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► **To cite this version:**

Adrien Revault d'Allonnes, Herman Akdag, Olivier Poirel. Trust-moderated information-likelihood. A multi-valued logics approach. *Computability in Europe - Computation and Logic in the Real World*, Jun 2007, Sienna, Italy. pp.1-6. hal-00600719

**HAL Id: hal-00600719**

**<https://hal.archives-ouvertes.fr/hal-00600719>**

Submitted on 6 Oct 2017

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# Trust-moderated information-likelihood. A multi-valued logics approach

Adrien Revault d'Allonnes<sup>1,2</sup>, Herman Akdag<sup>1</sup>, and Olivier Poirel<sup>2</sup>

<sup>1</sup> Laboratoire d'Informatique de Paris 6 - LIP6

{Adrien.Revault-d'Allonnes, Herman.Akdag}@lip6.fr

<http://webia.lip6.fr/~allonnes/?lg=en>;

<http://webia.lip6.fr/~akdag>

<sup>2</sup> Office National d'Études et de Recherches Aérospatiales - ONERA

{Adrien.Revault\_dAllonnes, Olivier.Poirel}@onera.fr

<http://www.onera.fr/english.php>

**Abstract.** This work's motivation is to evaluate an information's certainty based on a confirmation criterion and weighted by its source's credibility. To simplify matters in order to keep this paper legible, we will suppose we have an incoming flow of information, each of which is either a confirmation of a known information, a contradiction of a known information or is unknown. With each piece of information an external estimation of its source's credibility will be given. Due to the uncertainty regarding the evolution of information certainty and that of its source's credibility, we have chosen to represent both in a multi-valued logics formalism. In this way, the constraints we will put on credibility evolution will be expressed in the same formalism as the actual evolution.

**Key words:** Information likelihood, multi-valued logic, uncertainty, trust, information fusion

## 1 Introduction

The application we are working on aims at giving a certainty score to information of different types and from various sources. The bias for calculating an information's certainty is that if it has been confirmed by other sources, it is probably more likely to be true. However, we would like to refine this idea and integrate a moderation with respect to the estimated trustworthiness of the information's source. This trustworthiness is an existing information and we believe that the more you trust someone, the more likely you are to believe what that person tells you. From this basic human tendency and using the information at hand, we wish to build a model that will favour trusted sources yet consider others as well.

Since we will be, in effect, calculating a confirmation score, we will need to compare incoming information. The actual comparison is beyond the reach of this paper, but if one considers that the sources are different and differently rated, that each source may give information of a different type than that already known, that the considered information may well confirm or disclaim the

information of interest but might do this only ‘to some extent’ one immediately sees how the data we wish to evaluate, to say nothing of the evaluation itself, is extremely uncertain. In addition to this, evaluating a source’s trustworthiness is also another potential cause for uncertainty. This explains why we have chosen to express our work in a multi-valued logics framework. Using the existing interpretation [2, 3] which states that  $x$  is  $v_\alpha A \Leftrightarrow$  “ $x$  is  $A$ ” is  $\tau_\alpha$ -true. In our problem this will be interpreted as both ‘Information  $i$  is likely is  $\tau_\alpha$ -true’ and as ‘Information  $i$ ’s source is trustworthy is  $\tau_\alpha$ -true’. The actual meaning of ‘ $\tau_\alpha$ -true’ will be briefly recalled in the next section. Section 2 will also introduce the notation we have chosen to express our model.

In section 3, we will start by introducing the principles of our algorithm (§3.1), and go on to show a more formal description in §3.2.

The following section, section 4, will discuss what we think our formalism represents and allows in terms of cognitive posture.

Finally in section 5 we will conclude this paper and offer some thoughts on our future works.

## 2 Multi-valued formalism

We shall be reasoning in a multi-valued logics formalism. This implies that we give ourselves a totally ordered scale of truth degrees,  $\mathcal{L}_M = \{\tau_0, \dots, \tau_{M-1}\}$ .  $\mathcal{L}_M$  is said to be totally ordered because  $\tau_i \leq \tau_j \Leftrightarrow i \leq j$ . This scale ranges from  $\tau_0$  which is considered to be ‘false’ to  $\tau_{M-1}$  or ‘true’, intermediate  $\tau_\alpha$ ’s values in between, such as ‘possibly true’ and the like.

Multi-valued scales, and hence qualitative degrees, offer a good way of modelling uncertain, ill-defined or poorly appraisable knowledge, something like Zadeh’s linguistic variables did [1]. Other works have added tools to reason on these uncertain values. Among these Darwiche and Ginsberg, Seridi and Akdag [4, 5] have built operators to combine intermediate truth values, and therefore model evolving cognitive processes. Following in Zadeh’s footsteps, Truck, Akdag and others [6, 9, 10] construct symbolic modifiers and other generalisations of useful operands.

