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Where is the pseudoscalar glueball?

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Abstract

The pseudoscalar mesons $\pi(1300)$, $K(1460)$, $\eta(1295)$, $\eta(1405)$ and $\eta(1475)$ are assumed to form the meson decuplet which includes the glueball as the basis state supplementing the standard $SU(3)_F$ nonet of light $q\bar{q}$ states ($q = u, d, s$). The decuplet is investigated by using the algebraic approach based on the hypothesis of vanishing exotic commutators (VEC) of $SU(3)_F$ "charges" and their time derivatives. This leads to a system of master equations (ME) determining: (a) octet contents of the physical isoscalar mesons, (b) the mass formula relating all masses of the decuplet and (c) the mass ordering rule. The states of the physical isoscalar mesons $\eta(1295)$, $\eta(1405)$, $\eta(1475)$ are expressed as superpositions of the "ideal" $q\bar{q}$ (N and S) states and the glueball G one. The "mixing matrix" realizing transformation from the unphysical states to the physical ones follows from the octet contents and is expressed totally by the decuplet meson masses. Among four one-parameter families of the resulting mixing matrices (multitude of the solutions arising from bad quality of data on the $\pi(1300)$ and $K(1460)$ meson masses) there is a family attributing the glueball-dominated composition to the $\eta(1405)$ meson. The pseudoscalar decuplet is similar in some respects to the scalar one: both are composed of the excited $q\bar{q}$ states and G ; the mass ordering of their N , S , G - dominated isoscalars is the same. Contrary to the Lattice QCD and other predictions, the mass $m_{G^{--}}$ of the pseudoscalar pure glueball state is smaller than the scalar $m_{G^{++}}$ one.

1 Introduction

•• The pseudoscalar glueball investigation has been initiated soon after it was realized that bound states of the gluons may play important role in the strong interactions [1, 2]. From the very beginning the glueball state was traced within structure of the η and η' mesons [3, 4]. At present, this is not the main purpose of the investigation, but is still continued, and not only within the meson structures [5, 6] but also within the baryon ones [7].

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The discovery of the ι meson [8, 9] rous hopes for existence of the glueball. ι meson has been detected in the gluon rich process of the J/ψ radiative decay and was immediately claimed to be a glueball. However, the glueball may exist as separate particle only if it has exotic quantum numbers; otherwise it should be mixed with the isoscalar $q\bar{q}$ states having the same signatures J^{PC} . The mass of the ι meson belongs to the region of higher-lying 0^{-+} multiplet and the states of this multiplet were (and still are) poorly known. That posed the question of how to certify such assignment. To this end several criteria have been invented which could be used in the cases of deficient multiplets. Some of them, concerning production, are pure qualitative like "creation in the gluon-rich environment", other ones, more regarding decay products, are semi-quantitative (big value of "stickiness" [10] and "gluiness" [11]). At the same time, the question has been risen whether the glueball is necessary for understanding data concerning the pseudoscalar mesons known at that time [12]. This question is still alive [13].

The trend of discussion has changed since the results of the Lattice QCD (LQCD) calculations became available [14, 15, 16]. They supported the very existence of the pseudoscalar glueball, but the mass attributed was about 2.3GeV - much above the ι . An attempt to lower the lattice prediction by including quark loops was not very successful [17]. Although the doubts were not dispelled (see e. g. [18]), this became a serious obstacle for ι to be recognized as the glueball candidate, because the results of the Lattice calculations are generally accepted. On the other hand, there is no candidate having the mass predicted by Lattice. Perhaps that induced several attempts to interpret the meson $X(1835)$ as the pseudoscalar glueball [19, 20, 21], although its mass might be regarded as too low also.

At the same time, starting from late 80's there was growing conviction that $\iota(1440)$ signal should be attributed to two different isoscalar mesons [22]. Much experimental effort was devoted to understanding the structure of the signal [23, 24, 25, 26, 27]. As a result, it has been split into $\eta(1405)$ and $\eta(1475)$. Hence, since 2004 three isoscalar pseudoscalar mesons has been listed in RPP within the narrow interval of mass [28]:

$$\eta_1 = \eta(1295), \quad \eta_2 = \eta(1405), \quad \eta_3 = \eta(1475). \quad (1)$$

Such three isoscalar mesons with similar masses in the vicinity of the isotriplet and isodoublet suggest overpopulation of a nonet and possible existence of a glueball which is hidden within the structures of three isoscalar states. The decuplet findings are very important because investigation of its properties is the most promising way of glueball search unless the glueball with exotic quantum numbers will be detected.

•• Information about the structures of the isoscalar mesons η_1 , η_2 , η_3 has been extracted from data on the reactions of their production and from branching ratios of their decays. Data suggest that the meson η_2 is a particle dominated by the glueball state [11, 28, 29, 31].

An unexpected objection has been risen against such a picture: the η_1 has been claimed to be not the $q\bar{q}$ state [13]. Even its very existence was considered uncertain. That implies non-existence of the decuplet and requires much more complicated spectroscopy of the pseudoscalar mesons. Therefore, we discuss this question in more detail.

The $q\bar{q}$ structure is put in doubt due to not occurrence of the η_1 in the reactions

$$p\bar{p}, \quad J/\psi, \quad \gamma\gamma \rightarrow \quad , \quad (2)$$

"at least not with the expected yields". The base of such expectation is not indicated.

However, this is not the only point of view concerning these reactions. The authors of recently published, very careful analysis of the experimental data on η_1 , came to the following conclusions [29]:

- (i) the charge exchange experiments $\pi^- - p \rightarrow n\eta\pi\pi$, $nK\bar{K}\pi^0$ definitively establish evidence of the η_1 ;
- (ii) a clear signal of η_1 is seen in the J/ψ radiative decay;
- (iii) there is an indication for the existence η_1 in $p\bar{p}$ annihilation;
- (iv) the LEP data on $\gamma\gamma$ reaction are compatible with existence of the η_1 signal.

So the η_1 is seen or not in the reactions (2). The three-body decays $\eta_1 \rightarrow \eta\pi\pi$, $K\bar{K}\pi$ are strongly suppressed by small phase space (cf $\omega \rightarrow \pi\pi\pi$) and that may be the reason why it is difficult to observe the η_1 . It is explicitly seen in the reaction $\pi^- - p \rightarrow \eta_1 n$, for which high statistics is available, but the number of events observed in reactions (2) is many times smaller [28]. Obviously, more measurements are needed to elucidate the situation. But this question has no relevance to the problem of the η_1 internal structure. As in the reaction (2) the η_1 is observed throughout the products of decay, the frequency of its registration depends on width; the subsequent measurements would verify the magnitude of the width. However, the definition of the multiplet does not depend on the widths. Therefore, the widths of the particles cannot be the basis for any conclusion about the structure of the multiplet. Also the width of the η_1 cannot be the base for conclusion about its $q\bar{q}$ structure. An attempt to call in question this structure resembles confusion which arose after denying the $q\bar{q}$ structure of the $f_0(980)$ meson motivated by its small width [30].

