



**HAL**  
open science

## Optimisation model for integration of cooling and heating systems in large industrial plants

Jarmo Söderman, Pekka Ahtila

► **To cite this version:**

Jarmo Söderman, Pekka Ahtila. Optimisation model for integration of cooling and heating systems in large industrial plants. *Applied Thermal Engineering*, 2009, 30 (1), pp.15. 10.1016/j.applthermaleng.2009.03.018 . hal-00593330

**HAL Id: hal-00593330**

**<https://hal.archives-ouvertes.fr/hal-00593330>**

Submitted on 14 May 2011

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## Accepted Manuscript

Optimisation model for integration of cooling and heating systems in large industrial plants

Jarmo Söderman, Pekka Ahtila

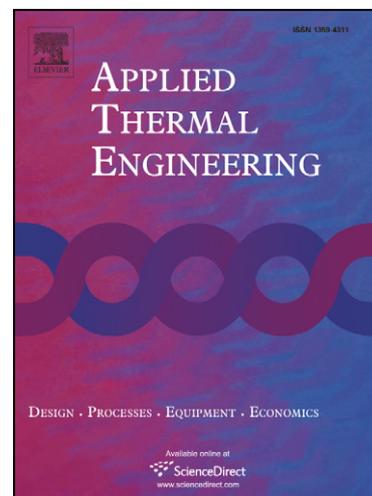
PII: S1359-4311(09)00100-8  
DOI: [10.1016/j.applthermaleng.2009.03.018](https://doi.org/10.1016/j.applthermaleng.2009.03.018)  
Reference: ATE 2767

To appear in: *Applied Thermal Engineering*

Received Date: 1 December 2008  
Revised Date: 25 March 2009  
Accepted Date: 26 March 2009

Please cite this article as: J. Söderman, P. Ahtila, Optimisation model for integration of cooling and heating systems in large industrial plants, *Applied Thermal Engineering* (2009), doi: [10.1016/j.applthermaleng.2009.03.018](https://doi.org/10.1016/j.applthermaleng.2009.03.018)

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



## **Optimisation model for integration of cooling and heating systems in large industrial plants**

Jarmo Söderman<sup>1</sup>, Pekka Ahtila<sup>2</sup>

1) Åbo Akademi University, Heat Engineering Laboratory, Biskopsgatan 8, 20500 Turku, Finland, e-mail: Jarmo.Soderman@abo.fi, tel: + 358 2 215 4964, fax: + 358 2 215 4792.

2) Helsinki University of Technology, Industrial Energy Engineering and Economics, P.O.Box 4100, 02015, HUT, Finland, e-mail: Pekka.Ahtila@hut.fi, tel: + 358 9 451 3622, fax: + 358 9 451 3674.

### **Abstract**

Large industrial plants have often hundreds of heating and cooling heat exchangers. A common situation is that cooling demands of the processes are satisfied without any deeper analysis of the overall impact of the cooling systems on the plant's economy or the environment. If cooling water is available it is used as much as needed and then pumped back to the river, some degrees warmer.

An optimisation model was developed for integration of cooling and heating systems to tackle the problem. An industrial cooling system is a complex energy system comprising different options of producing cooling, distribution pipelines for cold media

and cooling storages. Integration of power generation and heating systems to the cooling systems was included in the model. An illustrative example is presented in the paper. 10 process streams with cooling demand and 10 streams with heating demand were chosen, situated at different locations at the plant site. The optimal matches between the streams were found together with the sizes of the heat exchangers and the demands of hot and cold utilities. The costs of pipelines and the pumping costs of the streams are included in the model. The model can be used in the design of greenfield and retrofit investments and in versatile what-if analyses of the plant design or operation.

*Keywords: industrial cooling systems, design, optimisation*

## **1. Introduction**

Process industry needs large amount of energy. Both heating and cooling demands must be satisfied. This leads often plant managers to have a park of hundreds of heating and cooling heat exchangers causing high capital, energy and maintenance costs. A typical situation is that if cooling water can be obtained sufficiently, the cooling demand of the processes is satisfied by the cold water without any deeper analysis of the overall impact of the cooling systems on the plant's economy or the environment. The water is used as much as needed in the process steps and pumped back to the river, some degrees warmer.

It has been questioned whether this kind of approach is the optimal way to handle cooling in different processes and plants. By applying suitable optimisation tools it could perhaps be possible to find better alternatives that could mean lower energy demands, lower cooling water demand and lower capital and running costs for the plant.

## 2. Problem definition

In this paper a generic model for cooling system optimisation is discussed. The model is developed especially for large industrial sites with a complex energy system comprising hot streams to be cooled, different options of producing cooling, distribution pipelines for hot and cold media, options for cooling storages and options for integration with plant's different heating demands. Integration of cold and hot systems gives the possibility to utilise the process streams in an optimal way. In large industrial sites the pipeline investment costs can be large, for instance if cooling demand at one part of the site is considered to be satisfied by a process stream with heating demand from another part of the site. Therefore, pipeline costs are a vital part of the model.

The costs of cooling comprise the pipeline investment costs, cold water pumping costs as well as the exchanger equipment costs at the user points. Cold water or different phase change materials can be used to store the cooling effect. With the model the optimal system structure can be obtained, i.e. which cooling units, distribution lines and storages shall be built, as well as what are the optimal design and locations of the equipment and the operational parameters.

The cooling machines can be compressor-driven machines or heat driven absorption machines. For a compressor-driven cooling machine the coefficient of performance (COP) is considerably higher, often around 3, than for an absorption cooling machine with COP usually around 1. Excess heat at a temperature near 100 °C is needed for an absorption cooling machine and cooling water is needed at the middle-temperature level of 30 to 40 °C. For a plant which is supplying district heating during the cold season to a nearby community nearly zero-cost excess heat is often available in the summer time.

An interesting option for the industry in the Nordic countries could be the use of ice or snow for cooling. The cost of cooling can be reduced substantially by collecting or making snow during the cold season and using the melting heat for cooling during the warm season. Seasonal cooling with snow has been applied successfully, for instance, in a large Scandinavian hospital.

### **3. Optimisation Model for Industrial Cooling Systems**

The paper deals with the development of a generic optimisation model for heating and cooling integration in large industrial sites. The model is based on the Mixed Integer Linear Programming (MILP) approach. Branch and Bound method can be applied in solving the defined optimisation problem. The objective in the optimisation is to minimise the overall cost of the integrated system. The overall cost is the sum of the running costs and the annualised investment costs of the included equipment. The aim of the development of the present MILP-model has been to take into account both the investment and running costs of the system. The model includes heat recovery

exchanger and pipeline investment costs and hot utility costs including pumping costs of the involved process streams. However, it is evident, that in practical design work many other costs and design criteria must be considered that are not included in the model. Therefore, a solution obtained by running the model is meant to be regarded as a concept or starting point for detailed design.

