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We demonstrate the existence of a breather mode in the self-consistent electron dynamics of a semiconductor quantum well. A nonperturbative variational method based on quantum hydrodynamics is used to determine the salient features of the electron breather mode. Numerical simulations of the time-dependent Wigner-Poisson or Hartree equations are shown to be in excellent agreement with our analytical results. For asymmetric quantum wells, a signature of the breather mode is observed in the dipole response, which can be detected by standard optical means.

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I. INTRODUCTION

The many-electron dynamics in nanoscale semiconductor devices, such as quantum wells and quantum dots, has recently attracted a great deal of interest, mainly in view of possible applications to the growing field of quantum computing.1 Particular attention has been devoted to intersubband transitions, which involve excitation frequencies of the order of the terahertz.2 On this time scale, various collective electronic modes can be excited. For instance, the electronic dipole response is dominated by a strong resonance at the effective plasmon frequency. This resonance (known as the Kohn mode)3 is characterized by rigid oscillations of the electron gas, which, for perfectly parabolic confinement, are decoupled from the internal degrees of freedom.

In this Brief Report, we show the existence of a new distinct resonance—a monopole or “breather” mode—which corresponds to coherent oscillations of the size of the electron gas around a self-consistent equilibrium. Breather modes have been described in many areas of physics, such as nuclear matter4,5 (where they are known as giant monopole resonances) and ultracold atom dynamics.6,7 In experiments on metallic nanoparticles, monopole oscillations of the ionic structure have been observed, which manifest themselves as slow modulations of the surface plasmon.8 However, to the best of our knowledge, previous investigations have not addressed the features of the breather mode in the self-consistent dynamics of a confined electron gas. Although quantum wells constitute a typical instance of such confined systems, the present approach should equally apply to metal nanoparticles and carbon-based systems such as fullerences.

II. MODEL

Because of the translational symmetry in the transverse plane, the problem reduces to a one-dimensional (1D) one in the $x$ direction.9,10 To model the electron dynamics, we use a self-consistent quantum hydrodynamic model (QHM) that was originally derived for quantum plasmas11,12 and metallic nanostructures.13 In the QHM, the evolution of the electron density $n(x,t)$ and mean velocity $u(x,t)$ is governed by the continuity and momentum equations

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0,$$

and

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{m_n} \frac{\partial P}{\partial x} - \frac{1}{m_*} \frac{\partial V_{\text{eff}}}{\partial x} + \frac{\hbar^2}{2m_*} \frac{\partial^2}{\partial x^2} \sqrt{n},$$

where $m_*$ is the effective electron mass, $\hbar$ is the reduced Planck constant, $P(x,t)$ is the electron pressure, and $V_{\text{eff}}(x,t)$ is the effective potential, which is composed of a confining and a Hartree term. The Hartree potential obeys the Poisson equation, namely $V_H = -e^2n/e$, where $e$ is the magnitude of the electron charge and $\epsilon$ is the effective dielectric permeability of the material. The term proportional to $\hbar^2$ on the right-hand side of Eq. (2) represents the quantum force due to the so-called Bohm potential.14

The above QHM can be derived from the self-consistent Hartree equations12—or equivalently from the phase-space Wigner-Poisson equations15—in the limit of long wavelengths compared to the Thomas-Fermi screening length. For the sake of simplicity, we shall neglect exchange/correlation corrections and assume Boltzmann statistics, which is a reasonable approximation at moderate electron temperatures $T$.9,16 We also stress that the 1D model relies on the separation of the transverse and longitudinal directions, which may be broken by collisional effects. However, such effects should not be dominant on the fast time scales considered here.17

The pressure $P(x,t)$ in Eq. (2) must be related to the electron density $n$ via an equation of state (EOS) in order to close our system of electron fluid equations. We take a polytropic relation $P = \frac{\hbar}{k_B} T(n/\bar{n})^\gamma$, where $k_B$ is the Boltzmann constant, $\gamma = 3$ is the 1D polytropic exponent, and $\bar{n}$ is a mean electron density. For a homogeneous system (where $\bar{n}=n_0$), this EOS correctly reproduces the Bohm-Gross dispersion relation12 in quantum plasmas. For the inhomogeneous electron gas considered here, the choice of $\bar{n}$ is subtler and will be discussed later.