In the present paper we admit the η_1 meson to be a $q\bar{q}$ state and assume that the examined pseudoscalar mesons form a decuplet. We thus focus glueball search again in the region of ι meson - this time being fully aware of the conflict with Lattice prediction.

The glueball assignment of the η_2 meson is also motivated on theoretical ground [32]. It is argued that η_2 meson is a natural pseudoscalar glueball candidate if the $f_0(1500)$ is the scalar glueball and the glueballs are described as the closed gluonic flux-tubes. Then the $f_0(1500)$ and the $\eta(1405)$ would be two parity related glueballs with equal masses. This description deserves attention in view of failure of the Lattice prediction, particularly if it can be treated not too literally.

It is thus interesting to make sure that this assignment can be confirmed by an argument based on the properties of the flavor multiplet as a whole.

•• We conclude the introduction with few comments concerning credibility of the approach we use in this paper. The credibility is especially important in evaluating the glueball contents of the decuplet isoscalar states.

It is currently known that broken $SU(3)_F$ symmetry predicts the existence of octets and nonets of light mesons. The multiplets are usually testified by the mass formula relating their masses. The Gell-Mann–Okubo (GMO) and Schwinger (S) mass formulae have been obtained by inclusion into the lagrangean the non-invariant mass term with regard to mixing of the octet isoscalar with

the unitary singlet.

Our model unifies and generalizes these mass relations. The model has been introduced at the University of Lodz in the middle of '80s [33, 34] and is based on requirement of vanishing the exotic commutators (VEC) of the "charges" and their time derivatives. Apart from the GMO and S mass formulae it gives additional insight into the properties of the multiplet. For the S nonet the model VEC determines the mixing angle and establishes the mass ordering rule which ensures the mixing angle to be a real number. There are two possible orderings. For one of them the mixing angle ϑ is smaller than ideal $\vartheta < \vartheta^{id}$, while for the other one it is bigger $\vartheta > \vartheta^{id}$ ($\vartheta^{id} \approx 35^\circ$).

The model predicts also the ideally mixed (ideal) nonet (I). This nonet has not been derived, as yet, from any other mixing description. In the quark model, where it is the basic object, it is postulated.

The S and I mass formulae are well obeyed by many nonets with various signatures J^{PC} comprising low mass mesons [35]. In general, the S nonets better describe data, although differences between I and S descriptions are small.

For the glueball quest the most important is the prediction of a decuplet [34] - a multiplet comprising three isoscalar mesons. The mass formula, mass ordering rule and the octet contents of the physical isoscalar states follow from the same constraints. The octet contents l_i^2 play a key role in determining mixing matrix of the isoscalar states, i.e. the contributions of glueball state to their structures. The orthogonal 3×3 mixing matrix can be parametrized by Euler angles. Absolute values of the trigonometric functions of these angles are expressed by the particle masses, i.e. by the masses.

The VEC model does not use additional assumptions nor introduces free parameters to describe multiplets. Its predictions are definite and applicable for decuplets of any signature J^{PC} . If fitted with required experimental input, it offers complete description of the decuplet states. Thus, it bestows quantitative meaning to the most obvious qualitative signature of the glueball presence - overpopulation of a nonet.

The model has appeared very effective in describing the 0^{++} mesons. It makes possible to sort out twenty scalar mesons among multiplets and attribute the glueball dominating structure to the $f_0(1500)$ meson [30]. To analyze the 0^{-+} decuplet we use essentially the same model. Our present analysis is proceeded in a different way because sample of the input data is different.

2 The decuplet of pseudoscalar mesons

2.1 The model of vanishing exotic commutators (VEC)

The following sequence of exotic commutators is assumed to vanish ¹

$$\left[T_a, \frac{d^j T_b}{dt^j} \right] = 0, \quad (j = 1, 2, 3, \dots) \quad (3)$$

where T is $SU(3)_F$ generator, t is the time and (a, b) is an exotic combination of indices, i.e. such that the operator $[T_a, T_b]$ does not belong to the octet representation. Substituting $\frac{dT}{dt} = i[H, T]$, and using the infinite momentum

¹formerly the model was called exotic commutator model (ECM)

approximation for one-particle hamiltonian $H = \sqrt{m^2 + p^2}$ [36], we transform eqs. (3) into the system:

$$\begin{aligned} [T_a, [\hat{m}^2, T_b]] &= 0, \\ [T_a, [\hat{m}^2, [\hat{m}^2, T_b]]] &= 0, \\ [T_a, [\hat{m}^2, [\hat{m}^2, [\hat{m}^2, T_b]]]] &= 0, \\ &\dots\dots\dots \end{aligned} \quad (4)$$

where \hat{m}^2 is the squared-mass operator.

For the matrix elements of the commutators (4) between one-particle states (we assume one-particle initial, final and intermediate states) we obtain the sequence of equations involving expressions $\langle x_8 | (m^2)^j | x_8 \rangle$ with different powers $j = 1, 2, 3, \dots$, where x_8 is the isoscalar state belonging to the octet. Solving these equations, we obtain the sequence of formulae for a multiplet of the light mesons. We find

$$\langle x_8 | (\hat{m}^2)^j | x_8 \rangle = \frac{1}{3}a^j + \frac{2}{3}b^j \quad (j = 1, 2, 3, \dots). \quad (5)$$

where a is the mass squared of the isovector meson π ; b is the mass squared of the subsidiary $s\bar{s}$ state,

$$b = 2K - a, \quad (6)$$

and K , in turn, is the mass squared of the isospinor K meson. The isoscalar octet state $|x_8\rangle$ can be represented as the linear combination of the physical isoscalar states

$$|x_8\rangle = \sum l_i |x_i\rangle. \quad (7)$$

The coefficients l_1, l_2, l_3, \dots determine octet contents of the physical isoscalar states $|x_1\rangle, |x_2\rangle, |x_3\rangle, \dots$. Substituting (7) into (5) we obtain *master equations* (ME) of the multiplet.

$$\sum l_i^2 x_i^j = \frac{1}{3}a^j + \frac{2}{3}b^j, \quad (j = 0, 1, 2, 3, \dots) \quad (8)$$

where the x_1, x_2, x_3, \dots are isoscalar meson masses squared. Normalization condition of the l_i coefficients is included into (8) as equation for $j = 0$.