Two earlier optimisation models have been the basis of the proposed approach. The temperature interval heat integration model, described in [1], and the model of optimal locations of the processes, storages and routing of the pipelines at the site, [2], have been combined. The model integrates hot and cold streams at different temperature levels and in different locations at the site, applying suitable temperature intervals, so that a global economical optimum solution for the energy system is found.

Mixed Integer Linear Programming optimisation of heat exchangers has been discussed in very large number of papers, for example in [3 - 6]. A comprehensive review of the heat exchanger network synthesis is given in [7]. The research on deterministic models for heat exchanger network synthesis is today very active, presented for example in recent papers [8 - 15]. Some works on optimisation of industrial process systems and regional energy planning, relevant to this paper, are given in [16 - 19].

Pinch technology has been applied in very many cases of heat integration and presented thoroughly in [20]. In the present approach, however, pinch technology is not needed to be applied. As examples of recent developments an application of pinch technology to

total site targeting has been given in [21] and combination of pinch technology and mathematical programming has been proposed in [22].

### 3.1 Heat exchange model

The heat exchange part of the model has been presented earlier in [1]. The approach is based on temperature intervals as in [3] and [4]. The target is to find an economically optimal heat integration for the plant. Two sets of streams are defined, a set of hot streams,  $H$ , and a set of cold streams,  $C$ . The cooling demand is defined by the needed temperature decrease of the hot streams and the heating demand by the temperature increase of the cold streams. The number of hot streams is denoted by  $n_H$  and the number of cold streams by  $n_C$ . Individual hot streams are denoted by subscript  $h$  and cold streams by  $c$ . The index sets for the hot and cold streams are

$$\begin{aligned} h \in H \quad H = \{1, 2, \dots, n_H\} & \quad \text{hot streams} \\ c \in C \quad C = \{1, 2, \dots, n_C\} & \quad \text{cold streams} \end{aligned} \quad (1)$$

The overall temperature range of the heat integration,  $\Delta\theta_{tot}$ , is defined as the difference of the highest temperature  $\theta_{max}$  and the lowest temperature  $\theta_{min}$  of the source and target temperatures of the involved streams.  $\Delta\theta_{tot}$  is then partitioned into a suitable number of temperature intervals, denoted by  $n_{TI}$ . The hot stream temperature intervals are denoted by subscripts  $i$  and the cold stream temperature intervals by  $j$ . The index sets are  $I$  and  $J$ , respectively

$$i \in I \quad I = \{1, 2, \dots, n_{TI}\} \quad \text{hot stream temperature intervals} \quad (2)$$

$$j \in J \quad J = \{1, 2, \dots, n_{TI}\} \quad \text{cold stream temperature intervals}$$

A hot stream interval  $(h, i)$  is cooled from the interval upper border temperature,  $\theta_{h,i,u}$ , to the interval lower border temperature,  $\theta_{h,i,l}$ . The heat flow that is transferred from the interval, denoted by  $\dot{Q}_{h,i}$ , is equal to the enthalpy decrease of the stream  $h$  at the temperature change  $\theta_{h,i,u} - \theta_{h,i,l}$ . Similarly, a cold stream interval  $(c, j)$  is heated from the interval lower border temperature,  $\theta_{c,j,l}$ , to the interval upper border temperature,  $\theta_{c,j,u}$  and the heat flow,  $\dot{Q}_{c,j}$ , that is transferred to the interval is equal to the enthalpy increase of the cold stream at the temperature change  $\theta_{c,j,u} - \theta_{c,j,l}$ .

The partition of the overall temperature range into temperature intervals is shown schematically in Figure 1.

**Figure 1.**

An example of the temperature interval formation and the corresponding hot and cold composite curves are shown in Figure 2. Heat transfer between a hot and cold interval can take place only when hot interval lower border temperature  $\theta_{h,i,l}$  is equal or higher than the cold stream interval upper border temperature,  $\theta_{c,j,u}$ .

**Figure 2.**

It is to be noted that specific heat capacities and heat transfer coefficients in different temperature intervals can vary strongly. This is the case, for instance, when a hot stream is humid air and condensation takes place inside a heat exchanger. The model takes into account these variations by calculating an individual heat transfer coefficient for each pair of temperature intervals.

### 3.2 Objective function

The objective for the optimisation is to minimise the sum of the running and investment costs of the integrated system, denoted by  $C_{tot}$ . The objective function is

$$\min C_{tot} = C_{oper} + C_{inv} \quad (3)$$

$C_{oper}$  is the sum of all running costs including the hot and cold utility costs, pumping costs, maintenance costs etc.  $C_{inv}$  is the sum of all investment costs, annualised with an annualisation factor that is defined by a chosen interest rate and a number of expected running years.

The investment costs of equipment are formulated with fixed and size-dependent costs. For instance, the investment costs of heat exchangers,  $C_{inv,E}$ , with linear heat exchanger price functions for a heat transfer match between  $h$  and  $c$  are formulated as

$$C_{inv,E} = \sum_{h \in H} \sum_{c \in C} c_{A(h,c)} \cdot A_{h,c} + \sum_{h \in H} \sum_{c \in C} F_{A(h,c)} \cdot y_{h,c} \quad (4)$$

where  $c_{A(h,c)}$  denotes the exchanger area unit cost,  $A_{h,c}$  heat transfer area,  $F_{A(h,c)}$  fixed cost for the heat exchanger and  $y_{h,c}$  binary variable, having value 1 at the existence, or 0 at the nonexistence, of the match.

For each optional match between hot and cold streams a pipeline route is defined prior the optimisation. Costs of the pipelines depend on the distances between the heat exchangers and the source and outlet points of the process streams and on the sizes of the pipelines.

The pipeline investment price  $P_L$  is formulated as

$$P_{L(h,c)} = u_{L(h,c)} \cdot L_{(h,c)} \quad (5)$$

where  $u_{L(h,c)}$  denotes the pipeline unit price defined per length unit for different diameters of the line and  $L_{(h,c)}$  denotes the length of a pipeline.

Line lengths are route lengths between two connected points at the site. Four pipelines are defined for each potential heat recovery match. Lines are from hot stream source point to exchanger, from exchanger to hot stream outlet point, from cold stream source point to exchanger and from exchanger to cold stream outlet point. A fixed location is given for each heat exchanger option.

Unit price coefficients of the pipelines are dependent on the size of the pipeline.