To model the evolution of our belief scores, we need to use operations on truth degrees. We will use those defined by Seridi and Akdag in [7] using Łukasiewicz’s implication, defined in a multi-valued context by:

$$\tau_\alpha \rightarrow_L \tau_\beta = \min(\tau_{M-1}, \tau_{M-1-(\alpha-\beta)})$$

### Notation

$K$ , in figure 1, represents the set of all known information.

$i \in K$  represents information  $i$  and any ulterior information confirming  $i$ .

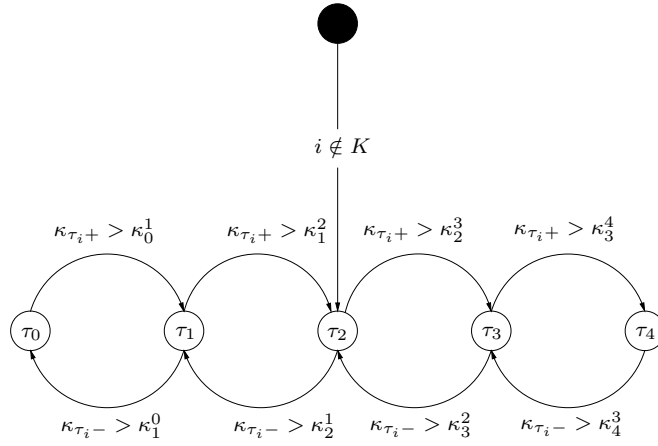
$\tau(i) \in \mathcal{L}_M$  is the current evaluation of information  $i$ ’s certainty.

Terms denoted using the letter  $\kappa$ , in general, refer to the evaluation of a source’s trustworthiness. In particular  $\kappa_{\tau_i+} \in \mathcal{L}_M$  is the current advancement in information  $\tau(i)$ ’s progression to the superior level, as  $\kappa_{\tau_i-} \in \mathcal{L}_M$  is  $\tau(i)$ ’s progression

to the inferior mark, if any. Also,  $\kappa_{\tau_\alpha}^{\tau_\beta}$  represents the threshold set to get  $i$ 's credibility from  $\tau_\alpha$  to  $\tau_\beta$ .

### 3 Algorithm

We want combined source-credibility to moderate the evolution information-score. Therefore the evolution of the truth degree of an information  $i$ , will be of the form illustrated in figure 1 and detailed hereafter.



**Fig. 1.** UML-like state diagram, representing the evolution of  $i$ 's likelihood and the various thresholds on the way in  $\mathcal{L}_5$ . Note that there is no end-state.

#### 3.1 Principle

We suppose, for legibility's sake, that we have a flow of information which either confirms a previously known information, contradicts it or is as yet unknown. Any such information will be denoted  $i$  hereafter, whether it be the original information or any other confirming it. Any contradicting information will be noted  $\neg i$ . We will suppose, for the time being, that an information confirms – or contradicts – another fully.

Now, suppose a  $\kappa_j$ -trustworthy ( $\kappa_j \in \mathcal{L}_M$ ) source gives us a new information  $i$ . As would any new information,  $i$  will be initially rated at the middle of our scale, to represent an uncertainty about its likelihood. What we want the process to do next is to have sufficient confirmation to go on to the next level of likelihood. We also wish to favour trusted sources over unknown or untrustworthy ones. For  $\tau(i)$  to move on to  $ADD(\tau(i), \tau(1))$ , i.e. a one-step increase in  $i$ 's likelihood (resp.  $SUB(\tau(i), \tau(1))$ , a one-step decrease in  $i$ 's likelihood ) in the case of a

confirmation (resp. contradiction), we will therefore require  $\kappa_{\tau_i+}$  (resp.  $\kappa_{\tau_i-}$ ) to reach a certain threshold  $\kappa_{\tau(i)}^{ADD(\tau(i),\tau(1))}$  (resp.  $\kappa_{\tau(i)}^{SUB(\tau(i),\tau(1))}$ ). Note that  $ADD$  and  $SUB$  are defined in the following section, §3.2.

The choice of parameters and its implications will be discussed in section 4.

### 3.2 Formal representation

To clarify the above described algorithm, the following representation describes each step along the way. Note that as long as we receive either a confirmation or a negation of a given piece of information  $i$ , we will loop through this algorithm. Obviously, if either end of the scale has been reached the progression in the corresponding direction will not evolve, but no information is permanently rated.