2.2 Master equations for the decuplet

The states of the decuplet belong to a reducible representation of the $SU(3)_F$

$$\mathbf{8} \oplus \mathbf{1} \oplus \mathbf{1},$$

where the octet and one of the singlets are considered as $q\bar{q}$ states while the second singlet is supposed to be a glueball G .

For the decuplet we have following system of the master equations [34, 33] which determines masses and mixings of the decuplet states [30]:

$$l_1^2 x_1^j + l_2^2 x_2^j + l_3^2 x_3^j = \frac{1}{3}a^j + \frac{2}{3}b^j, \quad (j = 0, 1, 2, 3) \quad (9)$$

Table 1: Pseudoscalar mesons merged into decuplet. Status of the particles and their masses (in MeV) are quoted after RPP [28]

$\bullet\pi(1300) \pm 100$	$K(1460)$	$\bullet\eta(1295)$	$\bullet\eta(1405)$	$\bullet\eta(1475)$
1300 ± 100		1294 ± 4	1410.3 ± 2.6	1476 ± 4

where x_1, x_2, x_3 are the masses squared of the isoscalar mesons η_1, η_2, η_3 . The coefficients l_1, l_2, l_3 , are real, as all isoscalar mesons are neutral particles. The ME (9) are considered as a system of linear equations with respect to unknown coefficients l_i^2 .

The solution is given by three kinds of relations [34, 30]:

a) the octet contents (OC) of the isoscalar states

$$l_1^2 = \frac{1}{3} \frac{(x_2 - a)(x_3 - a) + 2(x_2 - b)(x_3 - b)}{(x_1 - x_2)(x_1 - x_3)}, \quad (10a)$$

$$l_2^2 = \frac{1}{3} \frac{(x_1 - a)(x_3 - a) + 2(x_1 - b)(x_3 - b)}{(x_2 - x_1)(x_2 - x_3)}, \quad (10b)$$

$$l_3^2 = \frac{1}{3} \frac{(x_1 - a)(x_2 - a) + 2(x_1 - b)(x_2 - b)}{(x_3 - x_1)(x_3 - x_2)}; \quad (10c)$$

b) the mass formula (MF)

$$f(a) + 2f(b) = 0, \quad (11)$$

where

$$f(x) = (x_1 - x)(x_2 - x)(x_3 - x) \quad (12)$$

is characteristic polynomial of the m^2 operator; the numbering of its eigenvalues is chosen such as to satisfy the inequality

$$x_1 < x_2 < x_3; \quad (13)$$

c) the mass ordering rule (MOR)

$$x_1 < a < x_2 < b < x_3. \quad (14)$$

The MF (11) is a linear equation with respect to each of the x_i , but it is a cubic one with respect to a and b .

The masses and experimental status of the mesons assigned to the decuplet are quoted in the Tab. 1. The table shows that masses of the isoscalar mesons η_1, η_2, η_3 are determined with good accuracy; the $\pi(1300)$ meson mass has large error; the K -meson is not yet established – its mass is unknown. Therefore, these two masses should be considered unknown. It is natural in this model to choose a and b (6) as unknown variables of the ME.

For solving the ME and constructing the mixing matrix of the decuplet the solution of the MF is needed. However, MF is a single equation and its solution cannot be unique. Yet, high precision of the data on isoscalar meson masses provides correct form of the characteristic polynomial $f(x)$ of the m^2 operator as well as precise values of the a and b bands which are required by MOR (14). We hope that restrictions of the model on decuplet states will reduce ambiguity of the solution.

The restrictions imposed by MOR (14) are obvious. The unknown variables a and b have to satisfy MOR which requires $a \in (x_1, x_2)$ and $b \in (x_2, x_3)$. That restricts also the K-meson mass. It follows from (6) and (14) that

$$x_1 + x_2 < 2K < x_2 + x_3, \quad (15)$$

or

$$1353MeV < m_K < 1443MeV. \quad (16)$$

From $a \in (x_1, x_2)$ we have

$$1294MeV < m_\pi < 1410MeV. \quad (17)$$

Comparing the bands (17) with the range of error of the $\pi(1300)$ meson mass we find that the MOR cuts off lower part of the error range and that the MOR-allowed region covers the upper part of it. This is consistent with treating a as an unknown quantity of the ME.

2.3 Families of solutions of the ME

Combining MOR with MF we restrict the unknown masses much stronger. Moreover, as will be seen below, the allowed masses can be attributed to the solutions of ME with explicit flavor properties.

Fig. 1 displays $f(x)$ and $-2f(x)$ (11) (c.f. [37]). The pair of the unknown variables (a, b) provides a solution of the MF if they are such that $f(a) = -2f(b)$. It can be seen from the figure that beside the MOR restrictions $a \in (x_1, x_2)$, $b \in (x_2, x_3)$ there also appears the MF restriction forbidding $a \in (x_P, x_Q)$. Hence, the allowed values of a belong to two narrow intervals: $a \in (x_1, x_P)$ and $a \in (x_Q, x_2)$. To each allowed value of a there correspond two values of b (obeying the mass formula) placed on the opposite sides of the point x_R . If we wish to have unique solution, we should divide the interval (x_2, x_3) into two parts: (x_2, x_R) and (x_R, x_3) . Then, we get four domains including unique pairs of values (a, b) making solutions of the MF:

$$A : \quad a \in (x_1, x_P), \quad b \in (x_R, x_3), \quad (18a)$$

$$B : \quad a \in (x_1, x_P), \quad b \in (x_2, x_R), \quad (18b)$$

$$C : \quad a \in (x_Q, x_2), \quad b \in (x_R, x_3), \quad (18c)$$

$$D : \quad a \in (x_Q, x_2), \quad b \in (x_2, x_R), \quad (18d)$$

where

$$x_P \simeq (1.320GeV)^2, \quad x_Q \simeq (1.365GeV)^2, \quad x_R \simeq (1.447GeV)^2. \quad (19)$$

These values correspond to

$$x_1 = (1.294GeV)^2, \quad x_2 = (1.410GeV)^2, \quad x_3 = (1.475GeV)^2. \quad (20)$$

The domains A, B, C, D are shown on the Fig. 2. We solve the ME (9) in each of them separately and express b as functions of a . The details of solving the MF as well as properties of the solutions are described in Appendix.