Coefficients are based on analyses of practical prices of installed pipelines. A linear model is used. A fixed unit price  $F_L$ , given in €/m, is used for small diameter pipelines

when the installation costs are dominant. After a limit pipeline size, defined with the cross section area  $A_{lb}$ , the unit price is defined as linearly dependent on the pipeline cross section area, denoted by  $A_{cr,L}$ .

$$u_L = F_L \text{ when } A_{cr,L} \leq A_{lb} \quad (6)$$

$$u_L = F_L + c_L \cdot (A_{cr,L} - A_{lb}) \text{ when } A_{cr,L} > A_{lb}$$

The running costs are defined with hot and cold utility prices and pumping costs.

Pumping effects are modelled as proportional to the mass flow of the stream, the chosen pressure difference  $\Delta p$  in the pipeline and efficiency factor. The costs are calculated with a given electricity price.

The costs that are taken into the consideration in a specific case shall be significant in terms of their impact on the structure of the optimal system. Evidently, cost must be given on same units.

### 3.3 Constraints

The constraints in the model are heat balances, heat transfer area constraints and the existence constraints etc.

#### *Heat Balances*

Heat balance constraints are built for both sets of streams, the hot and the cold streams.

Heat can be transferred from a hot stream interval to all cold stream intervals having lower temperature than the hot stream interval. Alternatively, heat can be transferred to a cold utility or discharged. The hot stream interval energy balance is given by

$$\dot{Q}_{h,i} = \sum_{c \in C} \sum_{j \in J_E} \dot{Q}_{h,i-c,j} + \dot{Q}_{h,i-CU} + \dot{Q}_{h,i-R} \quad \forall h \in H \quad \text{and} \quad i \in I_Q \quad (7)$$

$\dot{Q}_{h,i}$  is the total heat flow from the interval  $(h,i)$ ,  $\dot{Q}_{h,i-c,j}$  the heat flow between the intervals  $(h,i)$  and  $(c,j)$ ,  $\dot{Q}_{h,i-CU}$  the heat flow that is transferred to cold utility and  $\dot{Q}_{h,i-R}$  the residual heat that is discharged. The index set  $I_Q$  denotes intervals  $i$  that have heat and the index set  $J_E$  intervals  $j$  that have temperatures allowing heat transfer.

If a hot stream residual heat is considered to be cooled by cold utility, e.g. cooling water, the cooling water demand increases as also the power demand for pumping. Instead, if the hot stream residual heat is discharged, the hot stream at the residual temperature is allowed to flow down to a waste water channel and the water is treated at the waste water treatment plant. If the hot stream is humid air the stream is often cooled first in heat recovery exchangers and then discharged to atmosphere.

Respectively, heat can be transferred to a cold stream interval from all hot intervals with higher temperature or from a hot utility. For a cold stream interval  $(c,j)$  with a heating demand  $\dot{Q}_{c,j}$  the energy balance is

$$\dot{Q}_{c,j} = \sum_{h \in H} \sum_{i \in I_E} \dot{Q}_{h,i-c,j} + \dot{Q}_{HU-c,j} \quad \forall \quad c \in C \quad \text{and} \quad j \in J_Q \quad (8)$$

$\dot{Q}_{h,i-c,j}$  is, as above, the heat flow between intervals  $(h,i)$  and  $(c,j)$ . The index set  $J_Q$  denotes the intervals  $j$  having a heat demand and the index set  $I_E$  intervals  $i$  having temperatures that allow the heat transfer.  $\dot{Q}_{HU-c,j}$  is the heat flow from a hot utility. Evidently, if no heat transfer is taking place between the two intervals, the cold interval heat demand must be satisfied solely by the hot utility.

Heat losses from the pipelines between hot stream source points to exchangers can be included in the model. Pipelines are usually insulated to minimise the losses. The losses can be modelled as a decrease of the stream temperature before the stream reaches the exchanger. Similarly, for a cold stream a temperature increase can be defined before the stream reaches the exchanger. The heat losses from exchangers can be modelled by defining an exchanger heat loss factor proportional to the heat content and the temperature of each interval.

#### *Heat Exchange Areas*

The heat exchange area  $A_{h,c}$  for the heat transfer from a hot stream  $h$  to a cold stream  $c$  is the sum of the areas needed for transferring the heat flows  $\dot{Q}_{h,i-c,j}$

$$A_{h,c} \geq \sum_{i \in I_Q} \sum_{j \in I_E} \frac{1}{U_{h,i-c,j} \cdot \Delta\theta_{h,i-c,j}} \cdot \dot{Q}_{h,i-c,j} \quad \forall h \in H \quad \text{and} \quad c \in C \quad (9)$$

The overall heat transfer coefficient  $U_{h,i-c,j}$  between the hot stream temperature intervals  $(h,i)$  and the cold stream temperature intervals  $(c,j)$  is defined for all interval matches.

Thereby, the heat transfer coefficient variations can be taken into account. The heat transfer coefficients are calculated from the heat transfer wall temperatures between the involved temperature intervals. If hot stream is moist air and condensation takes place inside a heat exchanger there is a strong increase in the heat transfer coefficient due to condensation.

Logarithmic mean temperature (LMTD) for counter-current heat exchanger is applied to define the temperature difference,  $\Delta\theta_{h,i-c,j}$ . LMTD is calculated as

$$LMTD = (\Delta\theta_U - \Delta\theta_L) / \ln(\Delta\theta_U / \Delta\theta_L) \quad (10)$$

$\Delta\theta_U$  is the upper and  $\Delta\theta_L$  the lower temperature difference in a counter-flow heat exchanger. However, if  $\Delta\theta_U$  and  $\Delta\theta_L$  are equal, logarithmic mean temperature can not be applied. In that case arithmetic mean temperature is used instead.

#### *Existence constraints*

The existence constraints for equipment in the system are built with binary variables  $y$ .

If  $y$  is 1, the equipment is included in the solution and if  $y$  is 0, the equipment is not

included. A sufficiently large number, denoted here by  $G$ , is used in the existence constraints to force  $y$  to become 1 if the equipment is involved in the heat transfer of the solution.

For instance, for cooling machines  $m$  the existence constraints are

$$\dot{Q}_m - G_m \cdot y_m \leq 0 \quad \forall \quad m \in M \quad (11)$$

$\dot{Q}_m$  is the cooling effect generated by the cooling machine  $m$  and  $G_m$  is sufficiently large number for the machines. The set of equipment in the problem is denoted by  $M$ .

