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Abstract—In this paper, a novel pilot-aided algorithm is developed for MIMO-OFDM systems operating in fast time-varying environment. The algorithm has been designed to work both with parametric L-path channel model (with known path delays) and equivalent discrete-time channel model to jointly estimate the multi-path Rayleigh channel complex amplitudes (CA) and Carrier Frequency Offset (CFO). Each CA time-variation within one OFDM symbol is approximated by a Basis Expansion Model (BEM) representation. An Auto-Regressive (AR) model is built for the parameters to be estimated. The algorithm performs estimation using Extended Kalman Filtering. The channel matrix is thus easily computed and the data symbol is estimated without Inter-sub-Carrier-Interference (ICI) when the channel matrix is QR-decomposed. It is shown that our algorithm is far more robust to high speed than the conventional algorithm, and the performance approaches that of the ideal case for which the channel response and CFO are known.

I. INTRODUCTION

Multiple-Input-Multiple-Output (MIMO) antennas with Orthogonal Frequency Division Multiplexing (OFDM) provide high data rates and are robust to multi-path delay in wireless communications. Channel parameters are required for diversity combining, coherent detection and decoding. Therefore, channel estimation is essential to design MIMO-OFDM systems. For MIMO-OFDM systems, most of the channel estimation schemes have focused on pilot-assisted approaches [1][2][3], based on a quasi-static fading model that allows the channel to be invariant within a MIMO-OFDM block. However, in fast-fading channels, the time-variation of the channel within a MIMO-OFDM block results in the loss of subcarrier orthogonality, and consequently intercarrier interference (ICI) occurs [4][5]. Therefore, the channel time-variation within a block must be considered to support high-speed mobile channels.

On the other hand, similarly to the single-input single-output (SISO) OFDM, one of the disadvantages of MIMO-OFDM lies in its sensitivity to carrier frequency offset (CFO) due to carrier frequency mismatches between transmitter and receiver oscillators. As for the Doppler shift, the CFO produces ICI and attenuates the desired signal. These effects reduce the effective signal-to-noise ratio (SNR) in OFDM reception such that the system performance is degraded [6] [7]. Most of the reported works consider that all the paths present identical Doppler shifts. Hence, they group together the Doppler shift and CFO due to oscillator mismatch to obtain a single offset parameter [8] for each channel branch. However, this model is not sufficiently accurate since separate offset parameters are required for each propagation path given that the Doppler shift depends on the angle of arrival, which is particular to each path. Recently, it has been proposed to directly track channel paths to take into account separate Doppler shifts for each path ([9][10] for SISO and [11] for MIMO). Those works estimate the equivalent discrete-time channel taps ([10]) or the real path Complex Amplitudes (CA) ([9][11]) which are both modeled by a basis expansion model (BEM). The BEM methods include Karhunen-Loeve BEM (KL-BEM), prolate spheroidal BEM (PS-BEM), composite exponential BEM (CE-BEM) and polynomial BEM (P-BEM).

However the CFO due to the mismatch between transmitter and receiver oscillators is not taken into account in those algorithms. The idea of joint channel and CFO estimation has been initially proposed for SISO-OFDM systems in [12] and then extended to MIMO-OFDM systems [13]. The authors proposed an algorithm based on Extended Kalman Filtering (EKF) and on equivalent discrete-time channel model. But the fast time-variation of the channel was not taken into account.

In this paper, we propose a complete algorithm capable of jointly estimating the CFO and the path CA, by taking into account the fast variation of each path CA in MIMO environment. Generally, it is preferable to directly estimate the physical channel parameters [14] [9][11] instead of the equivalent discrete-time channel taps [10]. Indeed, as the channel delay spread increases, the number of channel taps also increases and a large number of BEM coefficients have to be estimated. This requires more pilot symbols. Hence, using a parametric channel model rather than an equivalent discrete channel model enables to reduce the signal subspace dimension [14]. Additionally, estimating the physical propagation parameters means estimating path delays and path CA. Note that in Radio-
Frequency transmissions, the delays are quasi-invariant over several MIMO-OFDM blocks [15] [4] (whereas the CA may change significantly, even within one MIMO-OFDM block). In this work, the delays are assumed perfectly estimated and quasi-invariant. It should be noted that an initial, and generally accurate estimation of the number of paths and delays can be obtained by using the MDL (minimum description length) and ESPRIT (estimation of signal parameters by rotational invariance techniques) methods [14][9].

This paper is organized as follows: Section II introduces the MIMO-OFDM system and the BEM modeling. Section III describes the state model and the Extended Kalman Filter. Section IV covers the algorithm for joint channel and CFO estimation of the number of paths and delays can be stacked in vector form:

\[ \mathbf{\nu} \triangleq \begin{bmatrix} \nu_{1,1}, \ldots \nu_{1,N_T} \end{bmatrix}, \ldots, \begin{bmatrix} \nu_{r,1}, \ldots \nu_{r,N_T} \end{bmatrix}, \ldots, \begin{bmatrix} \nu_{N_R,1}, \ldots \nu_{N_R,N_T} \end{bmatrix} \]

(1)

After transmission over a multi-path Rayleigh channel, the received MIMO-OFDM block \( \mathbf{y}_n \) is stacked in vector form:

\[ \mathbf{y}_n = \mathbf{H}_n \mathbf{x}_n + \mathbf{w}_n \]