- Suppose we learn information  $i$  from a source whose trustworthiness is estimated at  $\kappa_j \in \mathcal{L}_M$ 
  - If  $i \notin K$ , then
    - \*  $\tau(i) = \tau_{\frac{M}{2}}$
    - \*  $\kappa_{\tau_i+} = \kappa_j$
  - Otherwise,
    - \*  $\kappa_{\tau_i+} = ADD(\kappa_{\tau_i+}, \kappa_j)$
  - If  $\kappa_{\tau_i+} \geq \kappa_{\tau(i)}^{\tau(i)+1}$  then
    - \*  $\tau(i) = ADD(\tau(i), \tau_1) = \neg\tau(i) \rightarrow_L \tau_1$
    - \*  $\kappa_{\tau_i+}$  is undefined
    - \*  $\kappa_{\tau_i-}$  is undefined
- Suppose, now, we learn information  $\neg i$  with a given source-credibility  $\kappa_j \in \mathcal{L}_M$ 
  - Since  $i \in K$ , then
    - \*  $\kappa_{\tau_i-} = ADD(\kappa_{\tau_i-}, \kappa_j)$
  - If  $\kappa_{\tau_i-} \geq \kappa_{\tau(i)-1}^{\tau(i)}$  then
    - \*  $\tau(i) = SUB(\tau(i), \tau_1) = \neg(\tau(i) \rightarrow_L \tau_1)$
    - \*  $\kappa_{\tau_i-}$  is undefined
    - \*  $\kappa_{\tau_i+}$  is undefined

## 4 Discussion

In this section, we will discuss the different parameters of our algorithm and their respective influence and range. We will then explain what we believe these parameters allow us to model and how.

First, we must note that we suggest to rate both information likelihood and source trustworthiness on the same scale  $\mathcal{L}_M$ . Obviously, this is possible only if the steps needed to distinguish different levels of trust are compatible with those required by the plausibility rating. That is to say that if we decide to rate source credibility on, say, a seven level scale (i.e.  $\mathcal{L}_7$ ), then there have to be seven steps in likelihood as well. The same consequence applies to the thresholds

$\kappa_{\tau_\alpha}^{\tau_\beta}$ . We could, of course, distinguish the two scales, since they only relate the granularity of truth we are allowing and because likelihood and trustworthiness are never compared nor combined. We have chosen to express both on the same scale to insist on the fact that they are truth values, distinct from the objects they relate to. However, whether both scores are evaluated on the same scale or not, it is important to note that they differ in interpretation. In fact the two factors differ in nature.  $\kappa_{\tau_i \pm}$  is an accumulated local evaluation of source credibility, whereas  $\tau(i)$  is an evolved evaluation on one information. When an information is rated from ‘impossible’ to ‘certain’ through ‘highly unlikely’ and the like, the aggregated credibilities of different sources cannot really be read as anything. However each credibility score, taken on its own, may well be anything from ‘untrustworthy’ to ‘completely trustworthy’, scaling through all and many degrees on the way.

The most important part of our method is the setting of the thresholds. We think that by not imposing any symmetry on  $\kappa_{\tau_\alpha}^{\tau_\beta}$  and  $\kappa_{\tau_\beta}^{\tau_\alpha}$  we can model different psychological postures. Indeed, we say that one way of modelling a suspicious character is to favour downward evolution over upward progression. Suppose that it would take three confirmations to get from ‘possibly true’ to ‘quite likely’, the next step up, and only two to go the other way. We think that this is typical of a mistrustful psychology. If, in addition to this, we fix the step down from ‘possibly true’ to ‘probably not true’ to two contradictions, we have a very hard to convince person. The fact that this probably increases the potential number of cycles in the evaluation of an information’s likelihood is not a problem, since, by construction, there is no end state. As long as confirmations or contradictions keep on coming, the credibility will keep on evolving, whatever the settings. The main consequence of having unequal thresholds is that the order of arrival of the information is important. We also think that, in a debate for instance, the order and the timing of arguments is of primary importance. Besides, we can eliminate this problem by setting all thresholds to the same value. Note that we have chosen to set  $\kappa_{\tau_i \pm}$  to nil when  $\tau(i)$  changes. We could have kept the overhaul to indicate which penchant we were on.

### Example

To try and make the above discussion clearer, we will consider a simple situation and look at its consequences. Suppose first of all that we will be using  $\mathcal{L}_5$  to judge the truth degrees. Table 1 gives the interpretations associated with each degree for the likelihood of an information and for the trustworthiness of any source.

Now suppose we have two different readers, as in table 2 and in figure 2, who will be rating the same flow of information, specified in table 3.

What we mean by a flow of information is a time-ordered list of information/source-rating pairs. To keep things handy, we will only list related informations, i.e. pieces of information either confirming or contradicting the original one.