Different values for  $G_m$  can be applied in the problem formulation for different equipment. The value of  $G_m$  has to be large enough so that the heat flow  $\dot{Q}_m$  can not in any situation be larger than  $G_m$ . On the other hand  $G_m$  should be chosen just so large that this demand can be satisfied. A tightly defined  $G_m$  has the advantage of making the computation time shorter.

The binary variable  $y_m$  can have value 0 or 1 in constraint (11) when the heat flow is zero. On the other hand, the investment cost of the equipment in the objective function is formulated so that the cost is dependent on the binary variable value. The overall cost is lower when  $y_m$  is zero. Therefore, the binary variable gets value 0, when the respective equipment is not included in the optimal solution.

Stream splitting is possible. The heat of a hot stream can be transferred to one or several cold streams and heating demand of cold streams can be satisfied by one or several hot streams.

The existence binary variables are used to formulate the problem in retrofit situations. Each existing unit of equipment to be taken into consideration is given a fixed binary variable value of 1, so the equipment will be forced to be among the solution equipment. Similarly, when a unit of existing equipment is formulated with a predetermined data, the data is added as constraints with fixed constant values to the problem formulation. For instance, an existing heat exchanger among the potential exchangers is given the binary variable value 1 and its actual surface area as an additional constraint in the formulation.

#### **4. Price functions**

An equipment price function is commonly defined as a nonlinear, concave curve to take into account the economy of scale. A small fixed part of the price is applied and the size-dependent part is applied with an exponent of 0.6 to 0.8 to the size variable. The disadvantage of using nonlinear price curves is that the problem becomes a Mixed integer nonlinear problem. Generally this means firstly much longer computation times than with linear price functions and secondly that the global optimum is not guaranteed to be found. In this study the linear or piecewise linear price functions, derived from concave price curves, are applied in order to assure global optimal solutions.

### *Linear price functions*

With linear price functions the problem can be solved in a relatively short computation time. Linear prices are also practical, as exchanger prices are often given as unit prices or as a fixed price for a certain exchanger size. Linear functions can be defined by first choosing an application range, and then placing a price line so that the difference between a concave price and a linear price is minimised, [23].

A schematic diagram of the formation of a linear price from a concave price curve is shown in Figure 3.

#### **Figure 3.**

For instance, for a heat exchanger the linear price is defined as

$$P_l = P_{f,l} + u_{A,l} \cdot A \quad (12)$$

$P_l$  is the exchanger price,  $A$  the exchanger area,  $P_{f,l}$  the fixed price and  $u_{A,l}$  the unit price.

### *Piecewise linear price functions*

A piecewise linear price function can be made to follow quite well a nonlinear price curve. With the piecewise linear price functions global optimal solutions can be guaranteed. Because of additional binary variables the computation time is longer than with linear price functions.

A piecewise linear price function can be built according to the approach given in [24].

A chosen application range is first divided into a number of steps and the heat exchanger price is defined as

$$P_{A,pl} = \sum_{s=1}^{n_s} \{d_s \cdot y_s + u_{A,s} \cdot S_s\} \quad (13)$$

The exchanger price is denoted by  $P_{A,pl}$ . The subscript  $s$  denotes a step and  $n_s$  the total number of steps. A binary variable is defined for each step  $s$  and denoted by  $y_s$ .  $S_s$  is the area in step  $s$ .  $u_{A,s}$  is the unit price in step  $s$  and  $d_s$  denotes a projected price with unit price for the step at the area value of  $A = 0$ .  $d_s$  is defined as

$$d_s = P_s - u_{A,s} \cdot A_s \quad \text{for } s = 1, 2, \dots, n_s \quad (14)$$

The area values at the step borders are denoted by  $A_s$  and the prices at these area points by  $P_s$ . The unit price between two area points is defined as

$$u_{A,s} = \frac{P_s - P_{s-1}}{A_{s1} - A_{s-1}} \quad \text{for } s = 1, 2, \dots, n_s \quad (15)$$

A schematic figure of the piecewise linear price is shown in Figure 4.

**Figure 4.**

The step areas are defined with the constraints

$$A_s \cdot y_{s-1} \leq S_s \leq A_s \cdot y_s \quad \text{for } s = 1, 2, \dots, n_s \quad (16)$$

As the match area must be inside one step the sum of binary variables  $y_s$  is 1

$$\sum_{s=1}^{n_s} y_s = 1 \quad \text{for } s = 1, 2, \dots, n_s \quad (17)$$

The area of the match is the sum of the step areas.

$$A = \sum_{s=1}^{n_s} S_s \quad (18)$$

If the prices at the area points are taken from a concave price curve, the piecewise linear price function is an under-estimator of the nonlinear price function.

## 5. Multi-period model

The developed optimisation model is extended to be applicable also for multi-period optimisation tasks. Different duration curves can be used, for instance, the outdoor air

or river water temperatures. The year can be divided into a suitable number of time periods and a mean value for the chosen variable can be defined for each period.

The formation of time periods is shown schematically in Figure 5, based on the outdoor air temperature duration curve. An index set  $P$  is defined as

$$p \in P \quad p = \{1, 2, \dots, n_p\} \quad (19)$$

Time periods are denoted by index  $p$  and the number of time periods by  $n_p$ . The mean temperatures for the periods are applied in the model.

**Figure 5.**

## 6. Illustrative example

**Figure 6.**

An example of large industrial sites is shown in Figure 6. The coordinates are given in kilometres. Some process streams of the site were chosen to illustrate the use of the model. The locations from which the cold streams to be heated and the hot streams to be cooled were taken are numbered in Figure 1 by 1, 2 etc.

The described optimisation model was tested with simulated flow rates and temperature values for a set of 10 hot streams to be cooled and 10 cold streams to be heated.

### 6.1 Stream and cost data

The locations, flow rates, temperatures and cooling and heating demands of the streams are given in Table 1.

#### Table 1.

Prices that were used in the problem are given in Table 2. Equipment prices was modelled from supplier price data. Hot utility prices were estimated from fuel prices. Cold utility prices were given in proportion to hot utility prices and discharge of heat was modelled as free.

#### Table 2.

The power demand of pumping was calculated with a flow velocity of 2 m/s, a coefficient of 300 Pa/m for the pressure difference in the pipelines taking into account the flow resistance in pipelines, level differences, valves, pipe elements etc. A pump efficiency factor of 0.75 was applied. The pumping costs were calculated with an electricity price of 60 €/MWh.

### 6.2 Solution

The illustrative problem was solved with linear price functions. With CPLEX 9.0 Solver and a Pentium M processor, 1.4 GHz, the CPU-time was 55 seconds.

An annual running time of 8000 hours and an annualisation factor of 0.2 were used to calculate the annual investment costs.