(2)

where \( \mathbf{w}_n \) is a white complex Gaussian noise vector of covariance matrix \( \mathbf{N}_T \mathbf{\sigma}_n^2 \mathbf{I}_N \). The matrix \( \mathbf{H}_n \) is a \( N_R \times N \times N_T \times N \) MIMO channel matrix given by:

\[ \mathbf{H}_n \triangleq \begin{bmatrix} \mathbf{H}_n^{(1,1)} & \ldots & \mathbf{H}_n^{(1,N_T)} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_n^{(N_R,1)} & \ldots & \mathbf{H}_n^{(N_R,N_T)} \end{bmatrix} \]

(3)

where \( \mathbf{H}_n^{(r,t)} \) is the \((r,t)\) branch channel matrix. The elements of channel matrix \( \mathbf{H}_n^{(r,t)} \) can be written in terms of equivalent channel taps [5] \( \{g_{l,n}^{(r,t)}(qT_s)\} \) or in terms of physical channel parameters [9] (i.e. delays \( \{\tau_{l,n}^{(r,t)}\} \) and CA \( \{\alpha_{l,n}^{(r,t)}(qT_s)\} \), yielding Eq. (4) and (5), respectively.

\[ L_{(r,t)}^{(r,t)} < N_g \] is the number of channel taps and \( L_{(r,t)} \) the number of paths for the \((r,t)\) branch. The delays are normalized by \( T_s \) and not necessarily integers \( \{\tau_{l,n}^{(r,t)}\} < N_g \). The \( L_{(r,t)}^{(r,t)} \) elements of \( \{\alpha_{l,n}^{(r,t)}(qT_s)\} \) are uncorrelated. However, the \( L_{(r,t)}^{(r,t)} \) elements of \( \{g_{l,n}^{(r,t)}(qT_s)\} \) are correlated, unless the delays are multiple of \( T_s \) as is commonly assumed in the literature. They are wide-sense stationary (WSS), narrow-band zero-mean complex Gaussian processes of variances \( \sigma_{\alpha_{l,n}^{(r,t)}}^2 \) and \( \sigma_{g_{l,n}^{(r,t)}}^2 \), with the so-called Jakes’ power spectrum of maximum Doppler frequency \( f_d [16] \). The average energy of each \((r,t)\) branch is normalized to one, i.e., \( \sum_{l=0}^{L_{(r,t)}-1} \sigma_{g_{l,n}^{(r,t)}}^2 = 1 \) and \( \sum_{l=0}^{L_{(r,t)}-1} \sigma_{\alpha_{l,n}^{(r,t)}}^2 = 1 \).

In the next sections, we present the derivations for the second approach (physical channel). The results of the first approach (channel taps) can be deduced by replacing \( L_{(r,t)}^{(r,t)} \) by \( L_{(r,t)} \) and the set of delays \( \{\tau_{l,n}^{(r,t)}\} \) by \( \{l, l = 0 : L_{(r,t)} - 1\} \).

B. BEM Channel Model

Let \( L \triangleq \sum_{n=1}^{N_R} \sum_{i=1}^{N_T} L_{(r,t)} \) be the total number of paths for the MIMO channel. There are \( N_b \) samples to be estimated...
\[
[H^{(r,t)}_n]_{k,m} = \frac{1}{N} \sum_{l=0}^{L^{(r,t)}_n-1} \left[ e^{-j2\pi \left( \frac{m-1}{N} \right) l} \sum_{q=0}^{N-1} e^{j2\pi \frac{\nu(T)}{N}} g_{l,n}^{(r,t)}(qT_s) e^{j2\pi \frac{n-k}{N} q} \right] 
\]

(4)

\[
= \frac{1}{N} \sum_{l=0}^{L^{(r,t)}_n-1} \left[ e^{-j2\pi \left( \frac{m-1}{N} \right) l} \sum_{q=0}^{N-1} e^{j2\pi \frac{\nu(T)}{N}} \alpha^{(r,t)}_{l,n}(qT_s) e^{j2\pi \frac{n-k}{N} q} \right] 
\]

(5)

for each path CA due to the fast time-variation of the channel, yielding a total of \(LN_b\) samples for the whole MIMO channel. In order to reduce the number of parameters to be estimated, we resort to the Basis Expansion Model (BEM). In this section, our aim is to accurately model the time-variation of \(\alpha^{(r,t)}_{l,n}(qT_s)\) from \(q = -N_g\) to \(N - 1\) by using a BEM.

