# Secondary instability of stratified Ekman layer roll vortices

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# SECONDARY INSTABILITY OF STRATIFIED EKMAN LAYER ROLL VORTICES

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We study the transition to turbulence of the Ekman flow in a linear density stratification, an exact solution of the Boussinesq equations occurring near boundaries in rotating fluids. The control parameters are the Richardson number $Ri = \frac{\delta^2 N^2}{G^2}$ , the Reynolds number $Re = \frac{G\delta}{\nu}$ and the Prandtl number $Pr = \frac{\nu}{\kappa}$ where $\delta = \sqrt{2\nu/f}$ is the Ekman length, $\nu$ is the kinematic viscosity, $f$ is the Coriolis parameter, $G$ is the geostrophic velocity far from the lower boundary and $\kappa$ is the thermal diffusivity. We consider $Re = 500$ and $Re = 1000$. In order to assess separately the effect the diffusion of momentum and heat, we consider two values of the Prandtl number : $Pr = 1$ and $Pr = 4$.

## 1 Nonlinear primary instability

The Ekman flow is subject to linear instability for $Ri < Ri_c(Re, Pr)$ where $Ri_c$ depends very little on $Pr$ (Brown, 1972). This instability develops into traveling Kelvin-Helmholtz roll vortices which we compute as a function of stratification as in Dubos et al. (2008). Fig. 1 presents the amplitude of the equilibrated vortices as measured by their kinetic, potential and total energy. While for $Pr = 1$ the roll amplitude vanishes at $Ri = Ri_c$ this is not the case at $Pr = 4$. So far only supercritical bifurcations had been identified in the Ekman layer stability diagram (Haeusser and Leibovich, 2003; Dubos et al., 2008). The subcritical bifurcation observed at $Pr = 4$ allows us to compute equilibrated vortices at $Ri > Ri_c$.

## 2 Linear secondary instability

Fig. 2 presents the growth rate $\sigma_2$ of infintesimal three-dimensional perturbations to the travelling rolls as a function of their horizontal wavenumber $k_y$. For $Ri = 0$ we recover at $k_y > 1$ the unstable modes identified by Dubos et al. (2008). New, less unstable modes branches are obtained for $0 < k_y < 1$. However as $Ri$ increases the modes for $k_y > 1$ become less unstable, become dominated by the unstable modes for $k_y < 1$ and eventually disappear. For $Pr = 1$ the overall maximum growth rate decreases as $Ri$ increases, but not for $Pr = 4$. This must be related to the fact that the roll vortices weaken as $Ri$ approaches $Ri_c$ if $Pr = 1$, but not if $Pr = 4$.

## 3 Conclusion

We have studied the influence of an ambient linear stratification on the secondary instability of the Ekman flow. This influence depends markedly on the Prandtl number. Our results reveal that the bifurcation at $Ri = Ri_c$ is subcritical $Pr = 4$. In this regime, finite amplitude vortices exist beyond $Ri = Ri_c$ and the growth rate of the secondary instability increases as the ambient stratification increases, instead of decreasing as occurs for $Pr = 1$ and as intuition suggests. A complementary study of the nonlinear development of the secondary instabilty is under way.

## References

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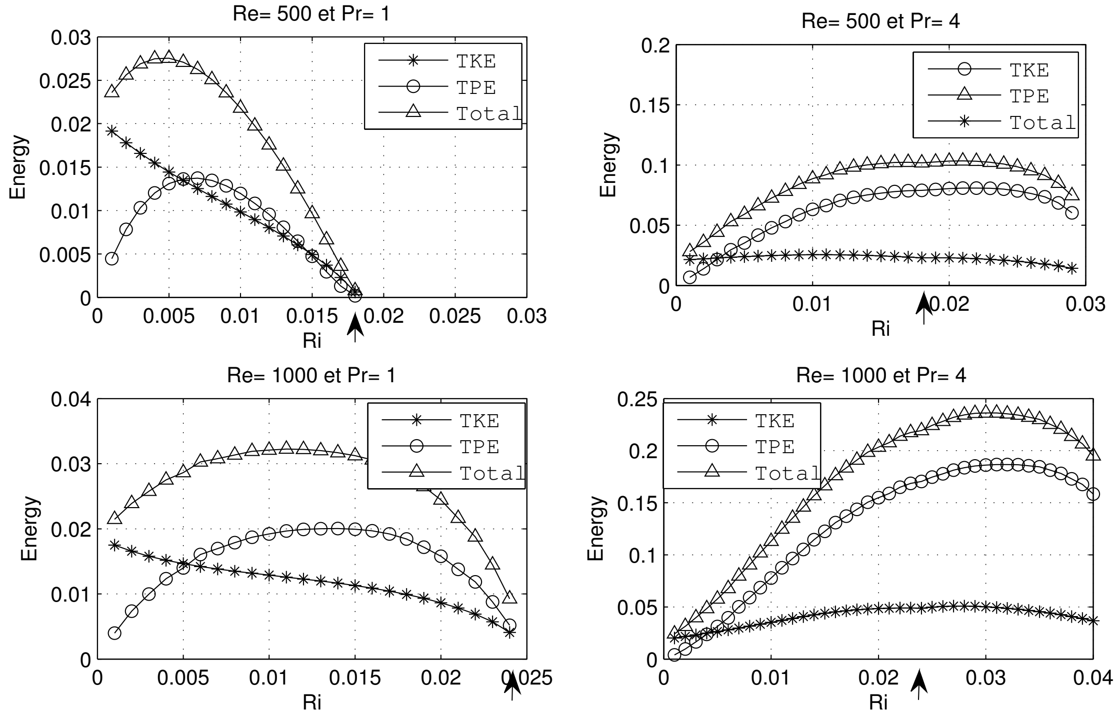