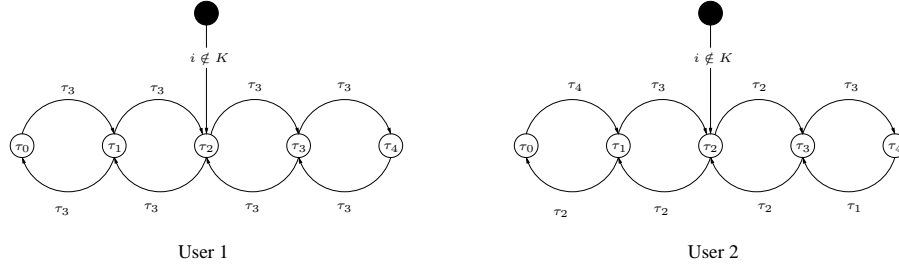
Table 3 shows the flow of incoming information and also the consequential evolution of both user’s rating. If User 1 is balanced and regular, he should not necessarily be seen as exceedingly trusting. Indeed he will only trust one

Likelihood	Degree	Trustworthiness
Totally unlikely	$\tau_0$	Absolutely untrustworthy
Rather unlikely	$\tau_1$	Rather untrustworthy
Possible	$\tau_2$	Possibly trustworthy
Rather likely	$\tau_3$	Quite trustworthy
Extremely likely	$\tau_4$	Completely trustworthy

**Table 1.** An example of possible truth-values in  $\mathcal{L}_5$

User 1				User 2			
↑		↓		↑		↓	
$\kappa_{\tau_0}^{\tau_1}$	$\tau_3$	$\kappa_{\tau_1}^{\tau_0}$	$\tau_3$	$\kappa_{\tau_0}^{\tau_1}$	$\tau_4$	$\kappa_{\tau_1}^{\tau_0}$	$\tau_2$
$\kappa_{\tau_1}^{\tau_2}$	$\tau_3$	$\kappa_{\tau_2}^{\tau_1}$	$\tau_3$	$\kappa_{\tau_1}^{\tau_2}$	$\tau_3$	$\kappa_{\tau_2}^{\tau_1}$	$\tau_2$
$\kappa_{\tau_2}^{\tau_3}$	$\tau_3$	$\kappa_{\tau_3}^{\tau_2}$	$\tau_3$	$\kappa_{\tau_2}^{\tau_3}$	$\tau_2$	$\kappa_{\tau_3}^{\tau_2}$	$\tau_1$
$\kappa_{\tau_3}^{\tau_4}$	$\tau_3$	$\kappa_{\tau_4}^{\tau_3}$	$\tau_3$	$\kappa_{\tau_3}^{\tau_4}$	$\tau_3$	$\kappa_{\tau_4}^{\tau_3}$	$\tau_1$

**Table 2.** Two different perspectives on persuasion



**Fig. 2.** Graphical representation of our two users, described in table 2

Information	Source Trustworthiness	User 1			User 2		
		$\tau(i)$	$\kappa_{\tau_i+}$	$\kappa_{\tau_i-}$	$\tau(i)$	$\kappa_{\tau_i+}$	$\kappa_{\tau_i-}$
$i$	$\tau_1$	$\tau_2$	$\tau_1$	-	$\tau_2$	$\tau_1$	-
$\neg i$	$\tau_2$	$\tau_2$	$\tau_1$	$\tau_2$	$\tau_1$	-	-
$i$	$\tau_2$	$\tau_3$	-	-	$\tau_1$	$\tau_2$	-
$\neg i$	$\tau_2$	$\tau_3$	-	$\tau_2$	$\tau_0$	-	-
$i$	$\tau_3$	$\tau_4$	-	-	$\tau_0$	$\tau_3$	-
$\neg i$	$\tau_2$	$\tau_4$	-	$\tau_2$	$\tau_0$	$\tau_3$	$\tau_2$

**Table 3.** A conflicting flow of information, the first line denoting the initial entry of the knowledge, the others either confirming it ( $i$ ) or contradicting it ( $\neg i$ )

source if it is rated as ‘quite trustworthy’. User 2, on the other hand, is rather mistrustful. Not only is he hard to convince, but changes of hearts will, in general, not be received very well. This simplified example was, obviously, constructed to enhance our point of view that different settings reflect different attitudes

towards trust. Yet we are convinced that in a more general context the same differences would be noted.

So, where Mendel and John [8] see the fuzzyfying of membership functions as an opportunity to allow for noise in the model, we think that our sort higher type multi-valued formalism may allow for different perceptions on the evolution of the truth-degrees.

## 5 Conclusion

In this paper we have used a multi-valued formalism to qualify both our belief in an information and in its source. We have used the latter to moderate the former's evolution. In so doing, we have constructed a qualitative estimation of the truth value, hence added some latitude to model uncertain processes. We have shown that different cognitive stands may be represented using this added degree of freedom.

In future works, we would like to investigate further in matters of comparison. Our model supposes that information are either unrelated or totally comparable. We think that it would benefit from the inclusion of a degree of similarity between compared objects. We would also like to work further on multi-valued scales. We think that, with all their convenient properties, they might benefit from being relaxed somewhat.

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