With the given cooling and heating demands, prices and coordinates for the site the optimal solution comprised 18 heat transfer matches from 100 optional matches. The heat transfer matches that are included in the optimal solution are shown in Table 3. The sum of heat transfer in the matches is about 96 MW and the overall heat transfer area about 12550 m<sup>2</sup>.

**Table 3.**

Heat transfer, discharged heat flows and hot utility demands in the solution are given in Table 4. About 64 % of the cooling demand is satisfied by the heat integration, the residual heat is discharged. The overall hot utility demand is 39 MW, i.e. 29 % of the heating demand. It can be noted that five of the ten cold streams are heated solely by the heat transfer from the hot streams, whereas one cold stream is solely heated by the hot utility. The remaining four cold streams are heated partly by the hot utility, partly by the hot streams.

**Table 4.**

Costs for the obtained solution are given in Table 5. The investment in heat recovery exchangers is about 3.3 M€ and in pipelines about 2.4 M€. With the annualisation factor of 0.2 the annual investment costs are about 1.15 M€. The annual hot utility costs are about 6.6 M€ and pumping costs about 0.1 M€. The annual running costs, when pumping costs and hot utility costs are taken into account are about 6.3 M€ and the overall annual costs about 7.5 M€.

**Table 5.**

## 7. Conclusions

The developed optimisation model for industrial cooling systems can be used to optimise the structure and operation of these systems. Solutions can give valuable guidance for enterprises on how their cooling systems should be expanded, what sizes should the new equipment have, where should the heat recovery exchangers be built, where and what sizes should the cold media storages be built, what new pipelines are needed, etc. The model can be used for analysis of existing systems and as a concept-level tool for the design of greenfield and retrofit investments. The approach enables the industry to improve the efficiency of the cooling systems, reduce the energy costs by effective integration of the energy systems and reduce the environmental effects of the plants.

## **Acknowledgements**

The study has been made in connection with a research project "Optimisation of industrial cooling systems with energy cogeneration". The Finnish Funding Agency for Technology and Innovation, Tekes, and the industrial companies participating in the research project are gratefully acknowledged for the support of the study.

## List of Symbols

### Variables

$A$	heat transfer area, m <sup>2</sup>
$C$	total cost, €/a
$P$	price, €
$\dot{Q}$	heat flow, kW
$y$	binary variable, -

### Parameters

$a$	annualisation factor, -
$c$	unit cost, €/m·a, €/m <sup>2</sup> ·a etc.
$F$	fixed cost, €/a
$G$	a large number, -
$L$	Length of pipeline, m
$n$	number, -
$n_m$	number of cooling machines, -
$n_C$	number of cold streams, -
$n_H$	number of hot streams, -
$n_{TI}$	number of temperature intervals, -
$n_P$	number of periods, -

$t$	time, s or h
$U$	overall heat transfer coefficient, $\text{W}/\text{m}^2\text{K}$
$u$	unit price, €/m, €/m <sup>2</sup> etc.
$\theta$	temperature, °C

### Subscripts and index sets

$H$	set of hot streams
$h$	hot stream
$I$	set of hot stream intervals
$i$	hot stream interval
$C$	set of cold streams
$c$	cold stream
$CU$	cold utility
$HU$	hot utility
$E$	heat exchange
$Q$	heat
$J$	set of cold stream intervals
$j$	cold stream interval
$P$	set of time periods
$p$	time period
$s$	area step
$T$	interval
$R$	residual

<i>u</i>	upper
<i>l</i>	lower
<i>f</i>	Fixed investment cost
<i>pline</i>	Pipeline
<i>m</i>	Equipment
<i>M</i>	Set of equipment
<i>inv</i>	Investment costs
<i>oper</i>	Running costs
<i>tot</i>	Total costs

## References

- [1] J. Söderman, T. Westerlund, F. Pettersson, Economical optimisation of heat recovery systems for paper machine dryer sections, Proceedings of 2nd Conf. on Process Integration, Modelling and Optimisation for Energy Saving and Pollution Reduction - PRES '99, (Ed. F. Friedler and J. Klemes), Budapest, Hungarian Chemical Society (1999), 607–612.
- [2] J. Söderman, F. Pettersson, Structural and operational optimisation of distributed energy systems, Applied Thermal Engineering 26 (2006) 1400–1408.

- [3] S.A. Papoulias, I.E. Grossmann, A structural optimization approach in process synthesis - II: Heat recovery networks, *Comp. Chem. Eng.* 7 (1983) 707–721.
- [4] T.F. Yee, I.E. Grossmann, Simultaneous optimization models for Heat Integration, II: Heat Exchanger Network Synthesis, *Comp. Chem. Eng.* 14 (1990) 1165–1184.
- [5] C.A. Floudas, A.R. Ciric, I.E. Grossmann, Automatic synthesis of optimum heat exchanger network configurations, *AIChE J.* 32 (1986) 276–290.
- [6] A.R. Ciric, C.A. Floudas, Heat exchanger network synthesis without decomposition, *Comp. Chem. Eng.* 15 (1991) 385–396.
- [7] K.C. Furman, N.V. Sahinidis, A Critical Review and Annotated Bibliography for Heat Exchanger Network Synthesis in the 20th Century, *Ind. Eng. Chem. Res.* 41 (2002) 2335–2370.
- [8] J. Kim, J. Kim, J. Kim, C.K. Yoo, I. Moon, A simultaneous optimization approach for the design of wastewater and heat exchange networks based on cost estimation, *J. of Cleaner Prod.* 17 (2009), 162-171.
- [9] J.M. Ponce-Ortega, A. Jiménez-Gutiérrez, I.E. Grossmann, Optimal synthesis of heat exchanger networks involving isothermal process streams, *Comp. Chem. Eng.* 32 (2008), 1918-1942.

- [10] A.J. Isafiade, D.M. Fraser, Interval-based MINLP superstructure synthesis of heat exchange networks, *Chem. Eng. Research and Design*, 86 (2008), 245-257.
- [11] H.G. Dong, C.Y. Lin, C.T. Chang, Simultaneous optimization strategy for synthesizing heat exchanger networks with multi-stream mixers, *Chem. Eng. Research and Design* 86 (2008), 299-309.
- [12] C.L. Chen, P.S. Hung, Synthesis of flexible heat exchange networks and mass exchange networks, *Comp. Chem. Eng.* 31 (2007), 1619-1632.
- [13] M. Errico, S. Maccioni, G. Tola, P. Zuddas, A deterministic algorithm for the synthesis of maximum energy recovery heat exchanger network, *Comp. Chem. Eng.* 31 (2007), 773-781.
- [14] M. Serna-González, A. Jiménez-Gutiérrez, J.M. Ponce-Ortega, Targets for Heat Exchanger Network Synthesis with Different Heat Transfer Coefficients and Non-uniform Exchanger Specifications, *Chem. Eng. Research and Design* 85 (2007), 1447-1457.
- [15] W. Verheyen, N. Zhang, Design of flexible heat exchanger network for multi-period operation, *Chem Eng. Science* 61 (2006), 7730-7753.
- [16] I.E. Grossmann, Mixed-integer optimization techniques for algorithmic process synthesis, *Advances in Chem. Eng.* 23 (1996) 171-246.

