and isotropic even over casually disconnected regions of space. The flatness problem asks what initial conditions could the Universe have had to end up with a flat Universe today. The matter and radiation Universes gave different behaviours for the evolution of $a(t)$ as a function of t. With a regular Big Bang we want a Universe that started flat and stayed flat. Does the cosmological "constant" start at a very large value and decay to the value we see today, with passage first to the radiation dominated Universe, then matter dominated Universe and next to the (re-)accelerating vacuum dominated Universe that we see today ?

To gauge the size of the effect, consider the exponential expansion factor

$$a(t) = \texttt{constant} \ . \ e^{\int H_{\rm infl} dt} \tag{55}$$

Then, with

$$t_{\rm infl} > \frac{140}{H_{\rm infl}} \tag{56}$$

a patch of initial Planck size $l_{\rm Pl} = \sqrt{G\hbar/c^3} \sim 1.616 \times 10^{-35}$ m expands to the size exceeding the present horizon size $l_{H_0} \sim 10^{26}$ m. In practice the Universe has expanded considerably also since the end of inflation and the minimum of number of required e-foldings (or power of e) is between 50 and 70, depending on the model.

A time dependent vacuum energy density is often modelled using an ultra-light scalar field [35, 36] with

$$\begin{aligned} \rho &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p &= \frac{1}{2}\dot{\phi}^2 - V(\phi). \end{aligned} \tag{57}$$

The equation of state $w_\phi = p_\phi/\rho_\phi$ gives values close to -1 when $V \gg \dot{\phi}^2$. Inflation occurs when the additional *slow roll* condition $H\dot{\phi} \gg \ddot{\phi}$ is satisfied. Observations require slow evolution of the potential $V' \ll V$ plus weak clustering ($V'' \ll V$) so that the new scalar field has very small mass

$$m < H_0 \sim 1/R_0 \sim 10^{-33} \text{ eV} \tag{58}$$

or a Compton wavelength bigger than the Hubble length $R_0 \sim 1/H_0$.

In this scenario the cosmological constant puzzle including the vacuum energy of Standard Model particle physics is assumed to be solved by counterterms or other (unknown) physics. The accelerating Universe is then described in terms of the properties of the new (elementary) time-dependent scalar field and the potential that comes with it. The potential $V(\phi)$ is supposed to match precisely the value of the dark energy density today starting from a very high-energy scale, typically the GUT scale $\phi \sim 10^{16}$ GeV, requiring fine-tuning of the large initial value of the potential. There are new puzzles that come with this approach: Why does the mass of the scalar not run away towards large values due to quantum corrections ? Coupling a near massless scalar to Standard Model particles will introduce a "fifth force" (which is not gauged unlike the other forces of nature). There is no experimental evidence for any additional interaction meaning that any such couplings must be zero or very small. Coupling to a time dependent scalar will introduce a time dependence to fundamental constants like the electron or proton mass, or the fine structure constant α. There are very strong experimental constraints on any possible time dependence [37, 38].

Additional ideas include a possible coupling between dark energy and dark matter [11], and whether a non-minimal gravitational coupling to an elementary Higgs scalar might drive inflation [39].

5.3. Extra dimensions and modified gravity

There are also attempts to modify gravity either in 4 dimensions or by treating the Universe with Standard Model fields as a membrane in a higher dimensional space [12]. For the first case the idea is to modify the action by including additional, non-linear terms in the Ricci scalar R, viz.

$$S = \int d^4x \sqrt{-g} \Big[f(R) + \mathcal{L}_M \Big] \tag{59}$$

– e.g. higher order corrections $f(R) = R + R^2 + R^3 + ...$ or introducing a new scalar-tensor coupling. There are phenomenological challenges with these models. For example, singularities which may exclude neutron stars. In higher dimensional models [40] the Universe of Standard Model particle physics is confined to live on a 4 dimensional membrane in a 5 dimensional Minkowski bulk space. The extra dimensions have infinite extension and there is gravity leakage into the bulk. The membrane breaks the rotational symmetry of the larger dimensional space and generates a small effective mass for the graviton, $\sim 10^{-33}$ eV, thus modifying gravitation at very large distances. The gravitational action in the model is

$$S = \int d^5x\sqrt{-g^{(5)}}R^{(5)} + L\int d^4x\sqrt{-g}R \tag{60}$$

where $R^{(5)}$ and R denote the 5 dimensional and 4 dimensional Ricci scalars, and L is the crossover scale which determines the behaviour of the gravitational potential. One finds $V \propto \frac{1}{r}$ for $r \ll L$ and $V \propto \frac{1}{r^2}$ for $r \gg L$. In this model 5-dimensional gravity dominates at large distances and 4-dimensional gravity is recovered at small distance scales (relative to L). Modification of gravity at very large distances could, in principle, reproduce the cosmic acceleration associated with dark energy. On the other hand, these models still involve fine tuning issues with quantum fields. There are also technical challenges to remove negative kinetic energy ghost fields that enter the theory. Modifications of General Relativity are predicted which could, in principle, be investigated using astronomical observations. General Relativity so far provides an excellent description of large scale gravitational phenomena wherever it has been tested.