We assume a parabolic confinement, with $V_{\text{conf}}(x) = \frac{1}{2} \omega_0^2 m_* x^2$, where the frequency $\omega_0$ can be related to a fictitious homogeneous positive charge of density $n_0$ via the relation $\omega_0 = (e^2 n_0/m_e)^{1/2}$. We then normalize time to $\omega_0^{-1}$, space to $L_0 = (k_B T/m_*)^{1/2}/\omega_0$, velocity to $L_0 \omega_0$, and energy to...
through $k_BT$, and the electron number density to $n_0$. Quantum effects are measured by the dimensionless parameter $H=h\omega_0/k_BT$.

We shall use typical parameters that are representative of semiconductor quantum wells: the effective electron mass $m_\text{e}=0.067\, m_\text{e}$ and $s=13\epsilon_0$, the equilibrium density is $n_0=4.7 \times 10^{22}\, \text{m}^{-3}$, and the filling fraction $\tilde{n}/n_0=0.5$. These values yield an effective plasmon energy $h\omega_0=8.62\, \text{meV}$, a characteristic length $L_0=16.2\, \text{nm}$, a Fermi temperature $T_F=51.8\, \text{K}$, and a typical time scale $\omega_0^{-1}=76\, \text{fs}$. An electron temperature $T=200\, \text{K}$ then corresponds to a value $H=0.5$.

### III. LAGRANGIAN APPROACH

In order to derive a closed system of differential equations describing the breather mode, we list the quantum hydrodynamical equations in a Lagrangian formalism. We stress that this approach is not based on a perturbative expansion, and thus is not restricted to the regime. The Lagrangian density corresponding to the system of Eqs. (1) and (2) reads as (normalized units are used from now on)

$$\mathcal{L} = \frac{1}{2} \left( \frac{\partial\mathbf{V}_H}{\partial t} \right)^2 - nV_H - n \left( \frac{\partial \theta}{\partial t} \right)^2 - \int^n W(n')dn' - \frac{1}{2} \left( \frac{\partial \theta}{\partial x} \right)^2 + \frac{H^2}{4n} \left( \frac{\partial n}{\partial x} \right)^2 - nV_\text{conf},$$

where the independent fields are taken to be $n$, $\theta$, and $V_H$. The velocity field follows from the auxiliary function $(\theta,x,t)$ through $u=\partial\theta/\partial x$. The quantity $W(n)$ in Eq. (3) originates from the pressure, $W=\int_x^x \frac{dp}{dn}(2n)^2$. Taking the variational derivatives of the action $S=\int_0^L \mathcal{L} dx dt$ with respect to $n$, $\theta$, and $V_H$, we obtain the Eqs. (1) and (2), as well as the Poisson equation for $V_H$.

The existence of a pertinent variational formalism can be used to derive approximate solutions via the time-dependent Rayleigh-Ritz trial-function method.

Using the above Ansatz, one obtains the Lagrangian

$$L = \frac{1}{\sqrt{2\pi} \Lambda \sigma} \int L \mathcal{L} dx = \frac{\dot{d}^2}{2} + \frac{d^2 + \sigma^2}{2} + \frac{\sqrt{2}}{2} A\sigma - \frac{\sqrt{2} A^2}{6\sigma^2\sigma^2} - \frac{H^2}{8\sigma^2},$$

which only depends on two degrees of freedom, namely the dipole $d$ and the variance $\sigma$. The Euler-Lagrange equations corresponding to the Lagrangian $L$ read as

$$\ddot{d} + d = 0$$

and

$$\ddot{\sigma} + \sigma = \frac{\sqrt{2} A^2}{2} + \frac{\sqrt{3} A^2}{3\sigma^2\sigma^2} + \frac{H^2}{4\sigma^3}.$$

The quantum-well potential $V_\text{conf}$ manifests itself in the harmonic forces on the left-hand side of both Eqs. (7) and (8). As expected, the equations for $d$ and $\sigma$ decouple for purely harmonic confinement. Equation (7) describes rigid oscillations of the electron gas at the effective plasmonic frequency, i.e., the Kohn mode.

Equation (8) describes the dynamics of the breather mode, which features coherent oscillations of the width of the electron density. The three terms in the right-hand side of Eq. (8) correspond to the Coulomb repulsion (Hartree term), the electron pressure, and the quantum Bohm potential, respectively. The breather Eq. (8) can be written as $\ddot{\sigma} = -dU/\sigma d\sigma$, where $U(\sigma)$ is a pseudopotential defined as $U = \sigma^2/2 - \sqrt{2} A\sigma/2 + \sqrt{3} A^2/(6\sigma^2\sigma^2) + H^2/(8\sigma^3)$. From the shape of the pseudopotential (Fig. 1), it
Bohm-Gross dispersion relation holds, which for long wave-length uniform electron density exactly neutralized by the ion density becomes flatter and flatter, due to the strong Coulomb repulsion. This is clearly visible in Fig. 3, where we represent the evolved density profiles for two values of $A$. The dashed lines represent Gaussian distributions with the same width and the same area as the numerical curves.