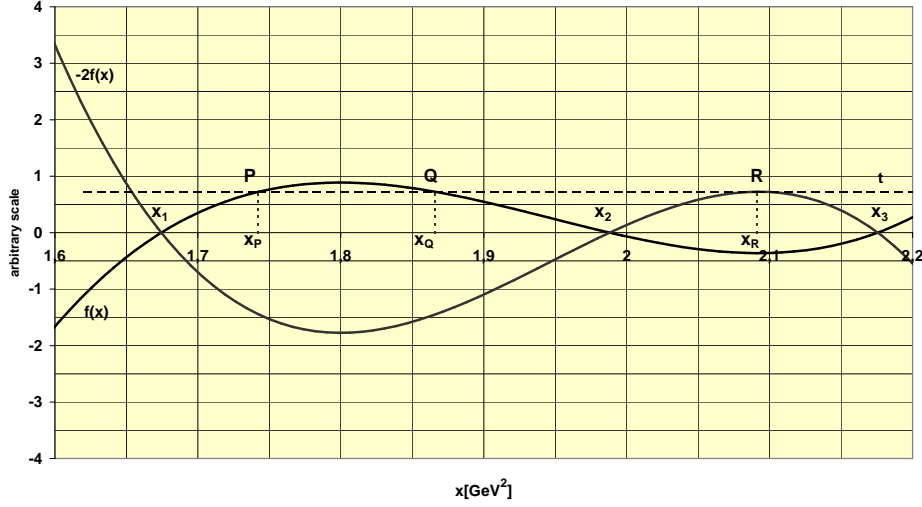


Figure 1: The allowed values of the unknown quantities a and b . The function $f(x)$ (12) is the characteristic polynomial of the m^2 operator. The eigenvalues x_1, x_2, x_3 are squared masses of the physical isoscalar mesons $\eta(1295)$, $\eta(1405)$, $\eta(1475)$. The function $-2f(x)$ is also shown. The a and b are restricted by ordering rule (14) and related by the mass formula (11): $f(a) = -2f(b)$. The horizontal line t which is tangent to the curve $-2f(x)$ at the point R of the local maximum ($x_R \in (x_2, x_3)$) crosses the curve $f(x)$ at the points P and Q . The figure indicates that the MF cannot be satisfied for $a \in (x_P, x_Q)$.

Next we calculate the octet contents of the physical isoscalar decuplet states l_1^2, l_2^2, l_3^2 (10) and construct the mixing matrix of the states η_1, η_2, η_3 [30]. The mixing matrix V is chosen such that

$$\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} = V \begin{bmatrix} N \\ S \\ G \end{bmatrix}, \quad (21)$$

where

$$N = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \quad S = s\bar{s}, \quad G - \text{glueball}. \quad (22)$$

So V expresses the states of the physical isoscalar mesons η_1, η_2, η_3 in terms of the decuplet ideal states N, S and G . The V is an orthogonal matrix. Its elements are defined by the masses.

In each of the domains A, B, C, D there is one point where the solution of the ME (9) is degenerate. The points are placed at the corners of domains A, B, C, D as is shown in the Fig. 2. In the first three domains we find three different ideally mixed $q\bar{q}$ nonets and a detached glueball; in the domain D we obtain degenerate decuplet composed of the octet states and two singlet states

detached from the octet:

$$A : \quad x_1 = a, \quad x_3 = b, \quad l_1^2 = \frac{1}{3}, \quad l_2^2 = 0, \quad l_3^2 = \frac{2}{3}, \quad (23a)$$

$$B : \quad x_1 = a, \quad x_2 = b, \quad l_1^2 = \frac{1}{3}, \quad l_2^2 = \frac{2}{3}, \quad l_3^2 = 0, \quad (23b)$$

$$C : \quad x_2 = a, \quad x_3 = b, \quad l_1^2 = 0, \quad l_2^2 = \frac{1}{3}, \quad l_3^2 = \frac{2}{3}, \quad (23c)$$

$$D : \quad x_2 = b = a = x_8, \quad l_1^2 = 0, \quad l_2^2 = 1, \quad l_3^2 = 0. \quad (23d)$$

For the wave functions one obtains:

$$A : \quad \eta_1 = \pm N, \quad \eta_2 = \pm G, \quad \eta_3 = \pm S, \quad (24a)$$

$$B : \quad \eta_1 = \pm N, \quad \eta_2 = \pm S, \quad \eta_3 = \pm G, \quad (24b)$$

$$C : \quad \eta_1 = \pm G, \quad \eta_2 = \pm N, \quad \eta_3 = \pm S, \quad (24c)$$

$$D : \quad \eta_1 = \gamma_1, \quad \eta_2 = \pm x_8, \quad \eta_3 = \gamma_2. \quad (24d)$$

Each of the degenerate solutions A, B, C points out its own candidate from among η_1, η_2, η_3 as a pure glueball. The solution D describes degenerate decuplet where η_2 is the octet isoscalar η_8 state and η_1, η_3 are scalar states built as superpositions of the $(q\bar{q})_{singlet}$ and G.

The intervals (x_1, x_P) and (x_Q, x_2) of the variable a allowed by MF and MOR are small. Also the intervals (x_2, x_R) and (x_R, x_3) of the variable b are small. Therefore, the domains A, B, C, D are also small and across any domain the solutions are not much different from the degenerate ones. The solutions of ME in any given domain constitute one-parameter family. To each of the domains there corresponds such a family. The solutions belonging to the same family are dominated by the same structure (N, S, G, η_8) which is pure in the degenerate solutions. That can be seen from the Tab. 2. Hence, the dominant structures of the η_1, η_2, η_3 in the domains A, B, C, D preserve the patterns of degenerate decuplet (24):

$$A : \quad \eta_1 \sim N, \quad \eta_2 \sim G, \quad \eta_3 \sim S, \quad (25a)$$

$$B : \quad \eta_1 \sim N, \quad \eta_2 \sim S, \quad \eta_3 \sim G, \quad (25b)$$

$$C : \quad \eta_1 \sim G, \quad \eta_2 \sim N, \quad \eta_3 \sim S, \quad (25c)$$

$$D : \quad \eta_1 \sim \gamma_1, \quad \eta_2 \sim \eta_8, \quad \eta_3 \sim \gamma_2, \quad (25d)$$

where η_8 is the octet isoscalar state and γ_1, γ_2 are superpositions of the $q\bar{q}$ singlet and G. Their contribution to the γ_1 and γ_2 states can be expressed by masses of the physical isoscalar mesons and are slowly varying functions inside the domain D.

Let us give the examples of the mixing matrix of the A, B, C, D solutions near degeneracy.