Suppose \(\alpha^{(r,t)}_{l,n}\) represents an \(N_b\times 1\) vector that collects the time-variation of the \(l\)th path of the \((r, t)\) branch within the \(n\)th MIMO-OFDM block:

\[
\alpha^{(r,t)}_{l,n} = [\alpha^{(r,t)}_{l,n}(-N_gT_s), ..., \alpha^{(r,t)}_{l,n}((N-1)T_s)]^T 
\]

(6)

Then, each \(\alpha^{(r,t)}_{l,n}\) can be expressed in terms of a BEM as:

\[
\alpha^{(r,t)}_{l,n} = \alpha_{BEM,n}^{(r,t)} + \zeta^{(r,t)}_{l,n} = \mathbf{B} \mathbf{c}_{l,n} + \zeta^{(r,t)}_{l,n} 
\]

(7)

where the \(N_b \times N_c\) matrix \(\mathbf{B}\) is defined as: \(\mathbf{B} \triangleq [\mathbf{b}_0, ..., \mathbf{b}_{N_b-1}]\). The \(N_b \times 1\) vector \(\mathbf{b}_j\) is termed as the \(j\)th expansion basis. \(\zeta^{(r,t)}_{l,n} \triangleq \alpha^{(r,t)}_{l,n}[0, ..., \alpha^{(r,t)}_{l,n}[N_c - 1]]^T\) represents the \(N_c\) BEM coefficients and \(\zeta^{(r,t)}_{l,n}\) represents the corresponding BEM modeling error, which is assumed to be minimized in the MSE sense [17]. Under this assumption, the optimal BEM coefficients and the corresponding BEM error are given by:

\[
\mathbf{c}_{l,n}^{(r,t)} = (\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H \alpha^{(r,t)}_{l,n} 
\]

(8)

\[
\zeta_{l,n}^{(r,t)} = (\mathbf{I}_{N_b} - \mathbf{S}) \alpha^{(r,t)}_{l,n} 
\]

(9)

where \(\mathbf{S} = \mathbf{B}(\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H\) is a \(N_b \times N_b\) matrix. Then, the MMSE approximation for all BEM with \(N_c\) coefficients is given by:

\[
\text{MMSE}_{\alpha^{(r,t)}_{l,n}} = \frac{1}{N_b} E \left[ \zeta_{l,n}^{(r,t)} \zeta_{l,n}^{(r,t) H} \right] 
\]

\[
= \frac{1}{N_b} \text{Tr} \left( (\mathbf{I}_{N_b} - \mathbf{S}) \mathbf{R}^{(r,t)}_{\alpha} [0] \right) (\mathbf{I}_{N_b} - \mathbf{S}) 
\]

(10)

where \(\mathbf{R}^{(r,t)}_{\alpha} [s] \triangleq E \left[ \alpha^{(r,t)}_{l,n-s} \alpha^{(r,t) H}_{l,n} \right]\) is the \(N_b \times N_b\) correlation matrix of \(\alpha^{(r,t)}_{l,n}\) with elements given by:

\[
[R^{(r,t)}_{\alpha}]_{k,m} = \sigma^{(r,t)}_{l,n}^2 \delta(k-m + sN_b) 
\]

(12)

Various traditional BEM designs have been reported to model the channel time-variatioan, e.g., the Complex Exponential BEM (CE-BEM) \(\mathbf{B}^{(r,t)}_{k,m} = e^{j2\pi \left( \frac{k-1}{N_b} \right) (m - \frac{1}{2})}\) which leads to a strictly banded frequency-domain matrix [18], the Generalized CE-BEM (GCE-BEM) \(\mathbf{B}^{(r,t)}_{k,m} = e^{j2\pi \frac{(k-1)}{N_b}} (m - \frac{1}{2})\) with \(1 < a < \frac{N_b}{}\) which is a set of oversampled complex exponentials [17], the Polynomial BEM (P-BEM) \(\mathbf{B}^{(r,t)}_{k,m} = \frac{(k - N_g)^m}{N_b}\) [9] and the Discrete Karhuen-Loeve BEM (DKL-BEM) which employs basis sequences that correspond to the most significant eigenvectors of the autocorrelation matrix \(\mathbf{R}^{(r,t)}_{\alpha}[0]\) [19]. From now on, we can describe the MIMO-OFDM system model derived previously in terms of BEM. Substituting (7) in (2) and neglecting the BEM model error, one obtains after some algebra:

\[
\mathbf{y}_n = \mathbf{K}_n (\nu) \mathbf{c}_n + \mathbf{w}_n 
\]

(13)

where the \(LN_c \times 1\) vector \(\mathbf{c}_n\) and the \(N_bN_c \times LN_c\) matrix \(\mathbf{K}_n (\nu)\) are given by:

\[
\mathbf{c}_n \triangleq \begin{bmatrix} \mathbf{c}^{(1,1)}_n, ..., \mathbf{c}^{(1,N_b)}_n, ..., \mathbf{c}^{(N_b,N_c)}_n \end{bmatrix}^T 
\]

(14)

\[
\mathbf{K}_n (\nu) \triangleq \text{blkdiag} \left\{ \mathbf{K}^{(1)}_n (\nu^{(1)}), ..., \mathbf{K}^{(N_b)}_n (\nu^{(N_b)}) \right\} 
\]

(15)

\[
\mathbf{K}^{(r,t)}_n (\nu) \triangleq \begin{bmatrix} \mathbf{K}^{(r,t)}_{n,0}, ..., \mathbf{K}^{(r,t)}_{n,N_b-1} \end{bmatrix} 
\]

(16)

where \(\nu^{(r,t)} \triangleq \begin{bmatrix} \nu^{(r)}, ..., \nu^{(N_b)} \end{bmatrix}\). Vector \(\mathbf{f}_{l}^{(r,t)}\) is the \(l\)th column of the \(N \times L^{(r,t)}\) Fourier matrix \(\mathbf{F}^{(r,t)}\) whose elements are given by:

\[
[F^{(r,t)}]_{k,l} = e^{-j2\pi \left( \frac{k-1}{N} \right) l} \cdot (r_{l},t) \cdot (t), 
\]