(VP) system, which is equivalent to the time-dependent Hartree equations. In the normalized variables, the Wigner pseudoprobability distribution $f(x,v,t)$ satisfies the evolution equation

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - i \int \frac{dx'dv'}{2\pi\hbar} \delta V_{\text{eff}}(v-v')x'/Hf(x,v',t) = 0,$$

and is coupled to the Poisson equation. In Eq. (9), $\delta V_{\text{eff}} = V_{\text{eff}}(x+x'/2,t) - V_{\text{eff}}(x-x'/2,t)$. It is important to note that this is a microscopic quantum mean-field model, much more general than the hydrodynamic model on which our Lagrangian theory was based.

The initial condition used in the simulations is a quantum canonical distribution for the harmonic oscillator at finite temperature, where the spatial width $\sigma_0$ has been adjusted to the value obtained from the Lagrangian approach to account for the Coulomb repulsion. This is very close, but not quite identical, to an exact equilibrium of the WP equations, so that the width of the electron density starts to oscillate. We then compute the evolution of the dispersion $(x^2)^{1/2} = (\langle fxx^2dx dv/f xdx dv \rangle)^{1/2}$ and its frequency spectrum, which generally shows a sharp peak at a dominant frequency.

The results of the WP simulations are plotted in Fig. 2 (squares) and agree very well with the theoretical curve based on the Lagrangian approach. The agreement slightly deteriorates for larger values of $A$, because the electron density deviates from the Gaussian profile due to strong Coulomb repulsion. This is clearly visible in Fig. 3, where we represent the evolved density profiles for two values of $A$. For $A=1$, the profile is still approximately Gaussian, whereas for $A=3$ an intricate internal structure has developed. Nevertheless, even in this case, the error on the frequency is still just over 3%.

Table I shows that the breather frequency depends weakly on the parameter $H$ (and hence on the electron temperature). Our theoretical results are in good agreement with the simulations, except for $H=1$. For this case, the frequency spectrum is particularly broad, denoting a significant fragmentation of the resonance.

IV. SIMULATIONS

In order to check the validity of the above results, we performed numerical simulations of the Wigner-Poisson

![FIG. 2. The breather frequency $\Omega$ as a function of $A$, for $H=0.5$. Solid line: analytical results from the Lagrangian method. Squares: the Wigner-Poisson (WP) simulations. The inset shows the mean electron density $\langle n \rangle = A/\sqrt{2}\sigma_0$ as a function of $A$.](image-url)
V. NONPARABOLIC WELLS

For a parabolic potential well, there is no coupling between the breather and dipole modes, which may render the experimental detection of the breather mode by optical means difficult to realize in practice.

It can be shown that nonparabolic corrections do not introduce any linear coupling if the confining potential is symmetric. A more interesting situation arises for asymmetric wells,21 which we model by adding a small cubic term to the confining potential, \( V_{\text{cub}}(x) = (K/3)x^3 \). The equations of motion are then

\[
\ddot{d} + d = -K(\sigma^2 + \dot{d}^2), \quad (10)
\]

\[
\sigma = \frac{\sqrt{2A}}{2} + \frac{\sqrt{3A^2}}{2\sigma^3} + \frac{H^2}{4\sigma^2} - 2K\sigma d. \quad (11)
\]

Linearizing Eqs. (10) and (11) around the stable fixed point \( (d_0, \sigma_0) \), we indeed find a coupling between the breather and the dipole, with resonant frequencies \( \Omega_\pm \).

We model the coupling to the laser field by instantaneously shifting the initial dipole of a small quantity, i.e., \( \delta(0)=\delta_0 + \tilde{d} \).

Figure 4 shows a typical spectrum obtained from the numerical solution of Eqs. (10) and (11), which is proportional to the optical absorption spectrum commonly measured in the experiments. The sharp peaks correspond to the resonant frequencies \( \Omega_\pm = 0.97 \) and \( \Omega_\pm = 1.58 \), which are rather close to those obtained for parabolic confinement.

The breather mode can thus be triggered using a purely dipolar excitation, and a clear signature of the breather frequency can be observed in the optical absorption spectrum. This opens the way to optically detecting the breather mode by means of standard pump-probe experiments. Finally, the methods used here could be readily extended to three-dimensional nanostructures, and may find applications in related areas such as quantum free-electron lasers.22

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