- [17] L.T. Biegler, I.E. Grossmann, A.W. Westerberg, Systematic Methods of Chemical Process Design, Prentice Hall, 1997.
- [18] A.L.S. Chan, T.T. Chow, S.K.F. Fong, J.Z. Lin, Performance evaluation of district cooling plant with ice storage, *Energy* 31 (2006) 2750–2762.
- [19] C. Weber, F. Maréchal, D. Favrat, Design and optimization of district energy systems, CD-ROM of Full Texts of 17th Int. Congr. Chem. and Proc. Eng., PRES 2006, File 0710, Lecture G 2.10, Prague, 2006.
- [20] B. Linnhoff, D.W. Townsend, D. Boland, G.F. Hewitt, B.E.A. Thomas, A.R. Guy, R.H. Marsland, User Guide on Process Integration. Rugby, IChemE, 1982, last edition 1994.
- [21] S. Perry, J. Klemes J., I. Bulatov, Integrating Waste and Renewable Energy to reduce the Carbon Footprint of Locally Integrated Energy Sectors, *Energy* 33 (2008), 1489-1497.
- [22] C. Bengtsson, M. Karlsson, T. Berntsson, M. Söderström, Co-ordination of pinch technology and the MIND method-applied to a Swedish board mill, *Applied Thermal Engineering* 22 (2002), 133-144.

[23] S.A. Papoulias, I.E. Grossmann, A structural optimization approach in process synthesis - I: Utility systems, *Comp. Chem. Eng.* 7 (1983) 695–706.

[24] C.A. Floudas, *Nonlinear and mixed-integer optimization*, pp. 248-256, Oxford University Press, 1995.

ACCEPTED MANUSCRIPT

**Table 1.** Cooling and heating demands for the included hot and cold streams.

	<b>Hot streams</b>	Flow kg/s	T1 °C	T2 °C	Moisture kg H <sub>2</sub> O/kg da	Cooling Demand MW	Location
H1	water	20	130	10		10.08	4
H2	water	35	85	10		11.03	4
H3	water	85	50	10		14.28	5
H4	moist air	30	70	10	0.156	12.75	8
H5	water	350	45	10		51.45	1
H6	water	20	70	10		5.04	6
H7	moist air	5	80	10	0.114	1.68	2
H8	water	280	35	10		29.40	2
H9	water	40	35	10		7.56	2
H10	water	50	50	10		8.40	8

**151.67**

	<b>Cold streams</b>	Flow kg/s	T1 °C	T2 °C	Moisture kg H <sub>2</sub> O/kg da	Heating Demand MW	Location
C1	water	70	5	110		30.87	8
C2	water	350	15	35		29.40	1
C3	water	100	45	75		12.60	1
C4	water	110	70	110		18.48	9
C5	water	85	15	35		7.14	4
C6	water	110	15	35		9.24	2
C7	water	15	5	20		0.95	8
C8	cold air	10	0	150	0.004	1.52	2
C9	water	200	25	50		21.00	7
C10	water	45	55	75		3.78	3

**134.98**

**Table 2.** Utility and exchanger prices.Utility prices

Steam 4 bar	20	€/MWh
Steam 16 bar	30	€/MWh
steam 38 bar	40	€/MWh
Cold utility	2	€/MWh
Heat discharge	free	

Exchanger pricesLinear prices

heat exchanger fixed price	80	k€
heat exchanger unit price	150	€/m <sup>2</sup>

Piece-wise linear prices

heat exchanger fixed price	20	k€			
heat transfer area	500	1000	2000	4000	m <sup>2</sup>
price at area	160	250	380	570	k€

**Table 3.** Heat exchanger areas and transferred heat of the heat exchanger matches in the solution.

Match	Exchanger area m <sup>2</sup>	Transferred heat MW	Match	Exchanger area m <sup>2</sup>	Transferred heat MW
H1 - C3	276	6.62	H5 - C2	2779	29.4
H1 - C9	238	1.79	H5 - C5	238	1.79
H2 - C9	544	4.52	H5 - C6	308	2.31
H2 - C10	322	3.57	H6 - C9	341	3.36
H3 - C9	952	7.14	H7 - C3	191	0.51
H4 - C1	1977	6.3	H7 - C8	555	0.46
H4 - C3	917	2.78	H8 - C5	595	5.36
H4 - C7	206	0.95	H8 - C6	686	6.93
			H9 - C9	560	4.2
			H10 - C1	866	8.4
			<b>Sum</b>	<b>12552 m<sup>2</sup></b>	<b>96.35 MW</b>

**Table 4.** Transferred and discharged heat flows and hot utility demands for hot and cold streams.

Solution							
	Transferred	Discharged	Sum		Heating	Hot	Transferred
Hot stream	heat	heat	MW	Cold stream	Demand	utility	heat
	MW	MW			MW	MW	MW
H1	8.40	1.68	10.08	C1	30.87	16.17	14.70
H2	8.09	2.94	11.03	C2	29.40	0	29.40
H3	7.14	7.14	14.28	C3	12.60	2.70	9.90
H4	10.02	2.73	12.75	C4	18.48	18.48	0
H5	33.50	17.96	51.45	C5	7.14	0	7.14
H6	3.36	1.68	5.04	C6	9.24	0	9.24
H7	0.97	0.72	1.68	C7	0.95	0	0.95
H8	12.29	17.12	29.40	C8	1.52	1.07	0.46
H9	4.20	3.36	7.56	C9	21.00	0	21.00
H10	8.40	0	8.40	C10	3.78	0.21	3.57
	<b>96.35</b>	<b>55.32</b>	<b>151.67</b>		<b>134.98</b>	<b>38.62</b>	<b>96.35</b>

**Table 5.** Solution cost data.

<u>Investment</u>			
	Heat recovery exchangers	3.323	M€
	Pipelines	2.421	M€
	Sum	5.744	M€
<u>Investment costs</u>	$C_{inv}$ (Annualisation factor 0.2)	1.149	M€/a
<u>Running costs</u>			
	Hot utility costs	6.212	M€/a
	Pumping costs	0.105	M€/a
	Sum = $C_{oper}$	6.317	M€/a
<u>Overall annual costs</u>	$C_{tot} = C_{inv} + C_{oper}$	7.466	M€/a

### Figure Captions

**Figure 1.** Partition of the overall temperature range into temperature intervals.

**Figure 2.** An example of temperature interval formation and corresponding hot and cold composite curves.

**Figure 3.** A linear price defined from a concave price function with a fixed price part and a size-dependent price part.