(17)

and \(\mathbf{M}^d_{d} (\nu)\) is a \(N \times N\) matrix whose elements are given by:

\[
[M^d_{d}(\nu)]_{k,m} = \sum_{q=0}^{N_b-1} e^{j2\pi \frac{\nu(T)}{N}} [\mathbf{B}]_{q+N_b \cdot d} e^{j2\pi \frac{n-k}{N} q} 
\]

(18)

Moreover, the channel matrix of the \((r, t)\) branch can be easily computed by using the BEM coefficients [4]:

\[
\mathbf{H}^{(r,t)}_n = \sum_{d=0}^{N_b-1} M^d_{d}(\nu) \mathbf{f}_{l}^{(r,t)} \mathbf{c}^{(r,t)}_{d} 
\]

(19)

where \(\mathbf{c}^{(r,t)}_{d} \triangleq \begin{bmatrix} \mathbf{c}^{(r,t)}_{d,0}, ..., \mathbf{c}^{(r,t)}_{d,N_b-1} \end{bmatrix}^T\). Eq. (17) will be used in the following to obtain an estimated channel matrix from the estimated CFO and BEM coefficients.

III. AR MODEL AND EXTENDED KALMAN FILTER

A. The AR Model for \(c_n\)

The optimal BEM coefficients \(\mathbf{c}^{(r,t)}_{n}\) are correlated complex Gaussian variables with zero-means and correlation matrix
given by:

\[
R^{(r,t)}_c[s] \triangleq \text{E}[c_{i,n}(r,t)c_{i,n-s}(r,t)^H]
\]
\[
= (B^H B)^{-1} B^H R^{(r,t)}_{\nu_n}[s] B (B^H B)^{-1}
\]  
(18)

Since the coefficients \(c_{i,n}^{(r,t)}\) are correlated Gaussian variables, their dynamics can be correctly modeled by an auto-regressive (AR) process [20] [21] [9]. A complex AR process of order \(p\) can be generated such that:

\[
c_{i,n}^{(r,t)} = \sum_{i=1}^{p} A^{(1)} \{c_{i,n-i}^{(r,t)} + u_{i,n}^{(r,t)}
\]  
(19)

where \(A^{(1)}, \ldots, A^{(p)}\) are \(N_c \times N_c\) matrices and \(u_{i,n}^{(r,t)}\) is a \(N_c \times 1\) complex Gaussian vector with covariance matrix \(U_{i,n}^{(r,t)}\). From [9], it is sufficient to choose \(p = 1\) to correctly model the path CA. The matrices \(A^{(1)} = A\) and \(U_{i,n}^{(r,t)}\) are the AR model parameters obtained by solving the set of Yule-Walker equations:

\[
A = R^{(r,t)}[1] \left[ R^{(r,t)}[0] \right]^{-1}
\]
\[
U_{i,n}^{(r,t)} = R^{(r,t)}[0] + \text{AR}[n(t)][-1]
\]  
(21)

Using (19), we obtain the first-order AR approximation for the dynamics of \(c_n\):

\[
c_n = A_c \cdot c_{n-1} + u_{en}
\]  
(22)

where \(A_c \triangleq \text{blkdiag} \{A, \ldots, A\}\) is a \(LN_c \times LN_c\) matrix and \(u_{en} \triangleq \{u_{0,1}^{(r,t)}, \ldots, u_{L(N_c, N_T) - 1,n}^{(r,t)}\}^T\) is a \(LN_c \times 1\) zero-mean complex Gaussian vector with covariance matrix \(U_{c} \triangleq \text{blkdiag} \{U_{0,1}^{(r,t)}, \ldots, U_{L(N_c, N_T) - 1,n}^{(r,t)}\}\).

### B. The AR Model for \(\nu_n\)

Let us write the general first-order AR model for \(\nu_n\) as follows:

\[
\nu_n = A_{\nu} \cdot \nu_{n-1} + u_{\nu n}
\]  
(23)

where the state transition matrix is of size \(N_R N_T \times N_R N_T\). Since the CFO can be assumed as constant during the observation interval, \(A_{\nu}\) is considered to be close to the identity matrix \(A_{\nu} = a I_{N_R N_T}\), where \(a\) is typically chosen between 0.99 and 0.9999 [22][13]. The \(N_R N_T \times 1\) state noise vector \(u_{\nu n}\) is assumed to be zero-mean complex Gaussian. The state noise covariance matrix is \(U_{\nu} = \sigma_{\nu}^2 I_{N_R N_T}\) where \(\sigma_{\nu}^2\) is the variance of the state noise associated with CFO. The value of the state noise variance depends on the parameter \(a\), as explained in Appendix.

### C. State equation

Now, let us write the state-variable model. The state vector at time instance \(n\) consists of the BEM coefficients \(c_n\) and the vector of CFO \(\nu_n\):

\[
\mu_n = [c_n^T, \nu_n^T]^T
\]  
(24)

There are \(LN_c\) BEM coefficients and \(N_T N_R\) CFO values in the state vector of dimension \(LN_c + N_T N_R \times 1\). Then, the linear state equation may be written as follows:

\[
\mu_n = A \cdot \mu_{n-1} + u_n
\]  
(25)

where the state transition matrix is defined as follows:

\[
A \triangleq \text{blkdiag} \{A_c, A_{\nu}\}
\]  
(26)

The \(LN_c + N_T N_R \times 1\) noise vector is such that \(u_n \triangleq [u_{en}^T, u_{\nu n}^T]^T\) with covariance matrix \(U \triangleq \text{blkdiag} \{U_c, U_{\nu}\}\).