In each example the value of parameter Δa is chosen such that the deviation of the $\pi(1300)$ meson mass m_a from its ideal value (i. e. from the η_1 or from the η_2 meson mass) is equal to $6MeV$. The choice of this number is to some extent arbitrary. We want to have a decuplet which is deviated both not too little and not too much from the degenerate one; 6 MeV is the difference between the mean RPP values of $\pi(1300)$ and η_1 masses.

Table 2: The range of changes of the glueball contents under variation of a within the domains A, B, C and the octet content within the domain D. The intervals of the K-meson mass allowed over these domains are also shown. All masses are in GeV.

A	$a \in ((1.294)^2, (1.318)^2)$	$1 \geq (V_{23})^2 \geq (0.764)^2$	$1.388 \leq m_K \leq 1.390$
B	$a \in ((1.294)^2, (1.318)^2)$	$1 \geq (V_{33})^2 \geq (0.842)^2$	$1.353 \leq m_K \leq 1.374$
C	$a \in ((1.377)^2, (1.410)^2)$	$1 \geq (V_{13})^2 \geq (0.810)^2$	$1.420 \leq m_K \leq 1.443$
D	$a \in ((1.377)^2, (1.410)^2)$	$1 \geq l_2^2 \geq (0.909)^2$	$1.403 \leq m_K \leq 1.410$

A. $a = (1.300\text{GeV})^2$, $b = (1.471652\text{GeV})^2$, $K = (1.388\text{GeV})^2$,
 $l_1^2 = 0.297415$, $l_2^2 = 0.112682$, $l_3^2 = 0.589903$,

$$V_A = \begin{pmatrix} 0.975488 & 0.021850 & 0.218966 \\ 0.217116 & -0.257600 & -0.941543 \\ 0.035833 & 0.966004 & -0.256030 \end{pmatrix}; \quad (26)$$

B. $a = (1.300\text{GeV})^2$, $b = (1.414537\text{GeV})^2$, $K = (1.358\text{GeV})^2$,
 $l_1^2 = 0.297415$, $l_2^2 = 0.689645$, $l_3^2 = 0.012941$,

$$V_B = \begin{pmatrix} 0.991872 & 0.033435 & 0.122771 \\ 0.059285 & -0.975166 & -0.213391 \\ 0.112588 & 0.218935 & -0.969222 \end{pmatrix}; \quad (27)$$

C. $a = (1.404\text{GeV})^2$, $b = (474088\text{GeV})^2$, $K = (1.439\text{GeV})^2$,
 $l_1^2 = 0.002985$, $l_2^2 = 0.374721$, $l_3^2 = 0.622294$,

$$V_C = \begin{pmatrix} 0.233852 & 0.098445 & 0.967276 \\ 0.971521 & -0.062752 & -0.228491 \\ 0.038204 & 0.993162 & -0.110316 \end{pmatrix}; \quad (28)$$

D. $a = (1.404\text{GeV})^2$, $b = (1.413137\text{GeV})^2$, $K = (1.409\text{GeV})^2$,
 $l_1^2 = 0.000598$, $l_2^2 = 0.996963$, $l_3^2 = 0.002439$,

$$V_D = \begin{pmatrix} 0.530736 & 0.345334 & 0.773992 \\ 0.595077 & -0.802101 & -0.050176 \\ 0.603492 & 0.487215 & -0.631205 \end{pmatrix} \quad (29)$$

Tab. 2 exhibits also intervals of the admissible K meson mass corresponding to these solutions. Its changes under variations of Δa are in all domains relatively small and the ranges of admissible values in different domains are strictly separated.

It follows that the states of the pseudoscalar mesons $\pi(1300)$, $K(1460)$, η_1 , η_2 , η_3 may constitute solution of the ME. The price to pay for ignorance concerning both $\pi(1300)$ and $K(1460)$ meson masses is the ambiguity of the solution: instead of unique solution we have four qualitatively different one-parameter families of solutions. These families are defined within four separated domains A, B, C, D of the (a, b) plane and can be distinguished due to the fact that the isoscalar physical states are dominated by one of the N , S , G or η_8 component. Hence the domination pattern enables us to distinguish between the families of the solutions A, B, C, D of the ME. With the present data on masses of the

$\pi(1300)$ and $K(1460)$ mesons we can only distinguish between the families. If one of these masses was known then there would be only two solutions (not two families of solutions !) as can be seen from the Fig. 1.

The data on flavor properties of the isoscalar mesons indicate the family A as the one which points out the meson η_2 as a particle dominated by the glueball state.

3 Comments on solutions of the ME

•• A decuplet of mesons is a multiplet such that the octet isoscalar state η_8 contributes to three isoscalar physical states η_i . The contributions of the isoscalar octet state to the physical states η_i , given by l_i^2 s (10), constitute solution of the ME. The coefficients l_i are real numbers, therefore, the following conditions should be satisfied

$$l_i^2 > 0 \quad /i = 1, 2, 3/. \quad (30)$$

This property is not guaranteed by solution (10). Requiring it, we put constraints on the masses of the decuplet.

The knowledge of the octet contents l_i^2 s provides very convenient way for constructing the mixing matrix.

This 3 x 3 orthogonal matrix can be parametrized by Euler angles. The absolute values of trigonometric functions of two of the angles can be expressed explicitly by l_i^2 s, i.e. by the masses. The requirement of the glueball flavor independence

$$\langle G|m^2|u\bar{u}\rangle = \langle G|m^2|d\bar{d}\rangle = \langle G|m^2|s\bar{s}\rangle, \quad (31)$$

relates them to the trigonometric functions of the third angle. However, some signs of the trigonometric functions cannot be determined if only masses are known. To find them we use available information on domination of the η_i states by one of the N , S , G , η_8 states. As a result, all the Euler angles, and consequently, all elements of the mixing matrix are uniquely determined [30].

•• The VEC description of the decuplet depends on the number of ME (9) which are taken into consideration. There are two kinds of decuplets [30].

i) A decuplet which is based on assumption that three exotic commutators vanish. Then four ME arise. If they were applied to the nonet, they would define the ideal (I) one. We may imagine the isoscalar states as superpositions of a glueball and the I nonet states. We say that this decuplet is of the kind I. It complies with one mass formula (11) which, together with conditions (30), defines explicit MOR restrictions for the masses (14). To admit also the degenerate solutions of the ME one must allow $l_i^2 \geq 0$ for some "i" and in the MOR " \leq " instead of the " $<$ ". The solutions of the ME do not include free parameters – all predicted quantities and relations are expressed by physical masses.

ii) A decuplet arising under assumption that two exotic commutators vanish. Then there are three ME. If applied to the nonet, they give the S one. The decuplet is of the kind S if it is formed as superposition of the glueball and the S nonet. In this case the mass formula and ordering rule do not arise. The restrictions on the masses are not so strict and follow from the conditions (30).