5.4. Anthropic arguments and the coincidence problem

Cosmology data tells us that 73% of the energy budget of the Universe is in the vacuum energy density term and 27% is in the matter density. It is fascinating to ask why these numbers should be so similar at the present time. Recalling Eqs.(31-33), the different contributions have very different dependence on $a(t)$: $\rho_{\text{vac}} \sim a^0$, $\rho_{\text{matter}} \sim a^{-3}$ and $\rho_{\text{radiation}} \sim a^{-4}$. This puzzle is called the Coincidence Problem. Is there something "special" about "today" ?

Weinberg has argued that (large scale) structure formation stops when ρ_{vac} starts dominating [41]. If ρ_{vac} were too large, there would be no galaxies. Supernova data tells us that the expansion of the Universe started to accelerate at redshift $z \sim 1$ (about 5 billion years ago). Life on Earth is believed to have begun with the first single cell prokaryotes (bacteria and archaea) around 3.5 billion years ago. The physical values of the fundamental constants seem to be finely tuned to the conditions necessary for "our" existence [42, 43].

In string theories it is possible to think that the observable Universe is just one of 10^{500} Universes in a grander Multiverse. The vacuum energy will have different values in different Universes, and in many or most it might indeed be large. But it must be small in ours because it is only in such a Universe that observers such as ourselves can evolve. It would be nice to have ideas how these theories might be tested in experiments.

5.5. Analogies with condensed matter physics

Ideas based on "emergence" phenomena in condensed matter physics have been suggested in Ref. [44]. Imagine the vacuum of particle physics as if it were a cold quantum liquid in equilibrium. Then its pressure must vanish, unless it is a droplet in which case there will be surface corrections scaling as an inverse power of the droplet size. Vacuum dark pressure scales with the vacuum dark energy and is measured by the cosmological constant which scales as the inverse square of the Hubble length (or "size" of the Universe) – that is, consistent with Eq. (45). In this picture the gauge fields (photons, gravitons, and gluons) must be viewed as collective excitations of the purported liquid. At high energies dispersion laws are not expected to be relativistic and fundamental symmetries like gauge invariance are probably not exact.

6. Towards possible understanding

Measuring the equation of state for dark energy is one of the biggest efforts in observational cosmology today. Is the acceleration of the Universe due to Einstein's cosmological constant or to a time varying (scalar) field ? Is the cosmological constant really constant or does it vary in time ?

On the experimental side, key information on the equation of state is expected from the PLANCK mission, the Atacama Cosmology and South Pole Telescopes and future dark energy missions (EUCLID and WFIRST/JDEM), as well as measurements at the next generation of 40m terrestial plus space telescopes. In parallel, particle physics experiments at the LHC and Tevatron will shed valuable information on electroweak symmetry breaking and the Higgs sector of the Standard Model. Are elementary scalar fields part of the solution or the main part of the problem [45, 46] ? In elementary particle physics, 24 parameters are associated with the Higgs sector [37]. An elementary Higgs brings with it the hierachy problem: What stabilises the mass of the Higgs boson against quantum corrections ? Supersymmetry is one possibility but this has (so far) not been observed and introduces many extra parameters into the theory which require explanation. If dark energy is due to a new elementary (time dependent) scalar, what stabilises the mass of this particle ? Why is it so very light and why does it not couple strongly to Standard Model particles ? Laboratory experiments in quantum optics are pushing the frontier with constraining possible time dependence of fundamental constants like the fine structure constant α. It is interesting that the energy scale associated with the cosmological constant is close to the value we expect for the light neutrino mass. Is this a clue ? The massive gauge bosons in particle physics which are expected to couple to the Higgs are also the gauge bosons that couple to the neutrino. The non-perturbative structure of chiral gauge theories is not well understood.

The cosmological constant puzzle involves two parts. Why is it finite, positive and so very small ? What suppresses the very large vacuum energies expected from particle physics in Einstein's equations of General Relativity ? Understanding these vital questions will tell us a great deal about the intersection of quantum fields and particle physics on the one hand, and gravitation on the other.

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