### D. Extended Kalman Filter (EKF)

The measurement equation (13) can be reformulated as:

\[
y_n = g(\mu_n) + w_n
\]  
(27)

where the nonlinear function \(g\) of the state vector \(\mu_n\) is defined as \(g(\mu_n) = K_n(\nu) \cdot c_n\). Non-linearity of the measurement equation (27) is caused by CFO. The BEM coefficients are still linearly related to observations. Since the measurement equation is nonlinear, we use the Extended Kalman filter to adaptively track \(\mu_n\). Let \(\hat{\mu}_{(n|n-1)}\) be our a priori state estimate at step \(n\) given knowledge of the process prior to step \(n\), \(\hat{\mu}_{(n|n)}\) be our a posteriori state estimate at step \(n\) given measurement \(y_n\), and \(P_{(n|n-1)}\) and \(P_{(n|n)}\) are respectively the a priori and the a posteriori error estimate covariance matrices of size \(LN_c + N_T N_R \times LN_c + N_T N_R\). We initialize the EKF with \(\mu_{(0|0)} = 0_{LN_c + N_T N_R}\) and \(P_{(0|0)}\) given by:

\[
P_{(0|0)} = \text{blkdiag} \{R_c[0], \sigma_{\nu}^2 I_{N_R N_T}\}
\]
\[
R_c[s] = \text{blkdiag} \{R_c(1,1)[s], \ldots, R_c(N_c,N_T)[s]\}
\]
\[
R_{\nu}[s] = \text{blkdiag} \{R_{\nu}(r,t)[s], \ldots, R_{\nu}(L,L-1,s)[s]\}
\]  
(28)

where \(R_c[r,t][s]\) is the correlation matrix of \(c_{r,t}^{(r,t)}\) defined in (18). To derive the EKF equations, we need to compute the Jacobian matrix \(G_n\) of \(g(\mu_n)\) with respect to \(\mu_n\) and evaluated at \(\hat{\mu}_{(n|n-1)}\):

\[
G_n \triangleq \nabla_{\mu_n} g(\mu_n) |_{\mu_n = \hat{\mu}_{(n|n-1)}} = \begin{bmatrix}
\nabla_{\nu_n} g(\mu_n) |_{\mu_n = \hat{\mu}_{(n|n-1)}}
\nabla_{c_n} g(\mu_n) |_{\mu_n = \hat{\mu}_{(n|n-1)}}
\end{bmatrix}
\]  
(29)

Let us define \(\mu^{(r)}_n \triangleq \begin{bmatrix} \mu_n^{(r,1)}, \ldots, \mu_n^{(r,L,L-1)} \end{bmatrix}^T\) and \(\nu^{(r)}_n \triangleq \begin{bmatrix} \nu_n^{(r)} \end{bmatrix}^T\). After computation, we find:

\[
G_n = \begin{bmatrix} G_n^{(1)}(\nu_n^{(1)}), \ldots, G_n^{(N_c)}(\nu_n^{(N_c)}) \end{bmatrix}
\]  
(30)

where

\[
V_n(\mu_n) = \text{blkdiag} \{V_n^{(1)}(\mu_n^{(1)}), \ldots, V_n^{(N_c)}(\mu_n^{(N_c)})\}
\]
\[
V_n^{(r)}(\mu_n^{(r)}) = \begin{bmatrix} V_n^{(r,1)}(\mu_n^{(r,1)}), \ldots, V_n^{(r,L,L-1)}(\mu_n^{(r,L,L-1)}) \end{bmatrix}^T
\]
\[
K_n^{(r,t)}(\nu_n^{(r,t)}) = \frac{1}{N} \begin{bmatrix} Z_n^{(r,t)}(\nu_n^{(r,t)}), \ldots, Z_n^{(L,L-1)}(\nu_n^{(r,t)}) \end{bmatrix}
\]
\[
Z_n^{(r,t)}(\nu_n^{(r,t)}) \triangleq M_n^{(r,t)}(\nu_n^{(r,t)}) \text{diag} \{x_n^{(r,t)}\} f_n^{(r,t)}
\]
\[
M_n^{(r,t)}(\nu_n^{(r,t)}) \text{diag} \{x_n^{(r,t)}\} f_n^{(r,t)}
\]
\[
M_n^{(r,t)}(\nu_n^{(r,t)}) \text{diag} \{x_n^{(r,t)}\} f_n^{(r,t)}
\]
The elements of the $N \times N$ matrix $M_d^{(\nu)}$ are given by:

$$M_d^{(\nu)}(t^r)_{k,m} = \sum_{q=0}^{N-1} j 2^{q} \frac{e^{j2\pi \nu (t^r)}}{N} [B]_{q+m-k} \frac{e^{j2\pi \nu q}}{N}$$

(31)

The EKF is a recursive algorithm composed of two stages: Time Update Equations and Measurement Update Equations, defined as follows:

**Time Update Equations:**

$$\hat{\mu}(n)|_{n-1} = A\hat{\mu}(n-1)|_{n-1}$$

$$P(n)|_{n-1} = AP(n-1)|_{n-1}A^H + U$$

(32)

**Measurement Update Equations:**

$$K_n = P(n)|_{n-1}G_n^H (G_nP(n)|_{n-1}G_n^H + N_T\sigma^2 I_{N} )^{-1}$$

$$\hat{\mu}(n) = \hat{\mu}(n-1) + K_n (y_n - g(\hat{\mu}(n)|_{n-1}))$$

$$P(n) = P(n)|_{n-1} - K_n G_n P(n)|_{n-1}$$

(33)

where $K_n$ is the Kalman gain. The Time Update Equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The Measurement Update Equations are responsible for the feedback, i.e., for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. The Time Update Equations can also be thought of as predictor equations, while the Measurement Update Equations can be thought of as corrector equations.

### IV. Joint Data Detection and Parameter Estimation

#### A. Proposed algorithm

The algorithm uses $N_p$ pilots subcarriers evenly inserted into the $N$ subcarriers. The pilot positions are the same for all the transmitter antennas, yielding the set of pilot indices $P = \{ nL_f + (t-1)N, \; n = 0 \ldots N_p - 1, \; t = 1 \ldots N_T \}$, where $L_f$ is the distance between two adjacent pilots. The data is detected with a QR-equalizer [9] with free Inter-Carrier-Interference (ICI) thanks to a QR-decomposition.

The general principle is as follows: to detect the data symbols $x_n$, we need to perform an equalization which requires the knowledge of the channel matrix $H_0$ (see Eq. (2) for the transmission model and Eq. (3) for the definition of the channel matrix). However, the data symbols $x_n$ are required to estimate the channel matrix. To alleviate this contradiction, a predicted version of the channel matrix $\hat{H}_{n}|_{n-1}$ obtained with $x_n$ unknown is computed. $\hat{H}_{n}|_{n-1}$ is subsequently updated into $\hat{H}_{n}|_{n}$ through the EKF measurement update equations (33) with the current received OFDM symbol $y_n$. The current data symbol $\hat{x}_{n}|_{n}$ is finally retrieved from this updated channel matrix $\hat{H}_{n}|_{n}$.

The algorithm for the $r$th OFDM symbol is depicted in details in Fig. 1. From the previous OFDM symbol $(n-1)$, we execute the EKF Time Update Equations (32) to obtain the prediction parameters $\hat{\mu}_{n}|_{n-1}$. The predicted version of the channel matrix $\hat{H}_{n}|_{n-1}$ is computed from $\hat{\mu}_{n}|_{n-1}$ instead of $\mu_n$ with Eq. (17). Therefore, the equalization task is now possible since a version of the channel matrix is available. Before this step, the contribution of the pilots to $y_n$ is removed:

$$y_n = y_n - \hat{H}_{n}|_{n-1} \cdot x_n$$

(34)

where the vector $x_n$ is a $N_T \times 1$ vector composed of the pilots at the pilot positions and 0 elsewhere. With this assumption that $H_{n}|_{n-1} \cdot x_n = H_n \cdot x_n$, we obtain a new version of the transmission model that only includes the data:

$$y_n = H_n^{data} \cdot x_n^{data} + w_n^{data}$$

(35)

where the $N_R \times N_T (N - N_p)$ matrix $H_n^{data}$ is obtained by removing the $N_T N_p$ columns of $H_n$ at the pilot positions $P$. $x_n^{data}$ and $w_n^{data}$ are $N_T (N - N_p) \times 1$ vectors built from $x_n$ and $w_n$, respectively, by removing the vector elements at pilot positions $P$.

Equalization is performed on this model, yielding a first version of the detected data symbols $\hat{x}_{n}|_{n-1}$. The Measurement Update Equations (33) are then computed by using $\hat{x}_{n}|_{n-1}$ instead of $x_n$ in Eq. (30). Finally, a new equalization is performed with the updated parameters $\hat{\mu}_{n}|_{n}$ to obtain the updated version of the data symbols $\hat{x}_{n}|_{n}$.

The algorithm is initialized with $\hat{\mu}_0(0) = 0_{LN}, N_R N_T, 1$, and $P_0(0)$ computed with Eq. (28).

#### B. Computational Complexity

The purpose of this section is to determine the implementation complexity in terms of the number of the multiplications needed for our algorithm. The matrices $\mathbf{F}^{(r,t)}$ are pre-computed and stored if the delays are invariant for a great number of OFDM symbols. The computational cost of computing the different terms and processes of the algorithm is given by Table I. The complexity analysis of Time Update Equations and Measurement Update Equations of the Kalman filter in Table I uses the fact that $\mathbf{A}$ is a sparse matrix. In practice, $L$, $N_T$, $N_R$ and $N_c$ are much smaller than $N$, therefore, the computational complexity of our algorithm is $O(N_R^3 N_c^3)$. So we can say that our proposed algorithm and the algorithm proposed in [13] have asymptotically the same complexity (same order of growth). The algorithm in [13] will be used for performance comparison in Section V.