**Figure 4.** A piecewise linear price function formed from a concave price function.

**Figure 5.** Time periods and mean temperatures derived from a temperature duration curve.

**Figure 6.** A schematic view of a large industrial site.

## ACCEPTED MANUSCRIPT

**Table 1.** Cooling and heating demands for the included hot and cold streams.

	<b>Hot streams</b>	Flow kg/s	T1 °C	T2 °C	Moisture kg H <sub>2</sub> O/kg da	Cooling Demand MW	Location
H1	water	20	130	10		10.08	4
H2	water	35	85	10		11.03	4
H3	water	85	50	10		14.28	5
H4	moist air	30	70	10	0.156	12.75	8
H5	water	350	45	10		51.45	1
H6	water	20	70	10		5.04	6
H7	moist air	5	80	10	0.114	1.68	2
H8	water	280	35	10		29.40	2
H9	water	40	35	10		7.56	2
H10	water	50	50	10		8.40	8
						<b>151.67</b>	
	<b>Cold streams</b>	Flow kg/s	T1 °C	T2 °C	Moisture kg H <sub>2</sub> O/kg da	Heating Demand MW	Location
C1	water	70	5	110		30.87	8
C2	water	350	15	35		29.40	1
C3	water	100	45	75		12.60	1
C4	water	110	70	110		18.48	9
C5	water	85	15	35		7.14	4
C6	water	110	15	35		9.24	2
C7	water	15	5	20		0.95	8
C8	cold air	10	0	150	0.004	1.52	2
C9	water	200	25	50		21.00	7
C10	water	45	55	75		3.78	3
						<b>134.98</b>	

**Table 2.** Utility and exchanger prices.Utility prices

Steam 4 bar	20	€/MWh
Steam 16 bar	30	€/MWh
steam 38 bar	40	€/MWh
Cold utility	2	€/MWh
Heat discharge	free	

Exchanger pricesLinear prices

heat exchanger fixed price	80	k€
heat exchanger unit price	150	€/m <sup>2</sup>

Piece-wise linear prices

heat exchanger fixed price	20	k€			
heat transfer area	500	1000	2000	4000	m <sup>2</sup>
price at area	160	250	380	570	k€

## ACCEPTED MANUSCRIPT

**Table 3.** Heat exchanger areas and transferred heat of the heat exchanger matches in the solution.

Match	Exchanger area m <sup>2</sup>	Transferred heat MW	Match	Exchanger area m <sup>2</sup>	Transferred heat MW
H1 - C3	276	6.62	H5 - C2	2779	29.4
H1 - C9	238	1.79	H5 - C5	238	1.79
H2 - C9	544	4.52	H5 - C6	308	2.31
H2 - C10	322	3.57	H6 - C9	341	3.36
H3 - C9	952	7.14	H7 - C3	191	0.51
H4 - C1	1977	6.3	H7 - C8	555	0.46
H4 - C3	917	2.78	H8 - C5	595	5.36
H4 - C7	206	0.95	H8 - C6	686	6.93
			H9 - C9	560	4.2
			H10 - C1	866	8.4
			<b>Sum</b>	<b>12552 m<sup>2</sup></b>	<b>96.35 MW</b>

## ACCEPTED MANUSCRIPT

**Table 4.** Transferred and discharged heat flows and hot utility demands for hot and cold streams.

Solution	Transferred	Discharged	Sum	Cold stream	Heating	Hot	Transferred
	heat	heat			Demand	utility	heat
Hot stream	MW	MW	MW		MW	MW	MW
H1	8.40	1.68	10.08	C1	30.87	16.17	14.70
H2	8.09	2.94	11.03	C2	29.40	0	29.40
H3	7.14	7.14	14.28	C3	12.60	2.70	9.90
H4	10.02	2.73	12.75	C4	18.48	18.48	0
H5	33.50	17.96	51.45	C5	7.14	0	7.14
H6	3.36	1.68	5.04	C6	9.24	0	9.24
H7	0.97	0.72	1.68	C7	0.95	0	0.95
H8	12.29	17.12	29.40	C8	1.52	1.07	0.46
H9	4.20	3.36	7.56	C9	21.00	0	21.00
H10	8.40	0	8.40	C10	3.78	0.21	3.57
	<b>96.35</b>	<b>55.32</b>	<b>151.67</b>		<b>134.98</b>	<b>38.62</b>	<b>96.35</b>

**Table 5.** Solution cost data.

<u>Investment</u>			
	Heat recovery exchangers	3.323	M€
	Pipelines	2.421	M€
	Sum	5.744	M€
<u>Investment costs</u>	$C_{inv}$ (Annualisation factor 0.2)	1.149	M€/a
<u>Running costs</u>			
	Hot utility costs	6.212	M€/a
	Pumping costs	0.105	M€/a
	Sum = $C_{oper}$	6.317	M€/a
<u>Overall annual costs</u>	$C_{tot} = C_{inv} + C_{oper}$	7.466	M€/a

### Figure Captions

**Figure 1.** Partition of the overall temperature range into temperature intervals.

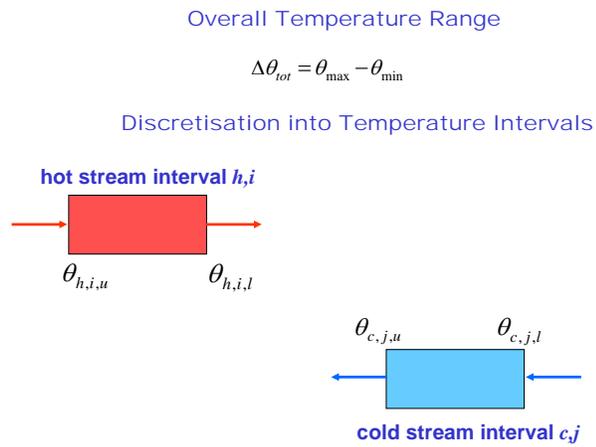
**Figure 2.** An example of temperature interval formation and corresponding hot and cold composite curves.

**Figure 3.** A linear price defined from a concave price function with a fixed price part and a size-dependent price part.