If the masses of ten mesons with proper quantum numbers are known and satisfy (30), but do not satisfy the MF, the decuplet is of the kind S. If one of

the masses is unknown then we can determine it from the MF and the decuplet becomes of the kind I, provided the masses satisfy (30). However, the states constituting these two distinct decuplets may be not very different.

•• The solutions A, B, C, D described above concern the decuplet I. We now summarize the main features of the solution.

The analysis of the pseudoscalar decuplet presented here does not give the unique result due to the fact that for determining two unknown masses we have only one MF. Moreover, the MF is represented by polynomial of the third degree with respect to each of these variables.

The masses of $\pi(1300)$ and $K(1460)$ mesons are unknown. So the values a and b which are natural variables in the VEC model are also unknown, but they are bounded by MOR from the below and above. These restrictions are helpful in choosing proper solution of the nonlinear equation (11). Further restriction is provided by the mass formula which cuts out the central part of the MOR-allowed interval of a and thus reduce it to two narrow disconnected subintervals (see Fig.1). To each a belonging to them there correspond two values of b . We divide the interval (x_2, x_3) of the values b to two parts. As a result, the whole domain of the values of a and b is reduced to four small domains A, B, C, D which are shown on the Fig. 2. In these domains solution is unique if one of the variables, a or b is known.

Hence, due to the restrictions of the ME on a and b , the solution is split into four one-parameter, qualitatively different families. The partition allows us to look for solution in each domain separately. Still we have two unknown masses and only one MF equation relating them, but the domains are small and the solutions are only slightly changing across them. The ranges of the masses of the a and K mesons over the domains can be found out from the Tab. 2.

The partition of the whole domain of variables (a, b) is especially helpful for describing properties of mixing matrix. To each of the domains A, B, C, D there is attributed a separate one-parameter family of solutions of the MF determining the decuplet – among them the degenerate one. A family of the MF solutions, in turn, induce one-parameter family of mixing matrices. All the matrices of the family preserve common dominance pattern. This pattern is determined by dominance of one of the N, S, G, η_8 amplitudes in the η_1, η_2, η_3 states and can be read out from the degenerate solution. In each domain the degenerate decuplet corresponds to a point at the outer corner of the domain (see Fig. 2). Across the domain the *pure* state of the degenerate decuplet is transformed into *dominating* one and all isoscalar states become mixed. Within the domains A, B, C the glueball dominates η_2, η_3, η_1 states, respectively.

In the domain D the dominance pattern is different. The degenerate decuplet consists of the octet of exact symmetry and two separate singlets being mixed states of the $(q\bar{q})_{\text{singlet}}$ and G . The η_2 is pure octet η_8 , while η_1 and η_3 are pure singlets. The rates of the $(q\bar{q})_{\text{singlet}}$ and G states in the structures of η_1 and η_3 mesons are comparable, slowly changing functions of the parameter Δa within the domain. In spite of identical flavor properties of the constituents, the properties of the η_1 and η_3 mesons should be different and the difference is changing across the domain. This is mainly due to the fact that they are opposite superpositions of the G and $(q\bar{q})_{\text{singlet}}$ amplitudes. In this family of solutions the glueball state is not apparent.

On account of so different properties of the families A, B, C, D the qualitative information on the isoscalar mesons is sufficient to make the choice. The proper

family can be chosen on the basis of flavor properties of the isoscalar mesons η_1, η_2, η_3 . As it has been pointed out, even if all masses are known and satisfy MF and MOR, such an extra information is necessary for constructing the mixing matrix. An exact solution of the ME corresponding to definite values of the $\pi(1300)$ and $K(1460)$ masses would be determined by suitable value of Δa .

•• The restrictions following from the Fig. 1 do not hold for decuplet of the type S. There is no mass formula in this case; therefore, there is no connection between the variables a and b and there is no mass gap (x_P, x_Q) of the a meson. Also the MOR (14) does not exist. Such situation can arise for the decuplet we discuss if the measured masses of the $\pi(1300)$ and $K(1460)$ mesons will not satisfy MF. However, to define a decuplet of any type we always need mesons having such masses that conditions (30) are satisfied. These conditions give weaker constraints on a and b . We find

$$x_1 < x_8 < x_3, \quad (32)$$

where x_8 is given by the GMO mass formula,

$$x_8 = \frac{1}{3}a + \frac{2}{3}b. \quad (33)$$

The glueball contents of the isoscalar mesons η_1, η_2, η_3 can be always calculated from (10) if the masses m_π and m_K satisfy $l_i^2 > 0$ / $i=1,2,3$ /.

Having known the l_i^2 s we can construct the mixing matrix. If the state of η_2 is predicted to be dominated by G the solution of ME should be similar to the one describing the states of the family A.

4 Pseudoscalar vs scalar meson multiplets

4.1 Parity related spectra of the spin 0 mesons

Having described the multiplets of pseudoscalar mesons we get the opportunity to confront its properties with the properties of the corresponding multiplets of scalar mesons [30]. The comparison may reveal some new features of the meson spectroscopy.

Let us compare the 0^{-+} and 0^{++} multiplets. The ground states form the nonets:

$$\pi, \quad K, \quad \eta, \quad \eta', \quad (34)$$

$$a_0(980), \quad K_0(1430), \quad f_0(980), \quad f_0(1710). \quad (35)$$

which are followed by the decuplets:

$$\pi(1300), \quad K(1460), \quad \eta(1295), \quad \eta(1405), \quad \eta(1475), \quad (36)$$

$$a_0(1450), \quad K_0(1950), \quad f_0(1370), \quad f_0(1500), \quad f_0(2200)/f_0(2330). \quad (37)$$

In both cases we have the same sequence of the multiplets. Some of the masses are not exactly known, but this does not spoil the general picture.

Let us observe that not only the sequences of the multiplets are similar - also the inner structures of the decuplets are; namely:
- the physical mesons $f_0(1500)$ and $\eta(1405)$ which are dominated by glueball

states are settled just between the remaining isoscalars which are expected to be mostly the N and S quark states,

- both decuplets involve excited $q\bar{q}$ states, hence both glueballs mix with the excited $(q\bar{q})_{isocalar}$ states.

The latter property suggests affinity of the glueball with the excited states. This is especially prompted by the mixing of the 0^{++} glueball. Its mass belongs to the region where the nonet ground states and the decuplet excited states are overlapping, but the glueball prefers mixing just with the excited $q\bar{q}$ states – there is no trace of mixing with the ground $q\bar{q}$ states [30].