### V. Simulation

In this section, the performance of our recursive algorithm is evaluated in terms of Mean Square Error (MSE) for joint CA and CFO estimation and Bit Error Rate (BER) for data detection. We consider two antennas at the transmitter and two antennas at the receiver ($N_T = N_R = 2$). A normalized 4-QAM MIMO-OFDM system with $N = 128$ subcarriers, $N_T = N_R = 2$ pilots (i.e., $L_f = 4$), and $\frac{1}{f_{c}} = 2MHz$ was used.

Both parametric and equivalent discrete channel models are being discussed. We recall that the derivations have been carried out for the parametric model, although the equations for the equivalent discrete-time channel model can also be obtained by substituting the set of delays $\{\tau^{(r,t)}\}$ by the tap indices (see Section II-A).
In Section V-A, the parametric channel model is being considered with a classical scenario with one base station and one mobile receiver, and one CFO parameter to be estimated. Section V-B deals with the equivalent discrete channel model and considers a more pessimistic scenario where each transmitter and receiver requires its own RF-IF chain. For this scenario, the number of CFO parameters to be estimated \((N_T N_R = 4)\) is the largest. This scenario could correspond to the area of coordinated base stations or network MIMO. Performance comparisons have been carried out with the algorithm proposed in [13].

### A. Parametric channel model

We assume that all the \((r,t)\) channel links, \(r = 1, \ldots, N_R, t = 1, \ldots, N_T\) share the same path delays and fading properties (i.e., the same number of paths, of \(\sigma_{n_r,t}^2\) and \(\tau_{l}^{(r,t)}\)) since the antennas are very close to each other, which is typical in practice. The Rayleigh channel model given in [9] [11] \((L^{(r,t)} = 6\) paths and maximum delay \(\tau_{\text{max}} = 10T_s\), see Table II) was chosen. The MSE will be computed for both path CA and CFO to evaluate the estimation performance. First, let us define:

\[
\tilde{\alpha}_{(n|n)}(t) = \text{blkdiag} \{ \tilde{B}(t) \} \quad \tilde{B}(t) = \left\{ \begin{array}{c}
\hat{Y}(t) \\
\end{array} \right\}_{n \in [1, \ldots, N_r, L-1]}
\]

\[
\alpha_n = \begin{bmatrix}
\alpha(1,1)^T, & \ldots, & \alpha(L_{(1,1)}-1,1)^T, & \ldots, & \\
\alpha(0,N_T)^T, & \ldots, & \alpha(L_{(N_N,N_T)}-1,n)^T
\end{bmatrix}^T
\]

\[
\bar{\nu}_{(n|n)} = \hat{\nu}(n|n) |_{[N_r L N_c L + N_c]}
\]

where \(N_c\) is the number of CFO to be estimated. The MSE of the path CA (denoted \(\text{MSE}_c\)) and the MSE of the CFO (denoted \(\text{MSE}_s\)) are computed as follows (we recall that \(L\) is the total number of paths for the MIMO channel, see Section II-B):

\[
\text{MSE}_c = \frac{1}{K} \sum_{n=0}^{K-1} \frac{1}{N_r} (\hat{\alpha}_{(n|n)} - \alpha_{n})^H (\hat{\alpha}_{(n|n)} - \alpha_{n})
\]

(36)

\[
\text{MSE}_s = \frac{1}{K} \sum_{n=0}^{K-1} \frac{1}{N_r} (\hat{\nu}_{(n|n)} - \nu_{n})^H (\hat{\nu}_{(n|n)} - \nu_{n})
\]

(37)

where \(K\) is set to 1000 in our simulations. The MSE and the BER were evaluated under a rapid time-varying channel with \(f_c T = 0.1\) (corresponding to a vehicle speed of 300 km/h at \(f_c = 5\) GHz). A GCE-BEM with \(N_c = 4\) was initially chosen to model the path CA of the channel and \(\nu = 0.1\).

The tracking capability of our proposed algorithm is first demonstrated as a function of time. Real and imaginary parts of one trajectory example of \(\alpha_{(r,t)}\) are plotted in Fig. 2 for \(r = 1, t = 1\) and \(l = 0, \ldots, 5\) at \(E_b/N_0 = 20\) dB. After an initial transient, the algorithm locks on to the true value of the
an expected result. On the other hand, it is seen that the gain in MSE performance is too small to impact the BER, which remains constant for any values of \( a \) (see Fig. 5). So it turns out that our system is relatively independent of \( a \).

Fig. 6 shows the CA MSE as a function of \( E_b/N_0 \). For reference, the MSE obtained in Data-Aided (DA) mode (knowledge of the data symbols) is also plotted. In addition to the MSE of the estimated CA (see Eq. (36)), we added the MSE obtained with the predicted CA by substituting \( \hat{\alpha}(n|n) \) with \( \hat{\alpha}(n|n-1) \) in Eq. (36). As expected, it is observed that both predicted and estimated MSE approach their DA curve when \( E_b/N_0 \) is increased. Indeed, for large \( E_b/N_0 \) values, the number of detection errors is small. On the other hand, it is seen that the estimated curve is far better than the predicted curve for each \( E_b/N_0 \). Hence, it can be concluded that the measurement update task (Eq. (33)) is still efficient, even when the equations are computed with the predicted data symbols \( \hat{x}(n|n-1) \) (see Section IV-A).