Figure 1: Energy of roll vortices as a function of ambient stratification. Arrows point to $Ri = Ri_c$

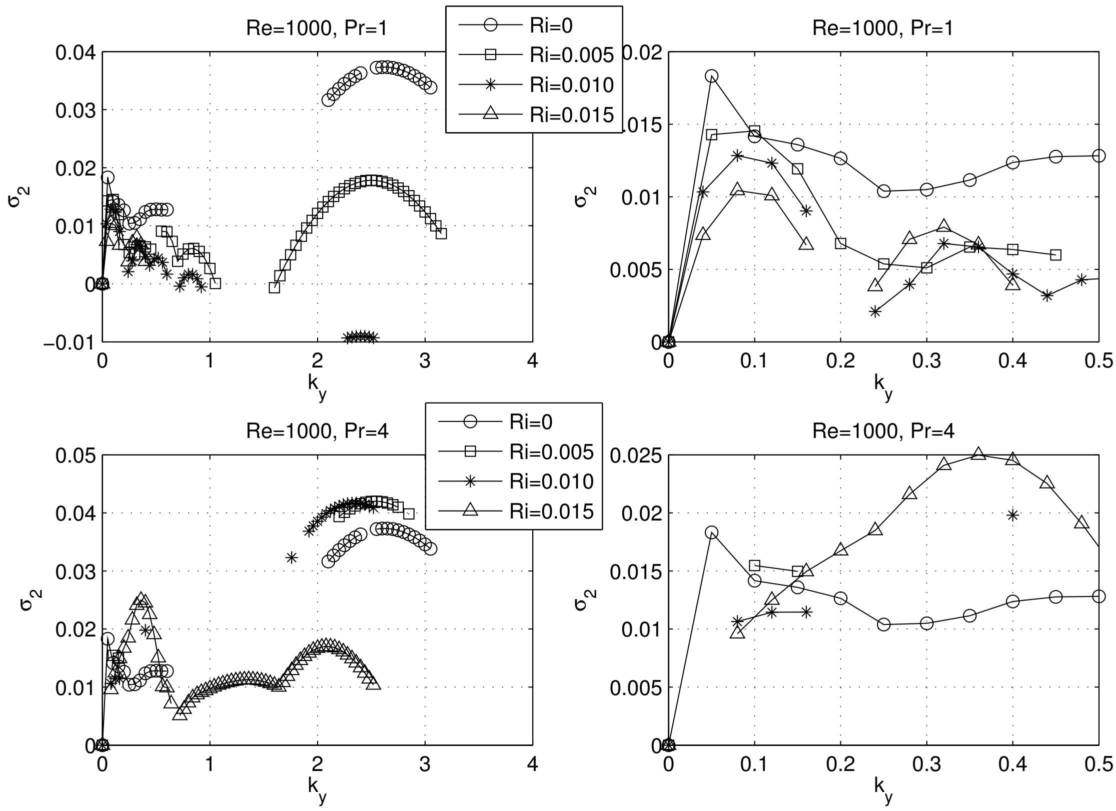


Figure 2: Growth rate $\sigma_2$ of secondary instability of Ekman roll vortices as a function of wave vector $k_y$ (right : zoom)