The 0^{-+} and 0^{++} mesons form the parity related spectra of multiplets (nonets and decuplets). The sequences of these multiplets differ only due to existence of the scalar meson $\sigma(600)$ which has no adequate pseudoscalar partner. But the nature of this meson is still a matter of discussion. Several authors suggest that its nature is different from the nature of other mesons [38, 39, 40]. By ignoring the $\sigma(600)$ we find that 0^{-+} and 0^{++} mesons form parity related *spectra of multiplets*.

The transparency of this picture confirms not only the opinion about the distinct nature of the $\sigma(600)$, but also supports correctness of sorting out the scalar mesons between the overlapping multiplets [30].

However, there is also a difference between these spectra. The mass spread of the 0^{-+} multiplets is shrinking for consecutive multiplets (nonet $137 \div 958$ MeV, decuplet $1295 \div 1475$ MeV, perhaps degenerate octet at 1800 MeV). The tendency of shrinking the mass spread of the higher lying multiplets is even more clearly expressed in the spectrum of 1^{--} mesons where all known multiplets above 1400 MeV (at 1400 MeV and 1800 MeV) are degenerate octets [35]. But this tendency is not seen in the spectrum of the 0^{++} multiplets - at least below 2300 MeV.

4.2 The masses of the spin 0 glueballs

The Lattice QCD (LQCD) calculations predict for the lower bound of the lightest 0^{++} glueball the mass [14, 15, 16]

$$m_{G^{++}} \approx 1.500 GeV. \quad (38)$$

Such mass allows to attribute the glueball nature to several mesons – among them to favored $f_0(1500)$.

For the lightest 0^{-+} glueball these calculations predict the lower bound at

$$m_{G^{-+}} \approx 2.300 GeV. \quad (39)$$

With this value no isoscalar meson discussed here can be assigned to be the glueball. Attempts to diminish this bound were unsuccessful.

The LQCD calculations predict also the lower bounds for the masses of many other glueballs with different J^{PC} . The result is that the mass (38) marks the minimum of these lower bounds. However, this result can be obtained also in other approaches (see, e.g. West's Theorem [41]). Therefore, it is considered more general and independent of particular approach.

The LQCD predicts masses of pure glueball states. Also the flux tube (FT) concerns such states.

The FT approach predicts, however, that masses of the 0^{-+} and 0^{++} glueballs should be equal. Since $f_0(1500)$ is the favored 0^{++} glueball candidate, we should expect the 0^{-+} glueball mass at about 1.500GeV . Hence, the LQCD and FT predictions on 0^{-+} glueball mass are contradictory.

The VEC prediction of the glueball mass has different source. It refers to broken unitary symmetry which collects the mesons in multiplets – the octets and the nonets. Several of them are well established in the low mass region at various J^{PC} . We assume that at higher masses the mesons are collected in multiplets as well. In the case where the glueball appears we expect the decuplet. The three isoscalar components of the decuplet are superpositions of the $q\bar{q}$ and G states. There is no pure glueball state but such state may dominate one of the isoscalars.

The mass formula for the decuplet relates physical masses. Also the mixing matrix is explicitly determined by physical masses. There is no ambiguity and only physical masses enter. Therefore, predictions are definite and, in favorable case, may help to perceive something new.

Using the mixing matrix we can calculate the mass of the pure glueball state. We can do this for the decuplets 0^{-+} and 0^{++} separately (not assuming any relation between them).

From the decuplet 0^{-+} , assuming Solution A and the mass input appropriate to the mixing matrix (26) we find

$$m_{G^{-+}} = 1.369\text{GeV}. \quad (40)$$

From the decuplet 0^{++} for Solution 1 [30] we get

$$m_{G^{++}} = 1.497\text{GeV}. \quad (41)$$

The difference between these predictions is approximately equal to the π meson mass.

$$m_{G^{++}} - m_{G^{-+}} = m_{\pi}. \quad (42)$$

Also observe that the inequality

$$m_{G^{-+}} < m_{G^{++}} \quad (43)$$

holds for all families A, B, C, D despite of LQCD calculations and West's Theorem predictions.

Let us comment.

The VEC search of the glueball is carried on within the isoscalar sector of the decuplet. Prediction of the glueball mass consists in setting all masses of the decuplet and fitting the mixings of the isoscalar components to their flavor properties. Hence, the mixings play important role in glueball determination.

The FT prediction of equality of the 0^{-+} and 0^{++} glueball masses is approximately obeyed. This may follow from the fact that G contributions to their structure are high and almost equal: for 0^{-+} decuplet (solution A (26))

$$V_A = \begin{bmatrix} xxxxxx & xxxxxx & xxxxxx \\ xxxxxx & xxxxxx & -0.94154 \\ xxxxxx & xxxxxx & xxxxxx \end{bmatrix} .; \quad (44)$$

for the 0^{++} decuplet [30]

$$V_1 = \begin{bmatrix} xxxxx & xxxxx & xxxxx \\ xxxxx & xxxxx & -0.88466 \\ xxxxx & xxxxx & xxxxx \end{bmatrix}. \quad (45)$$

Really the masses of the 0^{-+} and 0^{++} glueballs are not identical. Difference between them, although small in the scale of the mass of these glueballs, is not negligible. This difference is a result of parallel but independent examination of the relations between physical masses of the decuplet particles. Perhaps, different masses of these glueballs suggest that different decuplets affect the glueball component not identically. That may concern not only the cases of different J^{PC} but also decuplets of the same J^{PC} in different mass regions (if such ones exist).

The West's Theorem is not fulfilled: the observed mass difference (left part of the (42)) has opposite sign. The absolute value of the difference is not predicted. Also the difference between the masses predicted by LQCD has wrong sign. Beside, the value of this difference is probably too large to be explained as the mixing effect.

5 Conclusions

1. Owing to the unknown masses of $\pi(1300)$ and $K(1460)$ mesons the solution of ME for decuplet $\pi(1300)$, $K(1460)$, $\eta(1295)$, $\eta(1405)$, $\eta(1475)$ is not unique. In spite of that due to the restrictions of the VEC model all solutions can be classified into four separate families; one of the families points out $\eta(1405)$ as the particle dominated by glueball state.
2. The scalar and pseudoscalar glueballs belong to the decuplets formed by mixing G with excited $q\bar{q}$ isoscalar states.
3. The spectra of the known multiplets of the 0^{-+} and 0^{++} mesons are parity related provided the $\sigma(600)$ is ignored.
4. The mass of the pure pseudoscalar glueball state $m_{G^{-+}}$ is smaller than the mass $m_{G^{++}}$ of the scalar glueball one.