Then, to evaluate the performance of our joint algorithm, the curves obtained with the perfect knowledge of the CFO are plotted. It is seen that the performance in terms of CA estimation are unchanged. So, it turns out that the CFO estimation does not impact the CA estimation.

Let us now discuss the CFO estimation. Fig. 7 shows the MSEs for the CFO obtained with the predicted and the estimated parameter. Similarly to the CA MSE, the curves in DA mode and with the perfect knowledge of the CA are shown. First, it is observed that the estimated curve is very close to the predicted one. This is due to the fact that the CFO is constant in our model, and so the AR-model is not very accurate. Unlike for the CA estimation task, the knowledge of the unwanted parameter highly increases the performance of the CFO estimation because the CA rapidly varies in time, yielding high MSE. The impact of their estimation, due to this high MSE, is not negligible on the CFO estimation task.

Figure 8 gives the corresponding BER curve. A lower bound for the BER performance is given by using the ideal channel
state information (CSI), i.e. perfectly known CA and CFO at the receiver. Together with this reference curve, we also plotted the BER curves obtained with the perfect knowledge of the CFO only, and the CA only. As expected, the parameter that degrades the most the performance is the CA, due to the high mobility of the channel.

Figures 9 and 10 show the impact of the number of BEM coefficients $N_c$ to the performance for different BEMs. The considered BEM are the P-BEM, the GCE-BEM, and the DKL-BEM (see Section II-B). For low $E_b/N_0$ values, the P-BEM is the most efficient in terms of MSE, but the gain is negligible on the BER. However, for large $E_b/N_0$ values, the gain in terms of MSE obtained with the GCE-BEM and DKL-BEM impacts the BER. Hence, it turns out that the best trade-off is to choose $N_c = 3$ and either the GCE-BEM or the DKL-BEM. Nevertheless, these two BEMs require some a-priori information (Doppler frequency $f_d$ for the GCE-BEM and correlation matrix for the DKL-BEM) which is not the case for the P-BEM. It is noteworthy that the BER would be more sensitive to the estimation errors with a higher order modulation (we recall that we used a QPSK modulation).

B. Equivalent discrete channel model - comparison with the algorithm of [13]

Here, we consider the equivalent discrete channel model where 4 CFO have to be estimated (one per sub-channel). This scenario could correspond to the area of coordinated base stations or network MIMO. The CFOs have been arbitrarily fixed to $0.1, 0.07, -0.1, -0.05$. For the sake of comparison, we also show the performance of the algorithm proposed in [13], called the classical algorithm from now on. This
First simulations for different speeds ranging from 30 km/h up to 300 km/h have been performed at 20 dB (see Fig. 11). For reference, the performance of the algorithm is given by using the ideal channel state information (CSI).

For the classical algorithm, the performance degrades rapidly as the speed is increased. This is expected since this algorithm does not take into account the ICI due to mobility. However, we observe that our algorithm is far more robust to speed. The prediction performance degrades with the speed but is clearly compensated by the estimation.

VI. CONCLUSION

In this paper, a new algorithm which jointly estimates path Complex Amplitudes (CA) and Carrier Frequency Offsets (CFO) in MIMO environments has been presented. The algorithm is based on a parametric channel model or equivalent discrete channel model. Within one OFDM symbol, each time-varying CA is approximated by a Basis Expansion Model (BEM) representation. The dynamics of the BEM coefficients and that of the CFO parameters are modeled by first-order auto-regressive processes. Parameter estimation is performed by Extended Kalman Filtering and the data recovery is carried out by means of a QR-equalizer. Compared to the conventional algorithm, simulation results show the good robustness of our algorithm to fading rate for normalized Doppler frequency values \( f_dT \) up to 0.1. For this very high mobility, the performance of the joint estimation algorithm in terms of Bit Error Rate is close to the performance obtained with perfect knowledge of channel and CFO as long as 3 BEM coefficients are used with either the GCE-BEM or the DKL-BEM.

APPENDIX

In this section, we detail the computation of the state noise variance \( \sigma_u^2 \). For the sake of simplicity, only the scalar case is performed. The vectorial case can be easily extended from this. The scalar version of (23) is as follows:

\[
\nu_n = a \cdot \nu_{n-1} + u_n 
\]  

(38)
First, let us define the correlation function of $\nu$:

$$R_{\nu}[m] = E[\nu[n] \nu[n-m]] \quad (39)$$

Using (38) in (39) yields:

$$R_{\nu}[m] = a \cdot R_{\nu}[m-1] + E[\nu[n] \nu[n-m]] \quad (40)$$

Then, we compute (40) for $m = 1$ and $m = 0$, yielding:

$$R_{\nu}[1] = a \cdot R_{\nu}[0] \quad (41)$$
$$R_{\nu}[0] = a \cdot R_{\nu}[-1] + \sigma^2_{u_{\nu}} \quad (42)$$

Note that the expectation $E[\nu[n] \nu[n-m]]$ equals zero for $m = 1$ since $\nu[n-1]$ only depends on $u_{n-1}$ (and not on $\nu[n-1]$) on the one hand, and on the other hand $u_{n-1}$ is zero-mean white Gaussian noise.

Combining (41) and (42) yields:

$$\sigma^2_{u_{\nu}} = (1 - a^2) R_{\nu}[0] \quad (43)$$

since $R_{\nu}[-1] = R_{\nu}[1]$. 

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