**Figure 4.** A piecewise linear price function formed from a concave price function.

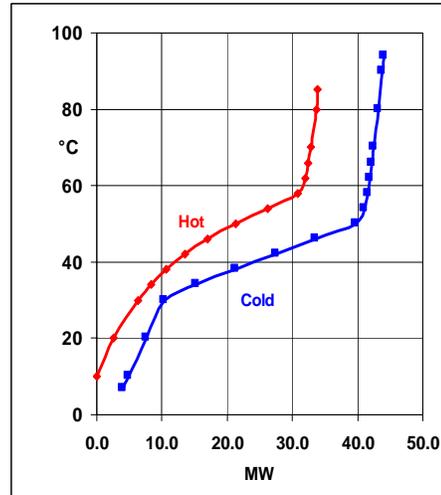
**Figure 5.** Time periods and mean temperatures derived from a temperature duration curve.

**Figure 6.** A schematic view of a large industrial site.

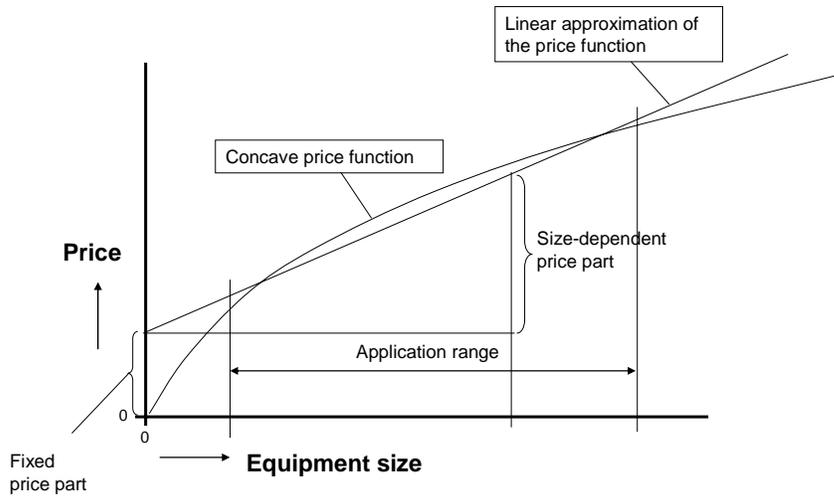


**Figure 1.** Partition of the overall temperature range into temperature intervals.

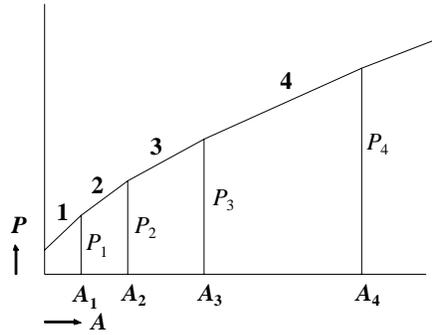
Upper and Lower Temp., °C	Interval Number	Hot streams			Cold streams		
		1	2	3	1	2	3
100 - 90	1	0	0	0	271	0	0
90 - 80	2	0	0	151	695	0	0
80 - 70	3	99	376	295	694	0	0
70 - 66	4	166	171	118	278	0	0
66 - 62	5	166	171	118	277	0	0
62 - 58	6	166	935	118	277	0	0
58 - 54	7	2146	2353	118	277	277	0
54 - 50	8	1877	1900	1115	277	1106	0
50 - 46	9	1531	1549	1143	277	1105	4736
46 - 42	10	1258	1273	940	277	1105	4735
42 - 38	11	1041	1054	778	277	1105	4733
38 - 34	12	866	877	647	277	1105	4733
34 - 30	13	725	734	541	277	1105	3513
30 - 20	14	1360	1377	1016	0	2763	0
20 - 10	15	917	929	685	0	2764	0
10 - 0	16	0	0	0	0	829	0



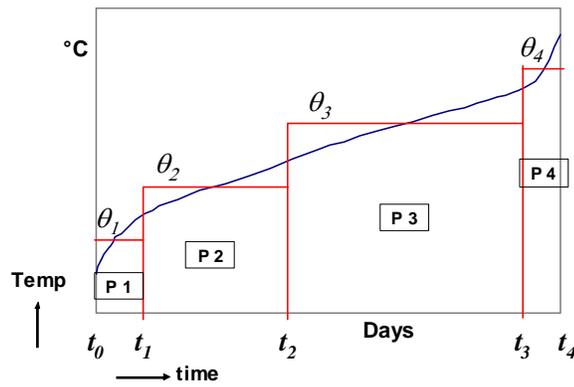
**Figure 2.** An example of temperature interval formation and corresponding hot and cold composite curves.



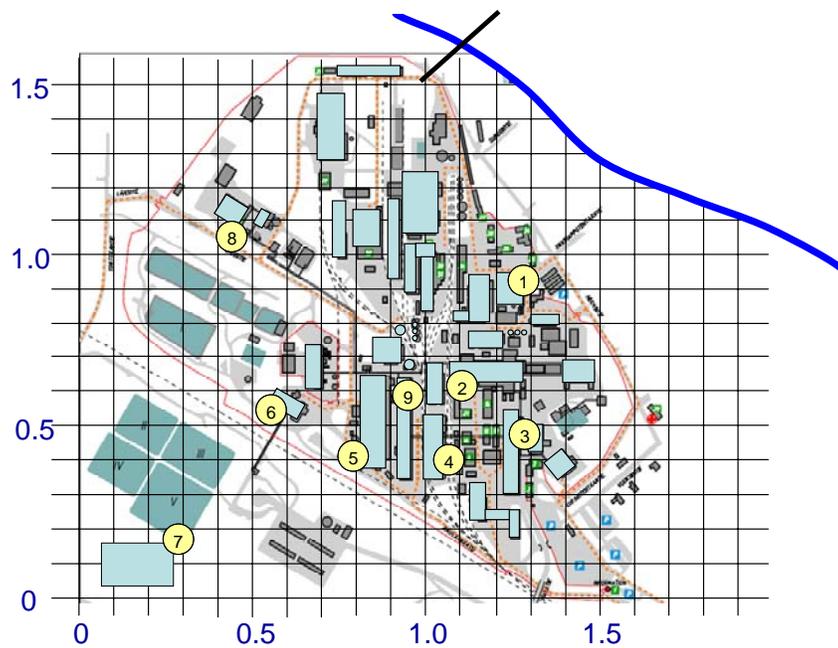
**Figure 3.** A linear price defined from a concave price function with a fixed price part and a size-dependent price part.



**Figure 4.** A piecewise linear price function formed from a concave price function.



**Figure 5.** Time periods and mean temperatures derived from a temperature duration curve.



**Figure 6.** A schematic view of a large industrial site.