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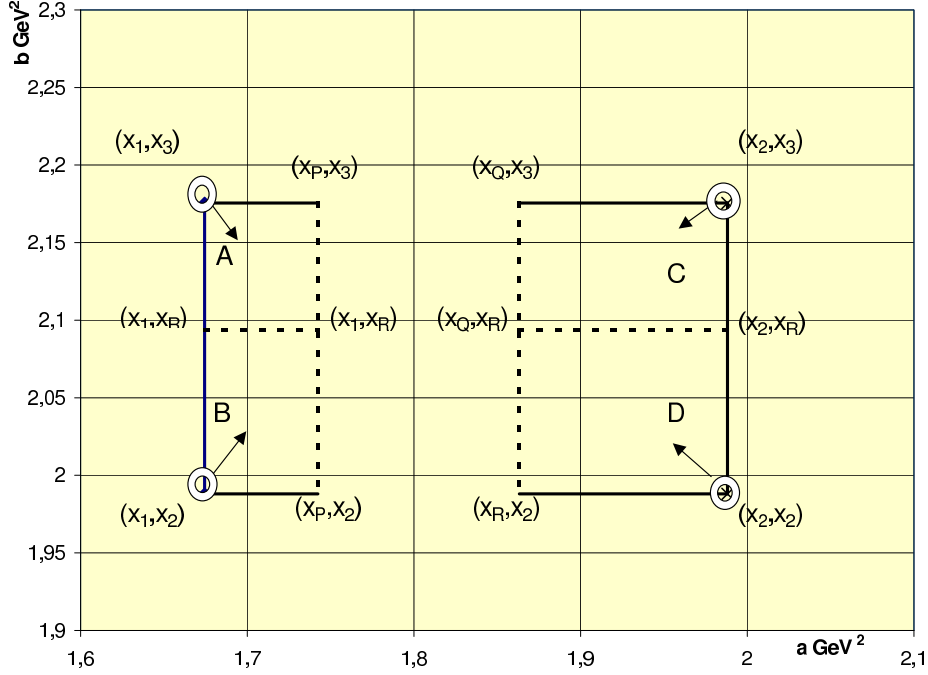


Figure 2: The domains A, B, C, D of the solutions of MF. The double circles at the outer corners of all domains correspond to degenerate decuplets. In each domain the pairs of values (a, b) , being the solutions of the MF, lie on a curve connecting this corner with the opposite one. The curves are close to diagonals of the domains. The arrows beginning at the points of degeneracy indicate directions towards the growing mixing of the states.

7 Appendix. Solving the MF

Introduce in each of the domains A, B, C, D shown on the Fig. 2. the small variables Δa and Δb . They can be chosen to be nonnegative ($\Delta a, \Delta b \geq 0$). Therefore, in the individual domains A, B, C, D, we have:

$$A : \quad a = x_1 + \Delta a, \quad b = x_3 - \Delta b, \quad (46a)$$

$$B : \quad a = x_1 + \Delta a, \quad b = x_2 + \Delta b, \quad (46b)$$

$$C : \quad a = x_2 - \Delta a, \quad b = x_3 - \Delta b, \quad (46c)$$

$$D : \quad a = x_2 - \Delta a, \quad b = x_2 + \Delta b. \quad (46d)$$

Further procedure is the following. Substituting a and b into (10), we express the coefficients l_i^2 as functions of the Δa and Δb . Putting a and b into MF (46) we get the relation between the Δb and Δa . This relation is a cubic equation with respect to any of these variables. For our purposes it is sufficient to find the approximate solution. We have

$$\Delta b = \Delta b(\Delta a, x_1, x_2, x_3). \quad (47)$$

Substituting this function into (10) we obtain the functions $l_i^2(\Delta a, x_1, x_2, x_3)$. If all these functions are positive we may consider the corresponding values of a

and b , together with the functions $l_i^2(\Delta a, x_1, x_2, x_3)$, as an approximate solution of the ME (9) in the appropriate domain. This procedure is to be performed for all the domains A, B, C, D.

In the domains A and B we may neglect all terms of (11) containing higher degrees of Δa or Δb and restrict ourselves to the linear dependence between them. The approximation is plausible for Δa covering all the interval (x_1, x_P) . We obtain

$$A : \quad \Delta b = \frac{\Delta a}{2} \frac{x_2 - x_1}{x_3 - x_1}, \quad (48a)$$

$$B : \quad \Delta b = \frac{\Delta a}{2} \frac{x_3 - x_1}{x_3 - x_2}. \quad (48b)$$

In the domains C and D we take into account also the term quadratic in Δb and all powers of Δa . This is to avoid $l_1^2 < 0$ in the domain C and to extend applicability of this approximation towards the largest values of Δa in the domain D. In these cases the expressions for Δb are the solutions of the quadratic equation, so they are simple but long and we do not write them out. Two solutions of the quadratic equation for Δb do not cause confusion, as only one of them complies with the condition $l_i^2 > 0$ for all $i=1,2,3$. In both regions the approximation is plausible everywhere, except the small surroundings of the point $b = x_R$.

In all the solutions A, B, C, D the value $\Delta a = 0$ implies $\Delta b = 0$. The degeneracy of the decuplet is destroyed if $\Delta a > 0$. An isoscalar state η_i having pure G or pure η_8 structure becomes mixed. However, it is still dominated by the same state provided Δa is sufficiently small. The mixing is intensified and the dominance is getting weaker as Δa is increasing. By examining the mixing matrix we can check whether the dominance is kept inside all domains.

Tab. 2 shows the range of change of the squared matrix elements V_{23} , V_{33} , V_{13} expressing contribution of the glueball to the η_2 , η_3 , η_1 respectively and the octet content l_2^2 under change of Δa in the solutions A, B, C, D. It can be seen that dominance of the G and η_8 states is kept over the whole domain of these solutions. N and S dominance of the other η_i states belonging to the same solution (not shown in the table) is preserved across the domains A, B, C as well; however, there is no dominance of η_1, η_2, η_3 by N, S, G in the case of the solution D. We thus find that within each domain the solution has specific *dominance pattern* which does not change under variation of Δa . (Obviously, the degree of the dominance does depend on the Δa). The dominance patterns of the solutions A, B, etc., are identical with the patterns of ideal structures of the degenerate decuplet (24). These structures correspond to the points at the outer corners of the appropriate domain.

To conclude, all solutions of the ME are split into four separate one-parameter families. The solutions belonging to the same family are slightly different. The solutions belonging to different families have different dominance patterns.

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